Geophysica (2014), 50(1), 49–58

On the Spatial Averaging of Paleomagnetic Data

Toni Veikkolainen[1*], *Kimmo Korhonen*[2] *and Lauri J. Pesonen*[1]

[1] Division of Materials Physics, Dept. of Physics, University of Helsinki, FI-00014 Helsinki, Finland
[2] Geological Survey of Finland, FI-02151 Espoo, Finland

*corresponding author, e-mail: toni.veikkolainen@helsinki.fi

(Received December 23, 2013; Accepted March 4, 2014)

Abstract

The spatiotemporally uneven sampling of paleomagnetic observations has remained a vexing problem in paleomagnetism. In analyses of Precambrian inclination data, a traditional method of binning the data has been based on the present-day geographic latitude-longitude grid and fixed-length intervals of geologic time. In this paper, using a simulation of synthetic and real paleomagnetic observations, we show that this method produces flawed estimates of averaged (binned) inclinations, leading to false inclination frequency distributions.

Keywords: I Geocentric Axial Dipole, sampling, binning, grid, simulation, paleomagnetism

1 Introduction

Paleomagnetic observations from geologically distant eras, especially from the Precambrian, show a strong concentration of records in geographically small areas (Fig. 1). Therefore it has been difficult to obtain an adequate global view of measures of the ancient geomagnetic field, such as the distribution of inclination data. The inclination frequency analysis (*Evans*, 1976), although proven to be the most efficient way of testing the Geocentric Axial Dipole (GAD) hypothesis, requires a critical assessment of the data available before analysis. Several researchers (e.g. *Piper and Grant*, 1989; *Kent and Smethurst*, 1998; *Grower*, 2005) have pointed out that anomalously high proportions of shallow and moderate inclinations are prevalent the Precambrian inclination distribution, thus questioning the validity of traditional GAD-based paleogeographic reconstructions. *Veikkolainen et al.* (2013a) investigated factors other than the non-GAD components which may alter the outcome of the inclination analysis. They demonstrated that the quality filtering, the sedimentary inclination shallowing, and the selection of rock types all have an influence on the inclination distribution, but do not completely remove the observed low-inclination bias.

Published by the Geophysical Society of Finland, Helsinki

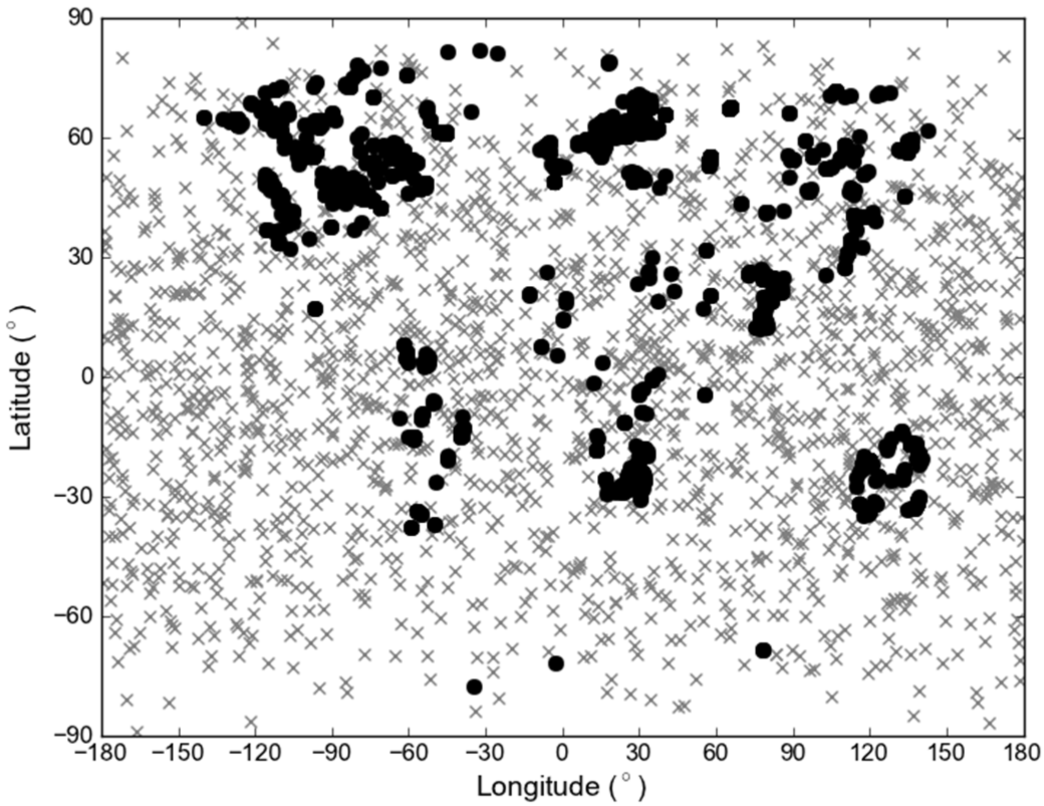

Fig. 1. The present-day geographical distribution of real paleomagnetic inclination data (MV $\geq$ 3, N=1855, solid circles) from the PALEOMAGIA database Precambrian paleomagnetic database (*Veikkolainen et al.* 2013b), compared with that of the randomly distributed global simulated dataset of same size (cross symbols). Miller projection.

To overcome the problem of geographically uneven sampling of paleomagnetic inclination data, *Kent and Smethurst* (1998) introduced the geographic grid-based spatiotemporal binning, with global data divided into slots with distinct temporal and spatial dimensions, and mean values calculated within. For example, in their analysis of absolute values of inclinations (|I|), *Kent and Smethurst* (1998) first divided the Phanerozoic into eleven geologic periods from Neogene up to Cambrian. For Precambrian data, they used fixed 50 Ma intervals. After this temporal binning, the spatial binning was done by using areas with 10° x 10° dimensions, and by calculating simple arithmetic mean inclinations within the time slots, giving rise to a better-defined inclination distribution for a given period. For example, the Mesozoic and Cenozoic inclination distributions showed high proportions of moderately high inclinations (40° $\leq$ |I| < 70°) prior to binning, but were turned much closer to GAD via binning, thus leading to the conclusion that the observed deviation from GAD in these eras is mainly caused by spatially insufficient sampling.

In the inclination analysis of *Kent and Smethurst* (1998), the Paleozoic and Precambrian binned data, unlike data from more recent eras, remained biased and were therefore interpreted as being in contradiction with the GAD hypothesis. Values of an axial quadrupole $G2 = g_2^0/g_1^0 = 0.10$ and an axial octupole $G3 = g_3^0/g_1^0 = 0.25$, with g_1^0,

g_2^0 and g_3^0 being zonal spherical harmonics, were needed to account for the anomalously large proportion of shallow inclinations in these data. Despite the fact that in the Cenozoic and Mesozoic the binned data seems to produce a more GAD-like distribution when compared with that of unbinned records, this may not only be due to the reduction in bias caused by concentrated sampling but it may also reflect errors inherently caused by the wrong implementation of the binning technique. Therefore, no attempt to apply spatiotemporal binning to paleomagnetic data can be considered robust unless the validity of the binning method has been confirmed with a set of simulated data based on the GAD model and compared with a real dataset of the same size. In this paper, the functionality of the binning method of *Kent and Smethurst* (1998) is tested and other ways of binning the data are discussed.

2 *Motivation and implementation*

Problems associated with the spatial averaging of Precambrian inclination data were previously discussed by *Veikkolainen et al.* (2013a). One of them is the fact that inclination observations from geologically coeval terranes, which are now docked together but were once far away from each other, cannot be correctly averaged in the analyses using present-day geographic locations. One of the best-known examples is the distinction of the Slave and Superior cratons in the early Proterozoic (*Buchan et al.,* 2012). On the other hand, it is evident that Laurentia and Baltica, the Precambrian continents with the largest number of inclination records in the Global Paleomagnetic Database (GPMDB; *McElhinny and Lock*, 1996; *Pisarevsky*, 2005) have been conterminous in paleogeographic reconstructions (*Pesonen et al.,* 2012), such as the long-lived Meso-Neoproterozoic supercontinent Nuna, or Columbia (*Evans and Mitchell*, 2011; *Zhang*, 2012), leading to closely matching inclination values in areas which are now separated by thousands of kilometres. The large variation in the quality of Precambrian paleomagnetic data is another source of error, and its extent can be estimated by filtering the data using a quality scheme (e.g. *Van der Voo*, 1990). The effect of the quality filtering on the inclination distribution is, however, minor and almost overshadowed by the statistically different igneous and sedimentary rock datasets (*Veikkolainen et al.*, 2013a).

To analyze the applicability of spatiotemporal binning, we have developed a Python script to enable a convenient way to compare simulated inclination data produced by zonal geomagnetic field models, such as the GAD, with real observations gathered from the new Precambrian paleomagnetic database PALEOMAGIA (*Veikkolainen et al.*, 2013b). Only observations satisfying three out of six of the modified Van der Voo criteria (MV) (*Veikkolainen et al.* 2013a) were considered, leading to a set of 1855 records to be used for the analysis, 1263 of which have been derived from crystalline rocks only. This is significantly larger than the number of unfiltered Precambrian paleomagnetic records (N=1277) analyzed by *Kent and Smethurst* (1998). Although the inclination flattening (*King*, 1955) is a problem that permeates in Precambrian sedimentary inclination data (*Veikkolainen et al.*, 2013a), we have followed the convention of *Kent and Smethurst* (1998), treating igneous, metamorphic and sedimentary rock records in equal manner to allow an easy comparison with previous results of binning and inclination frequency analysis.

Our script produces a simulated set of random inclination values following a user-defined zonal field model with the maximum spherical harmonic degree n=3 and shows both the binned and unbinned (simulated) inclination distributions in comparison with the real binned and unbinned data. The output of the script is provided in the form of figures, showing the geographic distribution of simulated and real data (Fig. 1), paleolatitude vs. inclination (λ vs. I) curve for unbinned and binned simulated and real data, and inclination distributions plotted in comparison with the pure GAD inclination model. In this approach, it is presumed that the continents have sampled the Earth adequately throughout their drifting history during the investigated time slot. This prerequisite, although questioned in some studies (e.g. *Meert et al.*, 2003; *Evans and Hoye*, 2007), serves as a useful proxy for inclination-based studies on the validity of GAD.

Whenever a random set of points is to be generated on the globe, they do not have any preferred longitudes, but their latitudes follow a sine-type distribution due to the spherical shape of the Earth. Therefore the calculation of the proportion of points between two fixed latitude values follows Eq. 1:

$$D(\lambda) = \sin \lambda_1 - \sin \lambda_2 \tag{1}$$

For example, with $\lambda_1 = 10°$ and $\lambda_2 = 0°$, it is shown that 17.3 % of points fall between these latitudes on both hemispheres. Assuming the GAD model of the geomagnetic field, the dipole equation (Eq. 2) can be applied to calculate the proportion of points between two fixed inclination values (Eq. 3):

$$\tan I = 2 \tan \lambda \tag{2}$$

$$D(I) = \sin(\arctan\left(\frac{1}{2}\tan(I_1)\right) - \sin(\arctan\left(\frac{1}{2}\tan(I_2)\right) \tag{3}$$

Despite showing the inclination distribution in the GAD field, Eq. 3 cannot be effectively applied for splitting the globe geographically into classes with equal number of inclination values, which can be used to disprove the binning based on equal latitudinal delimiters as done by *Kent and Smethurst* (1998). However, the solution follows the numerical integration of the dipole equation according to Eq. 4, where ds is the length of a section dλ, limited by λ_1 and λ_2) (Eq. 4)

$$ds = \int_{\lambda_1}^{\lambda_2} \sqrt{1 + \left\{\frac{d[\arctan(2tan\lambda)]}{d\lambda}\right\}^2}\, d\lambda \tag{4}$$

In our implementation of Eq. 4 (Fig. 2), the tan I = 2 tan λ curve has been split into 18 equal-length sections, meaning that both hemispheres have 9 sections, corresponding to the 9 classes used in the inclination frequency analysis. With the dipole equation in consideration, 1/9 (11.1 %) of I observations should plot between $0° \le |\lambda| \le 6.6°$, another 11.1 % between $6.6° < |\lambda| \le 13.5°$, and the steepest 11.1 % between $77.0° \le |\lambda| \le 90.0°$, as shown by the solid symbols in Fig. 2. These latitude delimiters serve as binning values in the case where the Earth is considered spherical and the GAD hypothesis valid.

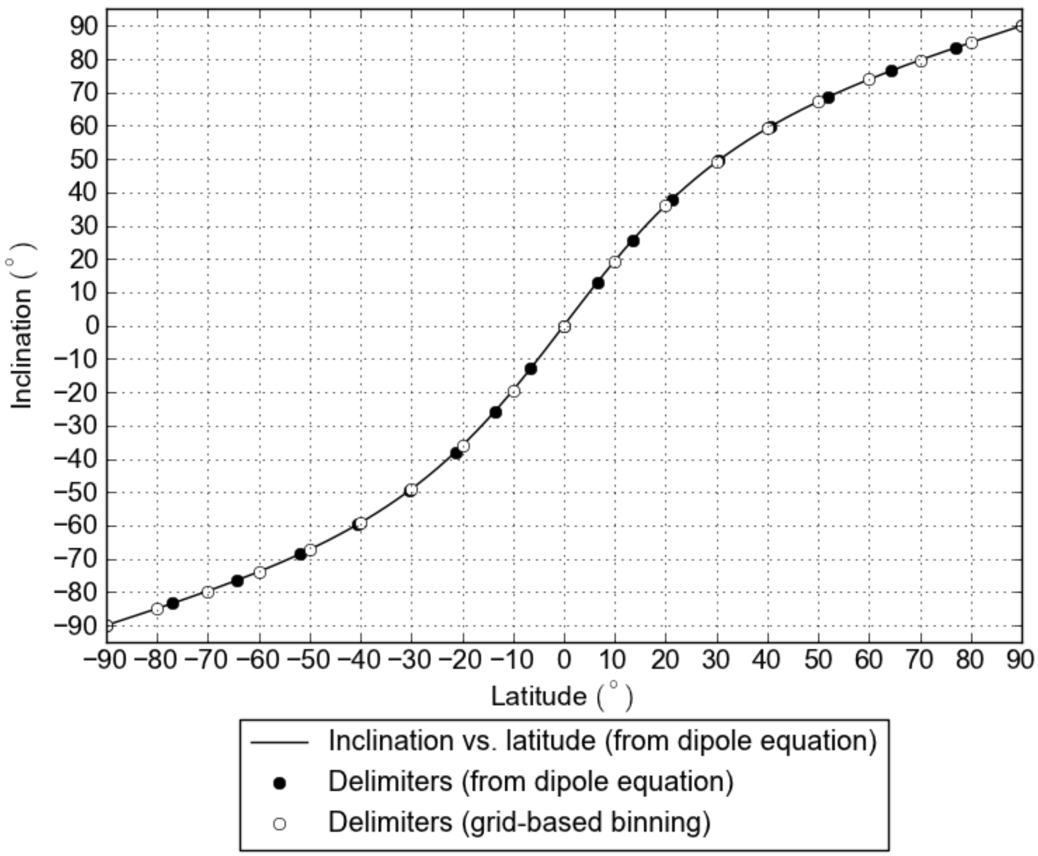

Fig. 2. The tan I = 2 tan λ model derived from the GAD field. The delimiters derived from the dipole equation show equal-length sections of the curve, whereas the delimiters based on Kent and Smethurst type binning incorrectly point to sections with different lengths. See also Table 2.

3 *Results*

Our modellings shows that the simulated, geographically unbinned inclination data (N=1855) give a nearly GAD-like inclination distribution, as expected (Table 1, Fig. 3). However, the binning of these inclination data by a latitude-longitude grid (e.g. 10° x 10°) gives rise to a flawed inclination distribution with a slight deficiency of shallow inclinations (0° ≤ |I| ≤ 30°), an even smaller proportion of moderate inclinations (30° < |I| ≤ 40°) and an overrepresentation of steep values (70° ≤ |I| ≤ 90°). This addresses a serious problem in using the grid-based inclination distribution. If the grid-based binning method were reasonable, only the number of observations in each interval would be altered by the binning, with the appearance of the inclination distribution remaining practically unchanged. It must also be noted that the number of observations in the binned dataset is highly dependent on the locations of the sampling sites in the present-day geography. Coincidentally, in our case the binned set of simulated data is slightly larger than the binned set of actual paleomagnetic observations, even though the unbinned sizes are the same.

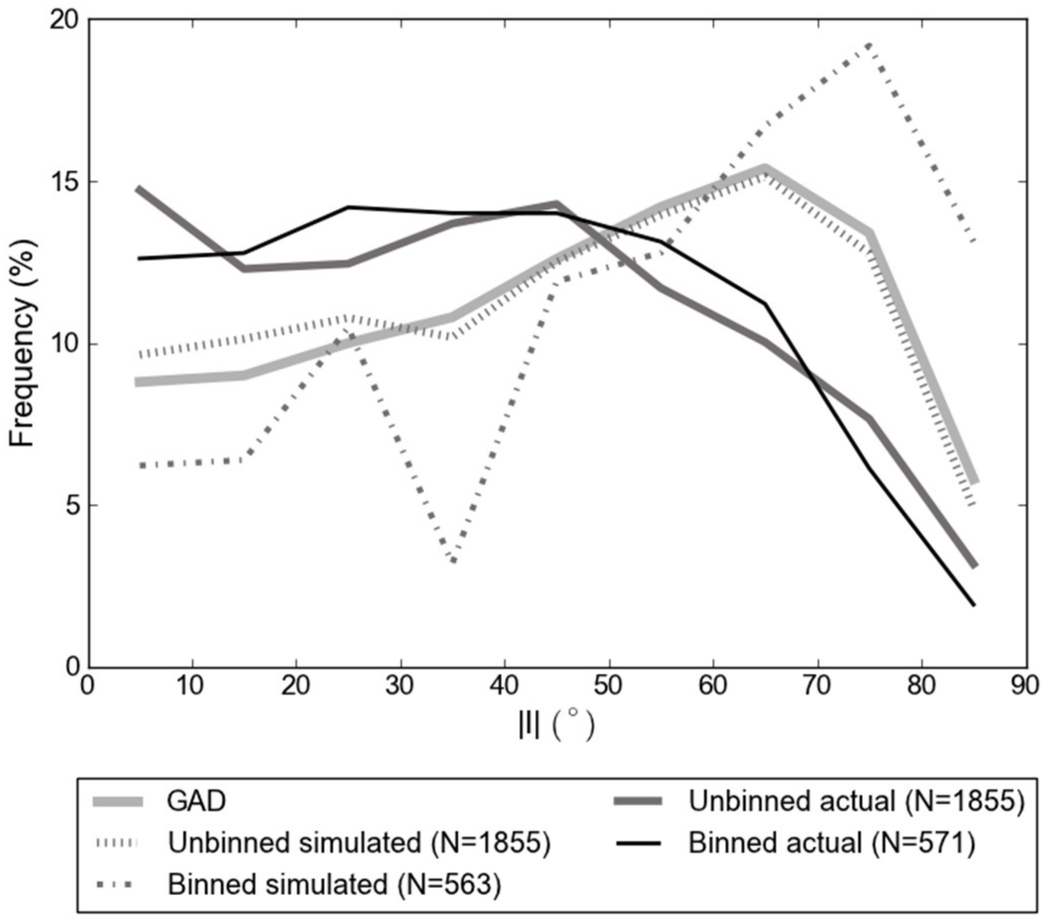

Fig. 3. Inclination distributions derived from datasets shown in Table 1.

A different view, yet still contradictory with the GAD, emerges when the simulated data is replaced by actual observations from our Precambrian paleomagnetic database (*Veikkolainen et al.*, 2013b). The method of spatial binning was predominantly same as that used for the simulated data, with the exception that the method was applied for 59 timeslots separately, starting from 540…590 Ma and ending to 3440…3490 Ma. The final inclination distribution was constructed simply by summing the spatially binned records over the entire Precambrian in each of the nine inclination intervals from $0° \leq |I| < 10°$ up to $80° \leq |I| < 90°$ separately. For example, as the 540…590 Ma slot had 13 binned inclination records in the interval $0° \leq |I| < 10°$, the 590…640 Ma slot had 8 of them, and finally, the 3440…3490 Ma slot had 6 of them, the sum 13+8+…6 was determined. The spatiotemporally averaged inclination distribution was constructed by applying this method to all nine inclination intervals from $0° \leq |I| < 10°$ up to $80° \leq |I| < 90°$. Although the binned distribution shows a significantly smaller value for chi-square statistic X^2 than the unbinned one does (Table 1), these datasets have very different sizes and thus they cannot be directly compared. Nevertheless, in Fig. 3 they quite closely resemble each other, having the typical tendency of inclination shallowing.

Table 1. Comparison of inclination distributions for a) GAD, b) unbinned simulated data (N=1855) produced assuming a GAD field, and c) binned simulated data (N=563) produced assuming a GAD field. Column d) shows the distribution derived from unbinned actual data and column e) that derived from binned actual data. The values of the test statistic X^2 have been calculated using GAD as a null hypothesis. See also Fig. 2.

interval	*a)*	*b)*	*c)*	*d)*	*e)*
0°≤|I| < 10°	164 (8.78 %)	179 (9.64 %)	35 (6.22 %)	273 (14.72 %)	72 (12.61 %)
10°≤ |I| < 20°	169 (9.12 %)	188 (10.13 %)	36 (6.39 %)	228 (12.29 %)	73 (12.78 %)
20°≤ |I| < 30°	183 (9.84 %)	200 (10.78 %)	59 (10.48 %)	231 (12.45 %)	81 (14.19 %)
30°≤ |I| < 40°	201 (10.94 %)	188 (10.18 %)	18 (3.20 %)	254 (13.69 %)	80 (14.01 %)
40°≤ |I| < 50°	232 (12.50 %)	232 (12.50 %)	67 (11.90 %)	265 (14.29 %)	80 (14.01 %)
50°≤ |I| < 60°	265 (14.28 %)	259 (13.95 %)	72 (12.79 %)	217 (11.70 %)	75 (13.13 %)
60°≤ |I| < 70°	285 (15.38 %)	281 (15.14 %)	94 (16.70 %)	186 (10.03 %)	64 (11.21 %)
70°≤ |I| < 80°	250 (13.46 %)	237 (12.77 %)	108 (19.18 %)	142 (7.66 %)	35 (6.13 %)
80°≤ |I| < 90°	106 (5.70 %)	91 (4.90 %)	74 (13.14 %)	59 (3.18 %)	11 (1.93 %)
combined	1855 (100 %)	1855 (100 %)	563 (100 %)	1855 (100 %)	571 (100 %)
$\mathbf{X^2}$	-	8.919	109.989	236.575	79.76
p value	-	0.349	<0.0001	<0.0001	<0.0001

4 Conclusions

A comparison between the unbinned and binned inclination distributions, both simulated and real ones, in the Precambrian points out that the traditional method of binning data using latitude-longitude grid results in biased inclination distributions. This does not rule out other ways of binning the data, such as the craton-based binning, which demands careful geological grouping of cratons and their building blocks (*Veikkolainen et al.*, 2013a). Regardless of the coordinate system, the size of a given craton remains unchanged during the continental drift, unless affected by the crustal shortening in collisional orogenies (*Halls*, 2013). Therefore binning the data cratonically and then making the temporal binning can be considered a reasonable approach in handling most of the Precambrian inclination data (*Veikkolainen et al.*, 2013a). However, it must be emphasized that the continental drift rate has been subject to change in the geological history, and therefore using a fixed-length temporal bin, such as 50 Ma, is to be viewed with caution. To overcome this problem, *Veikkolainen et al.* (2013a) used a variable temporal bin length for each craton separately, paying attention to the density of data along the respective apparent polar wander path (APWP) and velocities therein. Although our simulated datasets are slightly different each time the model run has been done, due to the inherent randomness in the modeling procedure, this does not have an influence on our main conclusion.

It has been observed that the spatiotemporal binning, as done using *Kent and Smethurst's* (1998) method, may in some occasions cause the original inclination distribution to turn closer to that of GAD. However, this phenomenon is artificial and simply based on an incorrect concept of spherical geometry. It was assumed 11.1 % of I,λ pairs plot between 0° ≤ |λ| ≤ 10°, another 11.1 % between 10° ≤ |λ| < 20° and so on, corresponding to the open symbols in Fig. 2. When applied, this method eventually leads to the situation where the data are binned within an incorrect binning interval, causing the accumulation of data in some bins and the corresponding lack of data in other bins. This is due to the fact that the GAD-based data are no longer evenly distributed on equal-length sections of the tan I = tan λ curve. For example, the underrepresentation of low- and moderate-inclination data (0° ≤ |I| ≤ 40°) renders gaps in the inclination vs. latitude curve (Fig. 4). Even though the spatial binning, when done correctly (Table 2), should not produce flaws of this kind, the problem of using present-day coordinate data for calculating average inclinations from ancient landmasses still remains, and recalls the need of binning the data cratonically in the Precambrian. For the recent intervals, such as the last five million years, using the revised grid-based binning method, as demonstrated in this paper, is yet reasonable, since no significant changes in the paleogeographic configuration of continents have occurred, and evidence points to a geomagnetic field with only minor departures from the GAD during this time (*McElhinny,* 2004; *Johnson et al.*, 2008).

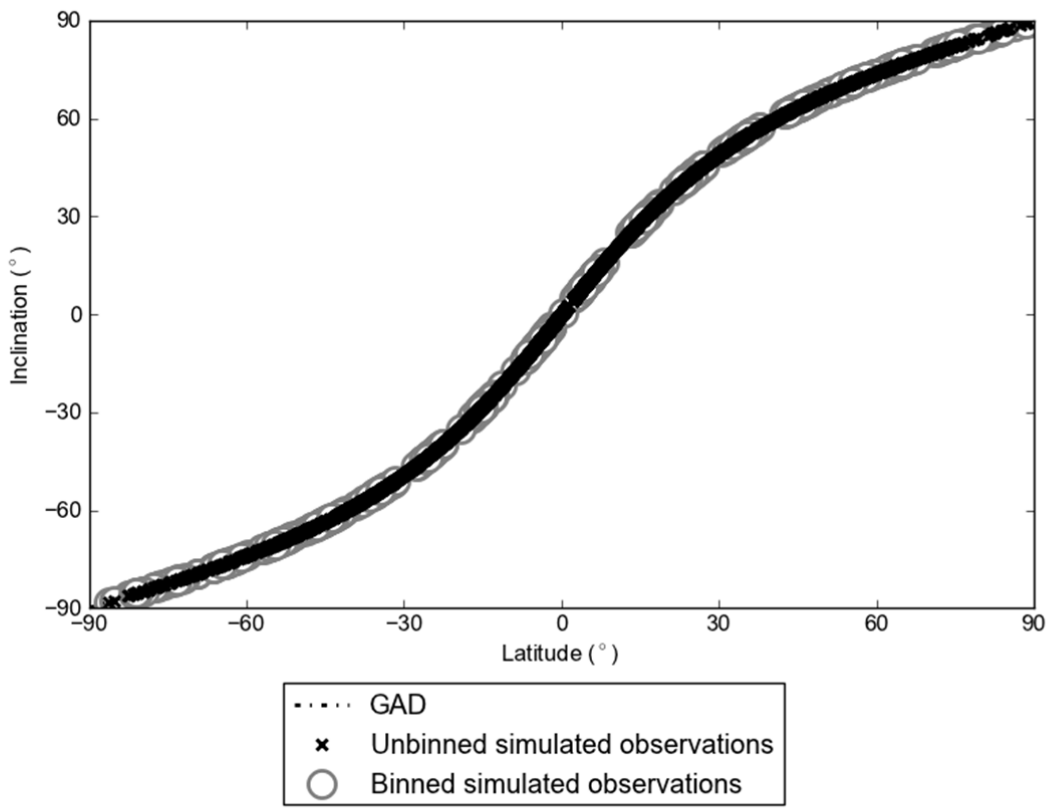

Fig. 4. Inclination vs. latitude derived from unbinned and binned simulated datasets in Table 1.

Table 2. a) Latitudinal bins in a) the Kent and Smethurst type binning, and b) in our revised way of binning the inclination data, corresponding to open and solid symbols in Fig. 3.

bin	*a)*	*b)*
1	0°≤|λ| < 10°	0°≤|λ| < 6.6°
2	10°≤ |λ| < 20°	6.6°≤ |λ| < 13.5°
3	20°≤ |λ| < 30°	13.5°≤ |λ| < 21.3°
4	30°≤ |λ| < 40°	21.3°≤ |λ| < 30.3°
5	40°≤ |λ| < 50°	30.3°≤ |λ| < 40.6°
6	50°≤ |λ| < 60°	40.6°≤ |λ| < 52.0°
7	60°≤ |λ| < 70°	52.0°≤ |λ| < 64.2°
8	70°≤ |λ| < 80°	64.2°≤ |λ| < 77.0°
9	80°≤ |λ| < 90°	77.0°≤ |λ| < 90°

Put together, both theoretical and experimental evidence shows that the Kent and Smethurst type binning of inclination data is unjustified. The method should no longer be used for any paleomagnetic data. Instead, cratonic binning, as applied by *Veikkolainen et al.* (2013a) is favoured for Precambrian data, and the revised binning method, as explained in this paper, should be used for spatial averaging of paleomagnetic observations from more recent eras.

Acknowledgements

The completion of the new Precambrian paleomagnetic database used in this work owes much to David A.D. Evans from Yale University, and Satu Mertanen from Geological Survey of Finland. Rob Van der Voo and Fabio Donadini are appreciated for their constructive comments.

References

Buchan, K., A.N. LeCheminant and O. Van Breemen, 2012. Malley Diabase Dykes of the Slave craton, Canadian shield: U-Pb age, paleomagnetism, and implications for continental reconstructions in the early Paleoproterozoic. *Canadian Journal of Earth Sciences*, **49**, 435–454.

Evans, M.E., 1976. Test of the dipolar nature of the geomagnetic field throughout Phanerozoic time. *Nature*, **262**, 676–677.

Evans, M.E. and G.S. Hoye, 2007. Testing the GAD throughout geological time. *Earth Planets Space*, **59**, 697–701.

Evans, D.A.D. and R.N. Mitchell, 2011. Assembly and breakup of the core of Paleo-Mesoproterozoic supercontinent Nuna. *Geology*, **39**, 443–446.

Grower, M., 2005. Closing Pandora's Box: Additional insights on inclination bias using a random walk approach. *Journal of Undergraduate Research, University of Florida*, **6(6)**, 16pp.

Halls, H., 2013. Crustal shortening during the Paleoproterozoic: Can it be accommodated by paleomagnetic data? *Precambrian Research*, in press.

Johnson, C.L., C.G. Constable, L. Tauxe, R. Barendregt, L.L. Brown, R.S. Coe, P. Layer, V. Mejia, N.D. Opdyke, B.S. Singer, H. Staudigel and D.B. Stone, 2008. Recent investigations of the 0-5 Ma geomagnetic field recorded by lava flows. *Geochemistry Geophysics Geosystems,* Q04032.

Kent, D.V and M.A. Smethurst, 1998. Shallow bias of paleomagnetic inclinations in the Paleozoic and Precambrian. *Earth and Planetary Science Letters*, **160**, 391–402.

King, R.F., 1955. The remanent magnetism of artificially deposited sediments. *Monthly Notices of the Royal Astronomical Society, Geophysical Supplement*, **7**, 115–134.

McElhinny, M.W., 2004. Geocentric Axial Dipole Hypothesis: A Least Squares Perspective. In: J.E.T. Channell, D.V. Kent, W. Lowrie and J.G. Meert (Eds.), Timescales of the Paleomagnetic Field, *American Geophysical Union Geophysical Monograph* **145**, 1–12.

McElhinny, M.W. and J. Lock, 1996. IAGA paleomagnetic databases with Access. *Surveys in Geophysics*, **17**, 575–591.

Meert, J.G., E. Tamrat and J. Spearman, 2003. Non-dipole fields and inclination bias: insights from a random walk analysis. *Earth and Planetary Science Letters*, **214**, 395–408.

Merrill, R.T., M.W. McElhinny and P.L. McFadden, 1998. The Magnetic Field of the Earth. Paleomagnetism, the Core and the Deep Mantle, Academic Press, San Diego, 531 pages.

Pesonen, L.J., S. Mertanen and T. Veikkolainen, 2012. Paleo-Mesoproterozoic Supercontinents - A Paleomagnetic View. *Geophysica*, **48(1-2)**, 5–47.

Piper, J.D.A. and S. Grant, 1989. A paleomagnetic test of the axial dipole assumption and implications for continental distributions through geological time. *Physics of the Earth and Planetary Interiors*, **55**, 37–53.

Pisarevsky, S.A., 2005. New edition of the Global Paleomagnetic Database. *EOS Transactions, American Geophysical Union,* **86**, 170.

Van der Voo, R., 1990. The reliability of paleomagnetic data. *Tectonophysics*, **184**, 1-9.

Veikkolainen, T., L. Pesonen, K. Korhonen and D.A.D. Evans, 2013a. On the low-inclination bias of the Precambrian geomagnetic field. *Precambrian Research*, in press.

Veikkolainen, T., L.J. Pesonen and D.A.D. Evans, 2013b. PALEOMAGIA, a PHP/MYSQL paleomagnetic database for the Precambrian. *Studia Geophysica et Geodaetica*, submitted.

Zhang, S., Z.-X. Li, D.A.D. Evans, H. Wu, H. Li and J. Dong, 2012. Pre-Rodinia supercontinent Nuna shaping up: A global synthesis with new paleomagnetic results from North China. *Earth and Planetary Science Letters*, **353-354**, 145–155.