

## The Bricks and Mortar for Contemporary Reimplementation of Legacy Nordic Transformations

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### Abstract

*We assert that re-implementation of legacy transformations using contemporary software platforms, is an efficient way of revitalizing old transformation code and improving its dissemination to end users. We successfully test this assertion on a case of legacy Danish systems, describe what was done, and investigate what additional (little) work is needed to support re-implementation of all known geodetic transformations for the Nordic region. Additionally, we outline related ventures into the implementation of dynamic reference frames and into the representation and interoperability of geodetic parameter data.*

*Keywords:* Geodetic transformations, PROJ, POSETTA, Historical geodesy

### I Introduction

This paper reviews some recent efforts to provide geodetic transformation functionality to end users in a timely, efficient and contemporary manner, by leveraging contributions from the open source geospatial community to the academic, operational and computational branches of geodesy.

Historically, transformations between geodetic systems were implemented by national geodetic authorities, using computational platforms reflecting the local practice of the implementing authority at the relevant time of history. Given the modest size of the global geodetic community, this was a perfectly rational way of doing things until a few decades ago.

But geodesists age, retire and, presumably, depart to survey the eternities of their respective denominations (e.g. *Anonymous*, c. 1270; *Lönnrot*, 1849). Meanwhile, with the rapid evolution of information technology, the long-term maintenance of their transformation code becomes the responsibility of younger generations with different skills and tools, in a different technology landscape.

But also society changes and becomes more spatially aware – and hence, more dependent on the ubiquitous availability of a broad spectrum of turnkey geodetic ser-

vices, ranging from real time positioning systems, to transformation systems integrating contemporary GNSS data with legacy map data. The latter is a key task for geodetic, cadastral, and mapping authorities, in their role as custodians of historical collections of spatial data. It is also crucial for long-term studies in scientific fields not typically related to geodesy: History, landscape ecology, etc.

While systems for disseminating real time positioning services are readily available, typically based on the Ntrip protocol (*Anonymous*, 2004), no comparable dissemination method is available for transformation services: ISO 19111 (*ISO*, 2007) and ISO 19162 (*ISO*, 2015) are the standards for (among other things) *describing* geodetic transformations, but no ubiquitous Ntrip like companion exists, for actually disseminating the transformations to the end user.

In 2015, two of us (Evers, Knudsen) were tasked with the mission of maintaining TrLib, a geodetic transformation library, mostly for legacy Danish systems (*Knudsen et al.*, 2012). During this work, we realized that the most realistic way of *both* keeping TrLib in shape *and* getting it into the hands of users, was to leverage the best available open source solution for dissemination of transformations, then improving and extending that solution with elements of TrLib, to support our specific transformation needs.

## 2 *Mixing the mortar*

From a 2015 point-of-view, the best available open source solution for dissemination was the PROJ transformation library. PROJ, originally a library for map projections originating at the US Geological Survey in the 1980s, had over the years, gained some limited functionality supporting datum transformations (*Evenden*, 1983, 1990; *PROJ contributors*, 2018).

Most importantly, however, the PROJ library was (and is) built into a huge number of geospatial application programs. So by extending and improving PROJ, the extensions and improvements automatically get into the hands of users, as developers of application programs update to newer versions of the library. This is the vector we decided to use to make generic transformation functionality ubiquitous.

As inspiration for the solution architecture needed, we took an archetypal example of geodetic work: The (simplified) transformation of a set of coordinates  $(\phi, \lambda, h)$  from the ED50 datum (based on the Hayford ellipsoid), to the ETRS89 (based on the GRS80 ellipsoid):

1. Convert geographical  $(\phi, \lambda, h)$  to Cartesian  $(X, Y, Z)$  coordinates, using the parameters of the Hayford ellipsoid.
2. Apply a 7-parameter Helmert transformation.
3. Convert back from Cartesian  $(X, Y, Z)$  to geographical  $(\phi, \lambda, h)$  coordinates, using the parameters of the GRS80 ellipsoid.

Based on this simple example, a number of observations can be made:

1. This transformation (and not just this, but in practice *any* transformation) is based on individual primitive operations.

2. The primitive operations in steps 1 & 3 comprise each other's inverse.
3. The input of step number  $n$  is identical to the output of step number  $n-1$ .
4. The transformation can be inverted by replacing each step by its inverse operation, then traversing the steps in reverse.

As a compelling metaphor, we will henceforth refer to the primitive operations as *building bricks* (or simply as *bricks*), for constructing more complex transformations.

One of the characteristics of the original 1980s design of PROJ is an overall architecture resembling this building brick metaphor (Evenden, 1983). In 2015, PROJ had reached version 4.9 and provided bricks for a large number of map projections. It even supported the symmetry noted in observation (2) by implementing a map projection and, when existing, its inverse, in a common sub-framework.

Hence, PROJ 4.9 provided a framework useful for implementing building bricks for geodetic transformations and their inverses, simply by treating each brick as a kind of map projection. So we had a paradigm for making bricks, but what was to be our mortar: How could we glue the bricks together into more complex transformations?

To solve this, let us look at observation (3), noting that the output of one brick comprises the input of the next. We used this property to implement a special brick, taking as its input *a number of bricks*, called steps, then joining the steps by executing them sequentially, while providing the output of each step to the input of the next.

Since this resembles the functionality of Unix shell pipes (McIlroy, 1987), the special brick, comprising the mortar of our solution architecture, was called *the pipeline*. The pipeline is closely related to the ISO 19111 concept “Concatenated Coordinate Operation” (ISO, 2007).

As hinted at in observation (4) above, when the inverse operation for a pipeline exists (which it does if the inverse exists for each step), it is also automatically available, by calling the inverse of each step in reverse sequential order.

With the release of PROJ version 5.0, we completed the solution architecture by introducing the pipeline brick. In addition, we filled in the architectural framework by identifying and implementing fundamental (and sometimes not-so-fundamental) building bricks, gradually re-implementing all functionality needed to reproduce everything available in TrLib.

### 3 Casting the foundation

When identifying the building bricks needed for a geodetic transformation system, it is useful to note that a large subset of commonly occurring geodetic transformations can be constructed from a surprisingly small number of foundational operations. These operations, which we deem essential for a geodetic transformation system, include:

1. Geodetic-to-Cartesian coordinate conversion, and its inverse.
2. Helmert transformations of various kinds (2D, 3D, 4D or, equivalently: 4 parameter, 3/7 parameter and 14/15 parameter).
3. The Molodensky transformation.

4. Horizontal grid shift (“NADCON-transformation”).
5. Vertical grid shift (ellipsoidal-to-orthometric height transformation).

Elements 1 through 3 are relatively simple to implement, well documented in the literature (e.g. *Hofmann-Wellenhof & Moritz*, 2006; *OGP*, 2014), and with ample test material available, making it easy to verify the proper operation of new implementations.

While elements 4 and 5 are, in principle, also simple and well documented (*Smith & Bilich*, 2017); implementing and testing software for reading the related grid file formats can be a surprisingly complex task. Fortunately, most of this functionality was already available in PROJ 4.9; we just needed to expose it for more general use cases. Hence, turning the mostly cartographical PROJ 4.9 into a quite capable generic geodetic transformation system was a minor effort.

Elements 1 through 5 above was, however, not enough to re-implement a number of TrLib transformations for Danish legacy systems, including “System 34” (*SDFE*, 2017), which for most of the 20th century was the legal basis of the Danish cadastral system.

To account for tensions in the original defining network, the transformations for System 34 (and a number of other important systems), include a step based on the use of high order polynomial mappings (i.e. multiple regression equations, cf. *Ruffhead*, 2016). The polynomial mapping transforms the raw legacy coordinates into UTM coordinates in a “modified ED50 datum” which, in loose terms, is defined as “ED50 if ED50 was defined by a perfect Helmert transformation from ETRS89” (*Evers and Knudsen*, 2017).

Fortunately, TrLib included a proven, highly efficient, and numerically stable implementation of a 2D polynomial evaluator, based on Horner’s scheme. It was a manageable task to integrate this code with PROJ. Unfortunately, however, it was a less manageable task to handle the large number of polynomial coefficients for the high order 2D polynomials. Essentially the number of polynomial coefficients corresponds to the number of grid nodes of a functionally equivalent grid based implementation. However, contrary to the grid case, no generally established file format exists for polynomial coefficients.

Eventually, we solved the problem by redefining and re-implementing the entire input subsystem of PROJ. This was a significant effort in a mostly non-geodetic subject, but a practical necessity for implementing the majority of transformations for Danish legacy systems in PROJ. With this practical necessity completed, the re-implementation of the actual transformations was a matter of compilation of parameters – a task that was mostly complete at the end of 2017 (*Knudsen et al.*, 2018).

#### 4 *Building the house*

Shortly after the publication of these improvements in dissemination of geodetic transformations (*Evers and Knudsen*, 2017; *Knudsen and Evers*, 2017), the presidium of the Nordic Geodetic Commission (NKG) appointed a study group to explore the poten-

tial of using PROJ in national and common Nordic transformations. The group, consisting of the authors of this paper, representing the national mapping agencies of Denmark, Norway, Sweden, Iceland and Finland, focused on three subjects:

1. Identifying bricks still not implemented in PROJ, but needed to re-implement legacy transformations in the Nordic countries (except for Denmark, where everything needed is now implemented).
2. Supporting the PROJ implementation of the NKG transformation between ITRF and national ETRS89 implementations in the Nordic and Baltic countries, by *Häkli et al.* (2016).
3. In general, improving the functionality for implementation of dynamic reference frames, and raising the awareness of the dissemination challenge.

#### 4.1 *Identifying missing transformation building bricks*

Most of the work on this task was carried out during a one-day all-hands workshop, where representatives from each country described the legacy transformations used at their respective institutions. While some quite peculiar examples showed up, the overall conclusion is that surprisingly few bricks are still missing in order to facilitate complete support for all practically used transformations between contemporary and legacy Nordic systems. The complete list reads as follows (note that no additional functionality is needed for Icelandic legacy systems):

##### 4.1.1 *The Fake 2D Helmert transformation*

This one easily takes the prize for most peculiar missing brick: Due to lack of general support for the 2D Helmert transformation in generic geospatial software, some Swedish transformations introduce a fake center meridian to handle a minor rotation of a misaligned system (*Reit*, 1997, 2010). This “projection transformation” approach provides a one-step transform from planar coordinates in a local/regional reference frame to geodetic coordinates in e.g. ETRS89. It works by manipulating the parameters of the Gauss-Krüger projection: Manipulating the central meridian absorbs a rotation, while manipulating the false northing and easting absorbs a translation. Finally, manipulating the scale factor corrects the scale error in the original coordinates and the scale change due to shift of central meridian and change of ellipsoid. This can be implemented in PROJ directly, either using the actually supported 2D Helmert transformation, or using the parameter-manipulation approach. It may, however, be facilitated by introducing appropriate wrappers – and at least it should be showcased occasionally to remind desperate geodesists that necessity is the mother of invention.

##### 4.1.2 *Transverse Mercator implementations*

PROJ supports etmerc, a high accuracy implementation of the Transverse Mercator projection, based on a derivation by *König and Weise* (1951). It was extended with additional series terms and originally implemented for use in TrLib, by *Poder and Engsager* (1998). Finnish systems, however, are based on the Transverse Mercator deriva-

tion by *Hirvonen* (1970, 1972) which may differ from that by König & Weise, although it will presumably do so only at the submillimeter level.

#### 4.1.3 TIN based corrections

In Sweden and Finland, triangular irregular networks (TINs) are used to represent high accuracy corrections for both plane and height coordinates. Although the mathematics of supporting TINs is trivial, PROJ does not currently support them. As in the case for the polynomial coefficients mentioned above, a crucial point is the lack of a file format sufficiently simple, sufficiently open, and sufficiently agreed-upon to implement in a widely used library as PROJ.

#### 4.1.4 Locally tailored Helmert transformations

A number of Norwegian transformations are implemented using Helmert transformations, locally tailored using sets of “common points”, i.e. fiducial points with coordinates in both relevant systems. For each point transformed, the 20 nearest common points are used to derive locally tailored Helmert parameters. This is an unusual, but apparently efficient approach, and again, for implementing it in PROJ, we run into the problem of selecting a sufficiently agreed-upon file format for that kind of material.

### 4.2 PROJ implementation of NKG transformations

The work on the NKG transformations was completed in December 2017 (*Evers, 2017*). It is based on the paper by *Häkli et al. (2016)*, which handles the fact that Fennoscandia is subject to significant glacial isostatic adjustment (GIA), not accounted for in the transformation between ITRF and ETRS89, as defined by EUREF (*Boucher and Altamimi, 2011*). Hence, for high accuracy applications, the EUREF transformations must be supplemented by intra-plate deformation models, *in casu* a model based on the NKG\_RF03vel model by *Nørbech et al.*, (2008), realigned to the ETRF2000 frame, as described by *Häkli et al. (2016)*.

The deformation model is used to estimate the additional shift in coordinates due to deformation of the plate, and hence the frame. To support this use case, we could, fortunately, repurpose existing grid data formats intended for traditional datum shift operations: While formally designed as containers for representation of linear shifts (i.e. physical quantities typically measured in meters or milliseconds-of-arc), these data formats can be used just as well for dynamic shifts, represented as speeds (meters/year or milliseconds-of-arc/year).

This solution is, however, not optimal: The deformation model is inherently three dimensional, while the datum shift grid formats are designed to support either 2D horizontal shifts or 1D vertical shifts. Hence, we need to split the deformation model into a horizontal and a vertical part, and handle the recombination of the elements through the building brick implementing the transformation.

Additionally, since the datum shift grid formats are designed to represent static shifts, they do not support the concept of “model epoch”: The building brick must han-

dle this by leveraging additional metadata supplied during brick instantiation. Discussions on defining a fully “3D+model epoch” grid data format for deformation models are, however, ongoing.

#### 4.3 *Support for dynamic reference frames*

The PROJ support for dynamic reference frames is related to, and comprises an extension of, the work mentioned under the NKG transformation subject above. It is, however, also of interest in tectonically active areas. Hence, the PROJ implementation of dynamic reference frames is of global interest, as a platform for both research and dissemination (i.e. academic and operational geodesy).

### 5 *Furnishing the garden*

An unexpected conclusion of the activities of the NKG PROJ study group is that in many cases, the major obstacle for geodetic interoperability is not algorithmic, but rather related to incompatibility of ingrained, but incompatible, data formats.

In general geoinformatics, this is a well-known problem, pragmatically solved by using a ubiquitous translation library, GDAL/OGR (*GDAL/OGR contributors*, 2018). For the much smaller geodetic community, where specific file formats are typically used in one institution only, introducing GDAL/OGR support for those formats is infeasible as it would inflict the geodetic pains on the entire geoinformatics community.

This led *Hjelle and Evers* (2018) to conclude that the time was ripe for a geodesy-specific GDAL/OGR counterpart, so they developed a feasible solution in the form of POSETTA – “A Rosetta Stone for Position Data”.

POSETTA stresses the point that, while it would be nice to have a common representation format for “all things geodesy” (as proposed by e.g. GeodesyML), the substantial problem to solve is interoperability. And in the short term, interoperability can be obtained pragmatically, by providing an easy to use, freely distributable, translation library.

In the intermediate term, this may lead to the proliferation of a small number of exchange formats for all kinds of geodetic data, not unlike the historical success of the RINEX and SINEX formats for (mostly) GNSS data (*Gurtner*, 1994; *Gurtner and Esteys*, 2007; *IERS*, 2006), and hence simplify the interoperability problem.

### 6 *Conclusion*

The PROJ implementation of Danish legacy transformations is an Open Source success story. It has allowed the Danish geodetic authority (SDFE, formally “Styrelsen for Dataforsyning og Effektivisering”, i.e. the “Danish Agency for Data Supply and Efficiency”), to retire a substantial amount of old code, and to share the maintenance efforts for the support software needed with other interested parties globally.

Inspired by this success, a survey of legacy transformations between geodetic systems used in the Nordic region revealed that all functionality for Icelandic transfor-

mations is already available in PROJ, while the additional effort for supporting *all* Nordic transformations will be minor; essentially consisting of the implementation of less than 5 new bricks.

The major hindrance for this to materialize is not geodetic, but related to lack of agreed-upon open file formats and data structures, needed to disseminate the data material in an efficient way. This interoperability problem cannot be solved entirely from within the geodetic community, as it pertains also to communities considering themselves mostly as *users* of geodetic infrastructure.

### *References*

- Anonymous, c. 1270. Grímnismál. Here quoted from the Danish translation in *Stavnem, Rolf*, (Ed.), 2018, Den Poetiske Edda, 582 pp, Copenhagen, Denmark, U Press.
- Anonymous, 2004. Networked Transport of RTCM via Internet Protocol (Ntrip) Version 1.0.  
[https://igs.bkg.bund.de/root\\_ftp/NTRIP/documentation/NtripDocumentation.pdf](https://igs.bkg.bund.de/root_ftp/NTRIP/documentation/NtripDocumentation.pdf)
- Boucher C. and Z. Altamimi, 2011. Memo: specifications for reference frame fixing in the analysis of a EUREF GPS campaign. Version 8, 18-05-2011
- Evenden, G.I., 1983. Forward and inverse cartographic projection procedures. Open-File Report 83-625. U.S. Geological Survey.  
<https://pubs.er.usgs.gov/publication/ofr83625> 76 pp.
- Evenden, G.I., 1990. Cartographic Projection Procedures for the UNIX Environment — A User's Manual. U.S. Geological Survey, Open File Report 90-284.  
<http://download.osgeo.org/proj/OF90-284.pdf> 64 pp.
- Evers, K. 2017. PROJ parameters for transformations defined by the Nordic Geodetic Commission.  
<https://github.com/NordicGeodesy/NordicTransformations/blob/master/README.md>
- Evers, K. and T. Knudsen, 2017. Transformation pipelines for PROJ.4. In *FIG Working Week 2017 Proceedings*.  
[https://www.fig.net/resources/proceedings/fig\\_proceedings/fig2017/papers/iss6b/ISS6B\\_evers\\_knudsen\\_9156.pdf](https://www.fig.net/resources/proceedings/fig_proceedings/fig2017/papers/iss6b/ISS6B_evers_knudsen_9156.pdf)
- GDAL/OGR contributors, 2018. GDAL/OGR Geospatial Data Abstraction software Library. Open Source Geospatial Foundation. <http://gdal.org>
- Gurtner, W., 1994. RINEX: The Receiver Independent Exchange Format, GPS World, July 1994, pp. 48–52.
- Gurtner, W. and L. Estey, 2007. RINEX: The Receiver Independent Exchange Format Version 2.11. <ftp://igs.org/pub/data/format/rinex211.txt>
- Hirvonen, R.A., 1970. The use of subroutines in geodetic computations. *Maanmittaus* 45, Helsinki, Valtion painatuskeskus, pp. 45–61.
- Hirvonen, R.A., 1972. Matemaattinen Geodesia, Teknillisen korkeakoulun ylioppilaskunta, Helsinki, 223 pp. (in Finnish)
- Hjelle, G.A. and K. Evers, 2018. Introducing Posetta. Poster presented at NKG General Assembly 2018 in Helsinki, Finland.  
[https://github.com/NordicGeodesy/posetta/blob/master/documents/presentations/201809\\_nkg\\_poster/nkg\\_posetta\\_poster.pdf](https://github.com/NordicGeodesy/posetta/blob/master/documents/presentations/201809_nkg_poster/nkg_posetta_poster.pdf)

- Hofmann-Wellenhof, B. and H. Moritz, 2006. *Physical Geodesy*. Vienna, Austria: Springer-Verlag, 403 pp.
- Häkli, P., M. Lidberg, L. Jivall, T. Nørbech, O. Tangen, M. Weber, P. Pihlak, I. Aleksejenko and E. Paršeliunas, 2016. The NKG2008 GPS campaign – final transformation results and a new common Nordic reference frame. *Journal of Geodetic Science*, **6**(1), 1–33. doi:10.1515/jogs-2016-0001.
- IERS, 2006. SINEX – Solution (Software/technique) INdependent EXchange Format Version 2.02 (December 01, 2006), 42 pp.  
[https://www.iers.org/SharedDocs/Publikationen/EN/IERS/Documents/ac/sinex/sinex\\_v202\\_pdf.pdf?blob=publicationFile&v=2](https://www.iers.org/SharedDocs/Publikationen/EN/IERS/Documents/ac/sinex/sinex_v202_pdf.pdf?blob=publicationFile&v=2)
- ISO, 2007. ISO 19111:2007 Geographic information – Spatial referencing by coordinates, <https://www.iso.org/standard/41126.html>, 78 pp.
- ISO, 2015. ISO 19162:2015 Geographic information – Well-known text representation of coordinate reference systems, <https://www.iso.org/standard/63094.html>, 85 pp.
- Knudsen, T., K. Engsager and K. Evers, 2018. PROJ implementations of current and legacy Danish coordinate systems. Source code.  
<https://github.com/NordicGeodesy/NordicTransformations/blob/master/resources/DK>.
- Knudsen, T., S.L. Kokkendorff and K.E. Engsager, 2012. A Vivid Relic Under Rapid Transformation. OSGeo Journal Volume 10 pp. 56–59.  
[http://svn.osgeo.org/osgeo/journal/volume\\_10/en-us/final\\_pdfs/OSGeoJournal\\_FOSS4G2011.pdf](http://svn.osgeo.org/osgeo/journal/volume_10/en-us/final_pdfs/OSGeoJournal_FOSS4G2011.pdf)
- Knudsen T. and K. Evers, 2017. Transformation pipelines for PROJ.4. Geophysical Research Abstracts, Vol. 19, EGU2017-8050.  
<https://meetingorganizer.copernicus.org/EGU2017/EGU2017-8050.pdf>
- König R. and K.H. Weise, 1951. *Mathematische Grundlagen der Höheren Geodäsie und Kartographie*, Erster Band, Springer, Berlin/Göttingen/Heidelberg, 522 pp.
- Lönnrot, E., 1849. Kalevala. Chapter 16 (Väinämöinen in Tuonela). Here quoted from John Martin Crawford, ed. (1888), The Kalevala: The Epic Poem of Finland, <http://www.gutenberg.org/ebooks/5186>
- McIlroy, M.D., 1987. A Research Unix Reader. Annotated Excerpts from the Programmer's Manual, 1971–1986. Technical report. Murray Hill, New Jersey. Bell Laboratories.
- Nørbech T., K. Engsager, L. Jivall, P. Knudsen, H. Koivula, M. Lidberg, B. Madsen M. Ollikainen M. and M. Weber, 2008. Transformation from a Common Nordic Reference Frame to ETRS89 in Denmark, Finland, Norway, and Sweden – status report. In: Knudsen P., ed, Proceedings of the 15th General Meeting of the Nordic Geodetic Commission (29 May – 2 June 2006), DTU Space, 68–75.
- OGP, 2014. Coordinate Conversions and Transformations including Formulas, Revised July 2014. OGP Publication 373-7-2: Geomatics Guidance Note Number 7, part 2. International Association of Oil & Gas Producers, 144 pp.
- Poder, K. and K. Engsager, 1998. Some Conformal Mappings and Transformations for Geodesy and Topographic Cartography. *Kort & Matrikelstyrelsen Publications* **4** (6), 63 pp.
- PROJ contributors, 2018. PROJ coordinate transformation software library. Open Source Geospatial Foundation. <https://proj4.org/>

- Reit, B.-G., 1997. A simple way of introducing a global reference frame in surveying and mapping. Survey Review, Vol. **34** no. 264, 87–90.  
<https://doi.org/10.1179/sre.1997.34.264.87>
- Reit, B.-G., 2010. On geodetic transformations. LMV-rapport 2010:1, Gävle, Sweden, Lantmäteriet. 52 pp.  
[https://www.lantmateriet.se/contentassets/4a728c7e9f0145569edd5eb81fececa7/rapport\\_reit\\_eng.pdf](https://www.lantmateriet.se/contentassets/4a728c7e9f0145569edd5eb81fececa7/rapport_reit_eng.pdf)
- Ruffhead, A.C., 2016. Introduction to multiple regression equations in datum transformations and their reversibility. Survey Review, vol. 50, no. 358, pp. 82-90.  
<https://doi.org/10.1080/00396265.2016.1244143>
- SDFE, 2017. System 34. Geomatics Notes 3, Styrelsen for Dataforsyning og Effektivisering. <https://sdfe.dk/media/2917582/s34s45.pdf>