

Studying Earth Rheology Using GNSS Permanent Stations and GIA Modelling Tools

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Abstract

We studied the solid Earth rheology using observations in the North American and Fennoscandian uplift regions, in the form of horizontal and vertical velocities from Global Navigation Satellite System (GNSS) permanent stations.

For North America the dataset from Sella and co-workers and for Fennoscandia the BIFROST dataset were used respectively. We performed GIA (Glacial Isostatic Adjustment) modelling with the SELEN program, varying model parameters in order to find the best fit, in the chi-square sense, to the GNSS-derived velocity values. In the GIA modelling, the ice models ICE-5G and KL05 were used. The main results from this research were fitted Earth model parameters for these ice models using data from the North American and Fennoscandian uplift areas. The application of this method to the North American uplift region for these two different ice models is novel.

Keywords: Earth rheology, GNSS, GIA modelling

1 Introduction

Glacial Isostatic Adjustment or GIA is known to cause a slow change in station coordinates in areas where continental ice sheets existed during the last glaciation, around 20 kyrs BP, a process which is ongoing to this day. GIA models can be used to predict rates of coordinate change in these areas.

GIA is nowadays observable by several techniques: GNSS, tide gauge and gravity observations, SLR (Satellite Laser Ranging) and VLBI (Very Long Baseline Interferometry). In this study we use GNSS observations, as they can measure both horizontal and vertical deformation rates in well-defined reference frames and they offer the dense coverage needed for detailed study. Horizontal position estimates can be achieved at an uncertainty level of a few millimetres, and vertical rates about twice as uncertain (Koivula *et al.*, 2006).

We focus on GIA processes in the North American and Fennoscandian uplift regions. Fennoscandia is well known for its good coverage by GNSS permanent stations.

The BIFROST (Baseline Inferences for Fennoscandian Rebound, Sea-level, and Tectonics) project was initiated already in 1993 and we use its results to validate our computations. The North American uplift area is several times larger (Fennoscandia is approximately the size of Hudson Bay), but the GNSS permanent station coverage there is much poorer than in Fennoscandia. The Fennoscandian uplift area has a long publication history in GIA studies (*Scherneck et al., 2001; Milne et al., 2001; Johansson et al. 2002; Milne et al., 2004; Lidberg et al., 2006; Lidberg et al., 2010; Zhao et al., 2012*).

Following these studies, we optimally fit, in the sense of χ^2 goodness of fit, Earth model parameters for both ice models, separately for both areas of investigation. While our Fennoscandian computations may be seen as a replication of results found in the literature, the novel part of our research is the application of our method to the North American uplift region for two different ice models, ICE-5G and KL05.

2 Data and methods

2.1 GNSS datasets

For North America the dataset from *Sella et al., (2007)* and for Fennoscandia, the dataset from *Lidberg et al., (2010)* (BIFROST project) were used. The data coverage and the velocity field obtained from GNSS measurements are presented in Fig. 1 for both target areas.

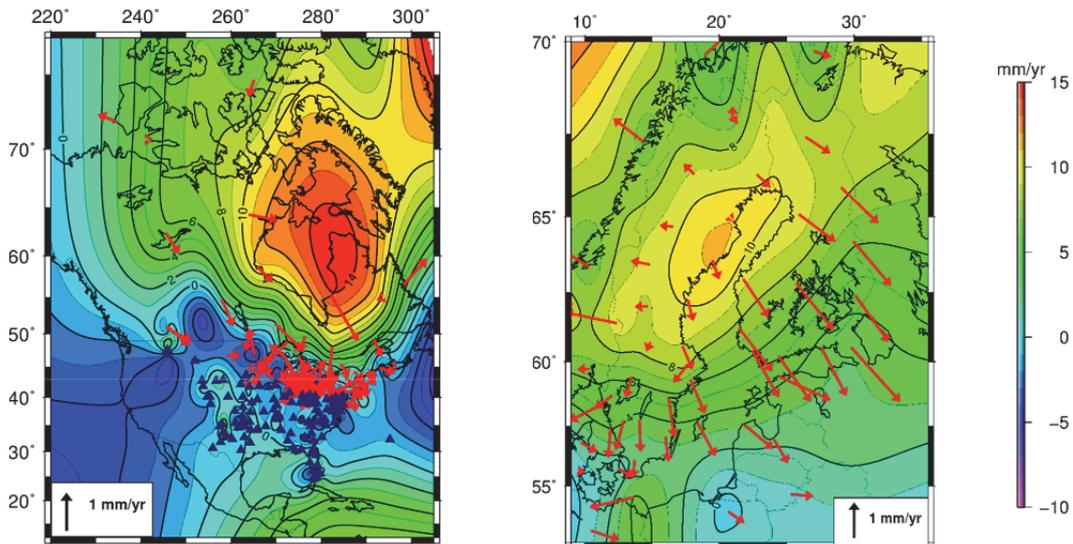


Fig. 1. The velocity fields from GNSS observations for North American (left) and Fennoscandian (right) uplift regions. In the North American figure only the GIA-affected stations are plotted, the stations in the rigid plate area are indicated with triangles (*Sella et al., 2007*).

2.2 Glacial Isostatic Adjustment (GIA)

When the load history (the time-dependent configuration of the ice masses) is known, it is possible from the observations to approximately reconstruct Earth rheology, in the form of the radial viscosity distribution (*Farrell and Clark, 1976*).

Maximum rates of the land uplift due to GIA at present are ~ 14 mm/yr and ~ 11 mm/yr for vertical motion in North America and Fennoscandia respectively (cf. Fig. 1). Horizontal motion does not exceed 3 mm/yr in either uplift area (Fig. 1).

We used a three-layer model called PREM (Preliminary Earth Model by *Dziewonski and Anderson, 1981*, spherical geometry) to describe the Earth's radial elastic and density structure. Lithosphere thicknesses typically range between 70 and 200 km, while upper mantle viscosities range between 10^{20} and 10^{21} Pa s, and lower mantle viscosities between 5×10^{21} and 10^{23} Pa s (*Whitehouse, 2009; Steffen and Wu, 2011*). GIA studies have shown that it is important to use a lithosphere thickness that represents the local situation in order to accurately fit the data (*Whitehouse, 2009*).

2.3 Ice models

The most widely used global ice models are ICE-5G (*Peltier, 2004*) and KL05 (*Fleming and Lambeck, 2004*). The ICE-5G ice model uses viscosity model VM2 and is an updated version of ICE-4G, where improved ice-dynamic modelling and new sea-level data were used (*Steffen and Wu, 2011; Whitehouse, 2009*). For the ICE-5G model the lower mantle viscosity is approximately 3×10^{21} Pa s, which gives the best fit to the observations which are used to constrain the ice model (*Peltier, 2004*). Moreover, the ice model ICE-5G contains significantly more mass of land-based ice, added outside of Hudson Bay (*Peltier, 2004*), than its precursors. A second ice model, which here is referred to as KL05, has been progressively developed at the Research School of Earth Sciences (RSES) of the National Australian University by Kurt Lambeck and coworkers (see *Fleming and Lambeck, 2004*, and references therein).

The mentioned ice models estimate a maximum ice thickness of 2400 m over Fennoscandia, but for the KL05 model the location of the ice maximum is placed over the Gulf of Bothnia, whereas ICE-5G maximum thickness is in central Sweden. Therefore the two ice models produce different patterns of rebound when used within a GIA model (*Whitehouse, 2009*).

3 Computations and results

In the computations the Fennoscandian dataset was used for testing the methodology used thereafter for North America. The benchmark study (*Spada et al., 2011*) showed that there are no differences due to the GIA computation strategy. However, the size of the study area as well as the locations of the GNSS permanent stations (*Wu et al., 2010*) can have an effect on the results obtained.

Although a newer and denser GNSS dataset for North America is available (cf. *Argus and Peltier, 2010*), their study showed that the results were similar to the study by *Sella et al., (2007)*. Unfortunately most of the new GNSS stations are outside the uplift area.

The computations were performed in two stages. Firstly, the sensitivity to the maximum harmonic degree included in the model was investigated. Secondly, GIA computations were performed finding the best fitting Earth model, in the maximum likelihood sense, to the GNSS-measured velocity field in the target areas. Finally, these best fitting models were compared with rates obtained from GNSS data.

In the research, the program SELEN (*Spada and Stocchi, 2007*), a code for solving the ‘‘Sea Level Equation’’, was used.

3.1 Sensitivity to the harmonic degree

Firstly the sensitivity to the maximum harmonic degree included was tested. In the following test, nominal viscosity values and lithosphere thicknesses were used (cf. Table 1).

The idea was to keep the computation time reasonable by limiting the harmonic degrees included. We determine the lowest cut-off harmonic degree which results in a similarly good χ^2 fit to the observations as all cut-off degrees larger than that. If the error measure does not change significantly when further increasing the maximum harmonic degree included (Fig. 2), then one may use that particular harmonic degree value as the bounding value (*Spada, 2001*).

Table 1. Ice models with their nominal parameters.

Ice model	Lithosphere thickness (km)	Upper mantle viscosity (10^{21} Pa s)	Lower mantle viscosity (10^{21} Pa s)
ICE-5G	90	0.5	2.7
KL05	65	0.3	10

In the sensitivity test computation the harmonic degrees from 12 to 164 were tested, from which a maximum harmonic degree of 72 was chosen to perform the calculations, as larger values did not significantly modify the results obtained (Fig. 2), and the computational time was kept reasonable. For a maximum harmonic degree of 72 the difference with the result for the maximum harmonic degree of 128 remained always below 3% for both ice models. The most notable effect was on computation time, which was reduced by 50% between the two computations.

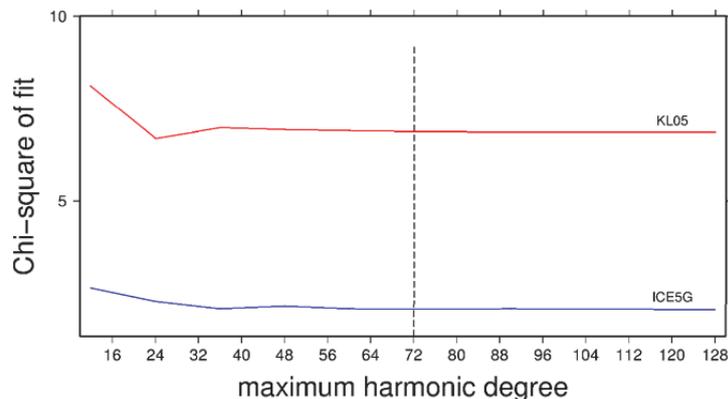


Fig. 2. Sensitivity to maximum harmonic degree for both ice models

3.2 Optimal Earth parameters

Secondly, GIA modelling was performed in order to find the best fitting Earth model parameters. In this study a three-layer Earth model was used (upper and lower mantle and lithosphere), and different viscosity and lithosphere thickness profiles for the Earth models were tested and their impact on horizontal and vertical velocity rates from GIA modelling was studied.

The global ice models ICE-5G and KL05 were used in GIA modelling. The SELEN program was used, which contains both ice models. The mantle viscosities for the first stage of the study varied in the range $0.1 \times 10^{21} \dots 5 \times 10^{21}$ Pa s for upper and $1 \times 10^{21} \dots 50 \times 10^{21}$ Pa s for lower mantle, respectively and the lithosphere thickness was held for the KL05 model at 65 km and for the ICE-5G model at 90 km. Step size used for upper mantle viscosity was 0.1×10^{21} Pa s, for lower mantle viscosity 1×10^{21} Pa s, and for lithosphere thickness 2 km.

In the computations firstly the lithosphere thickness was fixed, and the search for viscosity values was performed. Thereafter the viscosity was kept within the range indicated in the Table 2, and the best fitting lithosphere thickness was found.

As reference, the velocity field from GNSS permanent station time series was used both for North America and Fennoscandia. The GIA computations were performed to find the model fitting with highest likelihood to the GNSS-based velocity field in the two uplift regions. For every tested model the χ^2 quality of fit (Eq. 1) for horizontal, vertical and three-dimensional velocity rates from the reference model was found (Milne *et al.*, 2001). In the χ^2 computation equal weighting for horizontal and vertical velocities was used, as for North America we found that the χ^2 values for these rates were similar. For consistency, we used the same weighting for Fennoscandia (cf. Fig. 4), although a different weighting might produce smoother χ^2 -fit plots.

The χ^2 quality of fit is expressed as:

$$\chi = \frac{1}{n-1} \sum_{i=1}^n \left(\frac{x_i^o - x_i^p}{\sigma_i^o} \right)^2, \quad (1)$$

where x_i^o is the observed value (from GNSS time series), x_i^p is the predicted value (from GIA model), σ_i^o is the error measure for GNSS observation, and n is the number of data points.

To find the Earth model parameters having the smallest χ^2 fit with the GNSS-data, we produced plots of χ^2 as a function of Earth model rheology. In Figs. 3 and 4 the χ^2 plots for the North American and Fennoscandian uplift areas are presented. These plots were produced using the 2D+1D approach, and should be taken qualitatively.

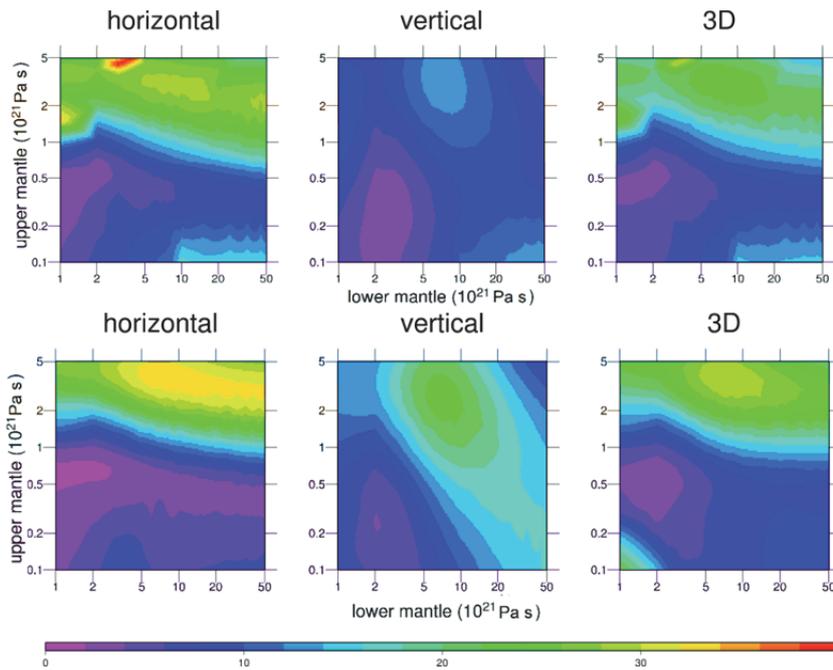


Fig. 3. χ^2 -fit for GIA modelling in North America, KL05 ice model (upper) and ICE-5G ice model (lower). Fixed lithosphere thicknesses were 65 km and 90 km for ice models KL05 and ICE-5G respectively.

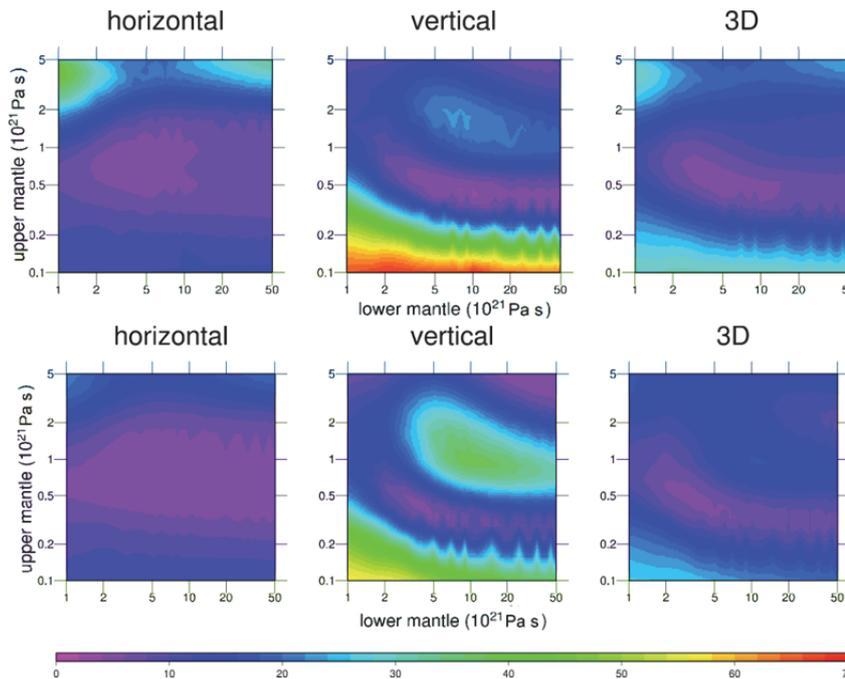


Fig. 4. χ^2 -fit for GIA modelling in Fennoscandia, KL05 ice model (upper) and ICE-5G ice model (lower). Fixed lithosphere thicknesses were 65 km and 90 km for ice models KL05 and ICE-5G respectively.

For North America the χ^2 for best fitting Earth model parameters is about 2...3 (cf. Fig. 3). For the Fennoscandian area, the χ^2 was in the range of 4.6...5.6 for both ice models (cf. Table 2, Fig. 4). This value agrees with other studies, which give around 4...5 (*Milne et al.*, 2001, *Milne et al.*, 2004, *Lidberg et al.*, 2010).

The next step was to find the best fitting lithosphere thickness. Doing the optimization in two separate steps is in our judgement a reasonable heuristic. For this step,

the viscosity values for upper and lower mantle, indicated in Table 2 were used and the lithosphere thickness was varied between 50 and 150 km. The overall results are presented in Table 2 together with the χ^2 -misfit and F-test values.

Table 2. Earth model parameters for Fig. 5 and Fig. 6. In the parenthesis is given the range which corresponds to the smallest χ^2 -misfit.

Region + ice model	Upper mantle viscosity (10^{21} Pa s)	Lower mantle viscosity (10^{21} Pa s)	Lithosphere thickness (km)	χ^2 -misfit	F-test
Fennoscandia, KL05	0.5 (0.4...0.6)	5 (4...7)	75 (70...85)	4.719 (4.7...5.6)	1.015
Fennoscandia, ICE-5G	0.4 (0.3...0.4)	4 (4...7)	95 (95...105)	4.650 (4.6...5.4)	
North-America, KL05	0.5 (0.4...0.6)	2 (2...3)	125 (115...125)	2.512 (2.5...3.0)	0.915
North-America, ICE-5G	0.5 (0.4...0.6)	2 (2...3)	65 (65...75)	2.742 (2.7...3.2)	

The resulting velocity field is presented for North America in Fig. 5 and for Fennoscandia in Fig. 6.

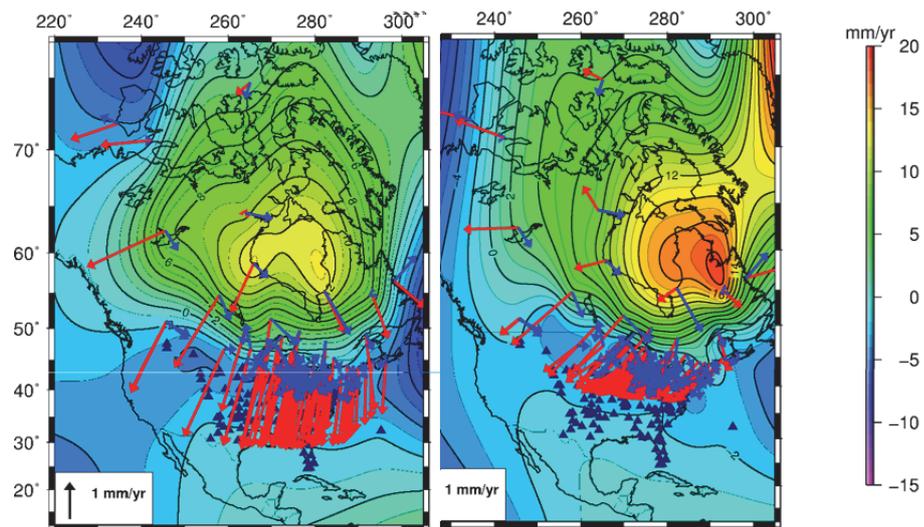


Fig. 5. Best fitting GIA model for North America, for ICE-5G ice model (left) and KL05 ice model (right), see Table 2 for numerical values. In the figure the GIA induced vertical velocity is represented by contours (from GIA model) and horizontal velocity by arrows, while red arrows indicate predictions (GIA) and blue arrows observations (GPS).

For the Fennoscandian area, the agreement between GNSS data and modelling results is fine for both ice models. In North America, on the other hand, the agreement is poor. Reasons for this include the sparseness of GNSS data and the huge size of the area where other phenomena besides GIA, like local tectonic movements, are in play (*Sella et al.*, 2007).

In an additional investigation we compared the 2-steps computation algorithm (hereinafter 2D+1D) and the 1-step computation algorithm (hereinafter 3D). The comparison was made for the Earth model parameters for the reduced ranges presented in Table 2. The results of this comparison are presented in Table 3.

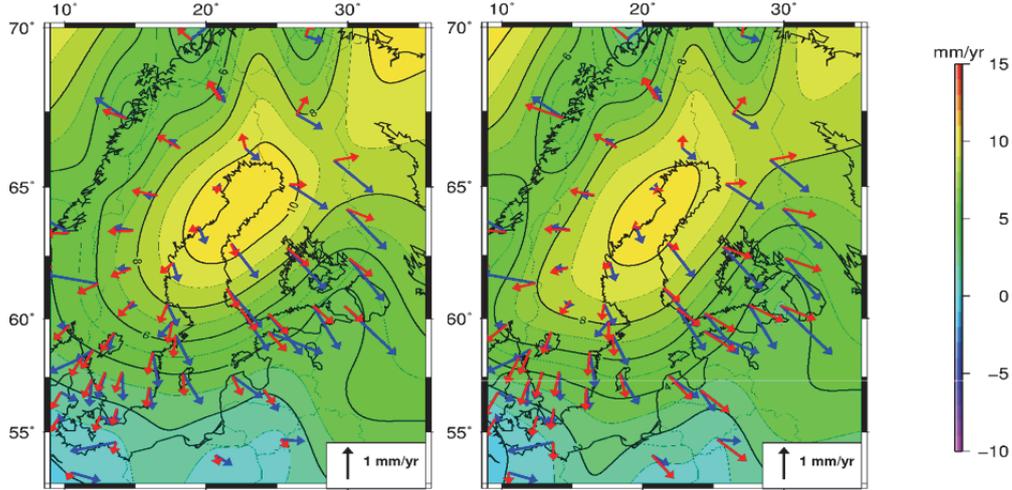


Fig. 6. Best fitting GIA model for Fennoscandia, for ICE-5G ice model (left) and KL05 ice model (right), see Table 2 for numerical values. In the figure the GIA induced vertical velocity is represented by contours (from GIA model) and horizontal velocity by arrows, while red arrows indicate predictions (GIA) and blue arrows observations (GPS).

Table 3. Comparison of the Earth model parameters: 2D+1D versus 3D computation approach

Region + ice model	2D+1D approach				3D approach			
	UM	LM	LT	χ^2 -misfit	UM	LM	LT	χ^2 -misfit
FS, KL05	0.5	5	75	4.719	0.55	4.5	75	4.703
FS, ICE-5G	0.4	4	95	4.650	0.4	4	85	4.633
NA, KL05	0.5	2	125	2.512	0.4	2.5	135	2.417
NA, ICE-5G	0.5	2	65	2.745	0.5	1.5	60	2.584

FS: Fennoscandia
 NA: North America
 LT: lithosphere thickness, km
 UM: upper mantle viscosity (10^{21} Pa s)
 LM: lower mantle viscosity (10^{21} Pa s)

As seen from Table 3, the Earth model parameters obtained from 3D approach differ only slightly. For the upper mantle, the change was about 0.1×10^{21} Pa s and for the lower mantle the change was about 0.5×10^{21} Pa s. For lithosphere thickness, the parameter changed about 0...10 km. The table also gives the optimum (smallest) χ^2 -misfit values for both computation strategies.

As the results show only small differences in the statistics, we discuss the results from the 2D+1D computation strategy (Table 2) in the following.

4 Discussion

Following (Paulson *et al.*, 2007; Zhao *et al.*, 2012) we used a three-layer Earth model (i.e. upper and lower mantle and lithosphere) in our modelling.

For additional testing the F-test was used to compare the variances from KL05 and ICE-5G ice models. The F-test statistic is expressed as the ratio of two χ^2 values (Riley *et al.*, 2002):

$$F = \frac{\chi_{KL05}^2}{\chi_{ICE-5G}^2}, \quad (2)$$

where χ_{KL05}^2 is the χ^2 of the KL05 ice model fit and χ_{ICE-5G}^2 is the χ^2 of the ICE-5G ice model fit. The number of degrees of freedom for Fennoscandia was 254 and for North America 698.

The F-test values are 1.015 and 0.915 for the two datasets, whereas the critical values for a two-sided confidence interval of 95% are for the Fennoscandian dataset 0.782...1.280 and for the North-American dataset 0.862...1.160 respectively. Results for the F-test (Eq. (2)) are shown in Table 2. From this analysis we cannot conclude in this confidence level that the distribution of the residual of fit would be different for the two models, for either dataset.

In Table 4 the Earth model rheology parameters obtained by other studies are tabulated with the results from this study.

Table 4. Overview of Earth model viscosity parameters (lithosphere thickness, upper mantle and lower mantle viscosities) from different studies (cf. *Steffen and Wu, 2011* Table 4). Results from the present study are shown in bold.

Study	Ice model	Lithosphere thickness (km)	Upper mantle viscosity (10^{21} Pa s)	Lower mantle viscosity (10^{21} Pa s)
<i>Fennoscandia</i>				
<i>Steffen and Kaufmann, 2005</i>	KL05	120	0.7	10
<i>Lambeck et al., 2010**</i>	KL05	65...100	0.3...0.4	5...20
<i>Lidberg et al., 2010</i>	KL05	120	0.5	5
<i>Zhao et al., 2012**</i>	KL05	93	0.5	12.2
	ICE-5G	106	0.35	7.26
<i>Steffen et al., 2010 *</i>	KL05	160	0.4	10
	ICE-5G	160	0.4	20
This study	KL05	75	0.5	5
	ICE-5G	95	0.4	4
<i>North America</i>				
<i>Zhao, 2013</i>	ICE-5G	130	0.35	1.71
<i>Zhao, 2013*</i>	ICE-5G	150	0.21	1.31
This study	KL05	125	0.5	2
	ICE-5G	65	0.5	2

* Study based on GRACE data

** improved model of KL05 was used, based mainly on relative sea-level data

4.1 Fennoscandian uplift area

In Fennoscandia we found that the best χ^2 -fit is achieved for the following viscosity values: for ICE-5G ice model 0.4×10^{21} Pa s for upper mantle and 4×10^{21} Pa s for lower mantle respectively; for KL05 ice model 0.5×10^{21} Pa s for upper mantle and 5×10^{21} Pa s for lower mantle respectively. The lithosphere thicknesses giving the

best results from χ^2 -fitting, were in the range of 95...105 km for ICE-5G and 70...85 km for KL05 ice models respectively (cf. Table 2).

We compared our results to other studies (cf. Table 4). We see that results for upper mantle viscosities from this study agree well with other studies for both ice models. Also the values found from *Steffen et al.*, (2010), using GRACE data, are similar.

The results for lower mantle viscosity agree less well. *Lambeck et al.*, (2010) estimates the lower bound of lower mantle viscosity being 5×10^{21} Pa s, and *Lidberg et al.* (2010), estimate a lower mantle viscosity of 5×10^{21} Pa s for ice model KL05, which agrees very well with results from this study. But *Zhao et al.*, (2012) estimates the lower mantle viscosity for KL05¹ to be about 7×10^{21} Pa s which is about 40% higher than the estimate from this study. For ice model ICE-5G the differences for lower mantle viscosity are about three times larger, although there is only one comparable study (*Zhao et al.*, 2012). One reason might be that the SELEN code uses an older version of the KL05 ice model than *Zhao et al.*, (2012). The results from *Steffen et al.*, (2010) show comparably large differences for lower mantle viscosity from all other studies.

The agreement for lithosphere thickness values (Table 4) is good, although the lithosphere thickness from this study for ice model KL05 is lower than estimates from other studies. *Lambeck et al.*, (1998) show that for an assumed lithosphere thickness of 75 km, the Earth rheology parameters are resolved with minimum error of fit. This agrees as well with the results from the present study.

Comparing our results with the Earth model values used in generating the ice model (i.e., nominal values) for ICE-5G and KL05 (Table 1), we find poor agreement for the Fennoscandian area for both upper and lower mantle viscosity values. The lithosphere thickness shows good agreement for both ice models.

As a conclusion, the methodology used in our approach does not produce large differences for Earth model parameters, except for estimates of lower mantle viscosity. As *Milne et al.*, (2004) have found, the GNSS data cannot be used to resolve more than three mantle layers. Therefore the values for lower mantle viscosity are more uncertain and show larger differences between different studies than those for upper mantle viscosity and lithosphere thicknesses (*Zhao et al.*, 2012).

4.2 North American uplift area

In North America the best χ^2 -fit is achieved for the following viscosity values: for 0.4×10^{21} Pa s for upper mantle and 4×10^{21} Pa s for lower mantle respectively; for KL05 ice model 0.5×10^{21} Pa s for upper mantle and 5×10^{21} Pa s for lower mantle respectively. The lithosphere thicknesses giving the best results from χ^2 -fitting, were in the range of 95...105 km for ICE-5G and 70...85 km for KL05 ice models respectively (cf. Table 2).

¹ *Zhao et al.*, (2012) and *Lambeck et al.*, (2010) use an improved version of the KL05 model based mainly on RSL data.

For North America there are not very many studies comparing GNSS data and GIA models for the ice models used by us. We found only one recent study (*Zhao*, 2013) which compares GIA modelling results with GNSS and GRACE data.

Comparison with *Zhao* (2013) shows that there is good agreement for upper and lower mantle viscosity values for ice model ICE-5G. Lithosphere thicknesses values, on the other hand, have very poor agreement, being about two times smaller for the present study (cf. Table 4). For ice model KL05 there are no other studies to compare with, but we see that the values estimated from our study agree well with estimates from *Zhao* (2013).

Comparing our results with the nominal values of the ICE-5G and KL05 ice models (cf. Table 1) we see that the viscosity values fit particularly well for ICE-5G. The nominal lower mantle viscosity for KL05 is much higher than the estimate from our study. Lithosphere thickness shows large differences from the nominal values (Table 1). There is “reverse” effect visible, i.e., the estimated best fit for ICE-5G is 65 km, while the nominal value is 90 km; and the best fit for KL05 is 125 km, while the nominal value is 65 km. This phenomenon indicates the need for more extended research in the future.

5 Conclusions

From this research, suitable Earth model parameters were found for ice models ICE-5G and KL05 in the North American and Fennoscandian uplift areas. Values obtained and test statistics are listed in Table 2. For Fennoscandia the results agree with those of other authors (Table 4). In North America the results are novel, but also much more uncertain.

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