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On the Fundamentals in the Methods of Wind Wave Forecasting

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Abstract

*In recent years for wind wave forecasting often begin to be used the so-called interactive models, where attempts have been made to take into account the feedback mechanisms of interaction between atmospheric boundary layer and wind waves. In such models, usually as the first step, atmospheric part of the models were formulated by using the assumption that the fluxes of momentum and energy from wind to waves doesn't significantly influence the structure of turbulent boundary layer above waves. However, with wind waves growing, it is suggested that there must be an adjustment of wind to new wind waves state called two-way coupling. The most popular and actually the necessary method in description of such two-way coupling is the emphasis of the variability of the sea surface roughness (*Kitaigorodskii*, 2003, 2004). Here we discuss the concept of aerodynamic roughness of the sea surface and generalization of empirical data on its variability in connection with so-called wind speed scaling in Kitaigorodskii similarity theory (*Kitaigorodskii*, 1962, 2013). It is shown that our present knowledge of this aspect of wind waves theory permits not only to use wind speed in atmospheric boundary layer at a given height as a governing parameter in wind wave forecasting, but use also a geostrophic wind in predictions of strong storms. As a conclusion to this paper, author considers also the question of applicability of the hypothesis about the existence of fully developed wind waves as an asymptotic regime for indefinitely large values of fetch and duration of the wind.*

Keywords: aerodynamic roughness, wave spectrum, Charnock constant, Kitaigorodskii scaling, mature waves, wave age

1 Introduction

Following my 1961 paper (*Kitaigorodskii*, 1962) I want here to discuss those aspects of the modern wind wave theory, which are of primary importance for numerical wave modelling and wind wave forecasts. From my point of view there are now two of them that deserve more detailed description and explanation.

The first one is about aerodynamic roughness of the sea surface and its variability during wind wave growth. The second is about the hypothesis of the existence of the asymptotic regime for fully developed (mature) wind waves. I am trying below not only to discuss these questions, but also present the latest experimental data and the arguments for their importance in wind wave forecasting.

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2 *The aerodynamic roughness of the sea surface*

Let us start with the so-called wind speed scaling as a main instrument for practical wind wave forecasts. Its origin can be dated to the times when the operation Overlord, the invasion of the Allied forces into Normandy, began during the World War II (*Kitaigorodskii*, 2007). The scientific problem that arose in this time in the British Admiralty wave forecasting section included forecasting of the heights and periods of ocean swell arriving from the Atlantic.

The ultimate goal was of course to forecast the height of the surf over specific coastlines. The Sverdrup and Munk diagrams and corresponding formulae were for so-called significant wave height, subsequently introduced by Sverdrup and Munk as an average height of the highest 1/3 of the waves. But only after considering wind waves as a realization of random processes, the general idea of wind wave forecasts – prediction of statistical characteristics of wind wave fields under their growth in different external meteorological conditions – was formulated (*Kitaigorodskii*, 1962). Among them of course on the first place has been considered wind speed. The basic deficiency of choosing wind speed as one of the governing parameters in the variability of wind generated waves was the need of additional indication of the height of its measurement. To eliminate this difficulty it was usually considered, that the atmospheric turbulent boundary layer above the sea surface can be modelled as a stationary logarithmic boundary layer occupying the half space $z > 0$.

In such model friction velocity U_{*a} is not the only velocity scale for turbulence of dynamic origin, but also can serve as a velocity scale for wind speed on the upper boundary of turbulent layer – so-called free stream velocity (in the real geophysical situation the latter scale simply became the velocity of geostrophic wind). The additional knowledge of the so-called roughness parameter of the sea surface Z_0 together with U_{*a} permits in such model to calculate the wind speed on any given height and vice versa. It is necessary to stress here that at those times the only constructive suggestion in the description of the sea surface roughness Z_0 was made by *Charnock* (1959), who assumed that

$$Z_0 = Z_0\,(U_{*a}, g) = m\,\frac{U_{*a}^2}{g} \tag{1}$$

where the nondimensional coefficient m later on was called Charnock constant. It follows from (1), that knowledge of m permits to find the needed relationships between the friction velocity U_{*a} and the wind speed $U_a(Z)$. That is the main reason why until now the description of the variability of Z_0 in framework of Charnock constant attracted so much attention. The Charnock formulation was initially considered as one related to *Phillips'* (1958) famous spectra of sharp crested waves. At least it was very tempting to do so and seemed to be logical at first sight. However, the simple and natural question, how you can receive a strongly wind dependent length scale (1) from a wind independent form of Phillips spectra, was never posed by modellers. The author was the first one to try to find another foundation for the determination of sea surface roughness Z_0 instead of the very attractive Charnock idea (1959). He suggested (*Kitaigorodskii et al.*, 1965; *Kitaigorodskii*, 1970) to describe Z_0 for the sea surface by two vertical length scales – the thickness of viscous sublayer $\delta_\nu = \frac{\nu}{U_{*a}}$ (ν - air viscosity) and the height of the roughness elements h_s. The latter was identified as the height of roughness elements responsible for the so-called flow separation. The treatment of the roughness of the sea surface by analogy with roughness of solid surfaces considered to be solidly based on small ratio of densities of air and water ($\frac{\rho_a}{\rho_w} \sim 10^{-3}$). Then for aerodynamically rough regime ($h_s >> \delta_\nu$) we get

$$Z_0 = A_s\, h_s \tag{2}$$

where A_s is a nondimensional coefficient, which can depend on different characteristics of roughness elements (the distance between them, for example). To close the problem we must make some assumptions about h_s for the sea surface. One of those can be based on Charnock idea and formulated as

$$h_s = h_s\,(U_{*a}, g) \approx \frac{U^2{}_{*a}}{g} \tag{3}$$

where the coefficient of proportionality, not shown in (3), now must be of order 1. Formulae (2) together with (3) will lead to Charnock formulae (1). The difference between our derivation of (1) and the initial Charnock idea was that not Z_0 but h_s must be considered as a function of only U_{*a} and g. This seemingly small detalization permits to connect Z_0 for the sea surface with the characteristics of the spectrum of wind-generated waves (*Kitaigorodskii*, 1973; *Kitaigorodskii et al.*, 1995; *Hansen and Larsen*, 1997).

The typical observed values of Z_0 for the sea surface in presence of wind waves were close to the sand type roughness elements in (2), when $A_s = \frac{1}{30}$, but not to the wavy solid surface like a washboard. This gives additional indication that the turbulent boundary layer structure above wind generated waves is not very different from classical turbulent shear flow above solid surfaces with rather regular roughness elements, when the largest vertical gradients of wind velocity are lying close to the underlying surface. However, it must not be forgotten that the analogy of air–sea interface with solid surfaces is not general dynamically satisfactory, because of the existence of energy flux through the moving liquid interface $\zeta(x,t)$, which is manifested as wind energy input to surface gravity waves. The latter is not limited to the airflow separation but includes a linear critical layer Miles mechanism. Only when the positions of critical layers are close to the surface, then on distances less or order h_s the analogy of the sea surface with moving roughness elements (*Kitaigorodskii*, 1968; *Hansen and Larsen*, 1997) can be considered dynamically justified. Nevertheless, one of the most important features of our empirical knowledge about Z_0 of the sea surface is that the proportionality coefficient in (1) – Charnock constant – is varied by more than a decade in different wind wave conditions, which is not possible to explain in the framework of assumptions (2, 3). The first attempt to explain the deviations of sea surface roughness from the sand type roughness elements was done in 1965 by the author (*Kitaigorodskii and Volkov*, 1965; *Kitaigorodskii*, 1968), who suggested to take into account that all roughness generating wavelets travel along the wind direction with their associated phase speed *C* and only those whose speed are larger than U_{*a} can contribute significantly to effective height of roughness elements. This leads to the appearance of the so-called Kitaigorodskii filter $\exp\left\{-æ\,\frac{C(k)}{U_{*a}}\right\}$ in the expression for h_s:

$$h_s = \{2\int_0^{\infty}\Psi(k)\exp\left[-2\,æ\,\frac{C(k)}{U_{*a}}\right]dk\}^{½} \tag{4}$$

where $\Psi(k)$ is the wave number spectrum (averaged over all directions of wave propagation). Here all wavelets were considered as moving roughness elements. The expression (4) indicates one very important fact – during wave growth and the shift of wave spectral peak to lower wavenumbers (frequencies) the contribution to the overall roughness length is transferred to high wavenumbers on spectral tail. For example for Phillips tail

$$\Psi(k) = Bk^{-4} \tag{5}$$

where *B* is Phillips constant, this leads to result

$$m \sim B^{½} \tag{6}$$

which was initially mistakenly considered as in indication of variation of Charnock constant due to variation of Phillips constant. However, as was correctly noticed by *Hansen and Larsen* (1997) *B* varies by about a factor 2 in the range 0.005 to 0.01, while *m* is found to vary more than decade for the same data set. So the Kitaigorodskii roughness length model (*Kitaigorodskii*, 1968, 1973) was not able to reproduce the large observed variation of roughness parameter of the sea surface. To explain this, *Hansen and Larsen* (1997) suggested first to consider random wavelets with different steepnesses *ak*, but assumed that flow separation occurs at a ratio of wave height to wave length $\frac{h}{\lambda} \geq 0.08$ with corresponding threshold steepness $\frac{\pi h}{\lambda} = 0.25$, and to use a more detailed expression for A_s (*Lettau*, 1969)

$$A_s = \alpha_L \frac{X}{A} \tag{7}$$

where α_L is now coefficient of order one and X/A is ratio of the areas occupied by roughness elements wide and far mean wind direction. Formula (7) does not change the foundation of aerodynamic classification (2) based on the classical Reynolds roughness number $Re_s = h_s/\delta_v = h_s u_{*a}/\nu$. The attempt to specify the value of h_s for the sea surface has been also made by Toba and Koga (1986). They suggested that $h_s = u_{*a}T$, where *T* is the time interval required for air particle with speed u_{*a} to cover the distance from bottom of the trough to the crest. Taking $T = 2\pi\omega_p^{-1}$ they recommend to use what they call breaking wave parameter $Rb = h_s/\delta_v = u_{*a}^2\omega_p/\nu$. They found that in some cases it could serve as a useful tool to describe the variability of air–sea interactions and drag coefficient and white cap coverage in particular (*Zhao and Toba*, 2001). For rather isolated roughness elements X/A = 0.1–0.04 in (7). It is not coincidental that the range of observed variation of Charnock constant m in (1) is exactly the same (see Fig. 1a, 1b).

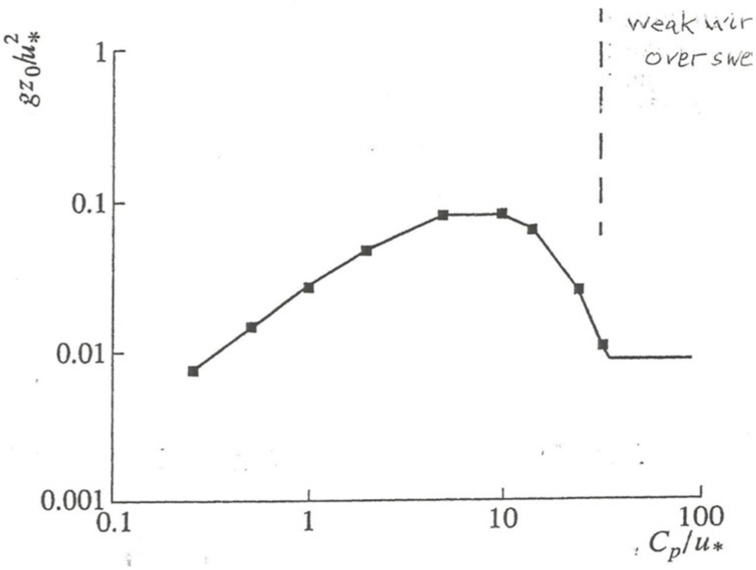

Fig. 1a. Nondimensional sea surface roughness gz_0/u_{*a}^2 vs. wave age C_p/u_{*a}. Comparison with observations on Fig. 1b.

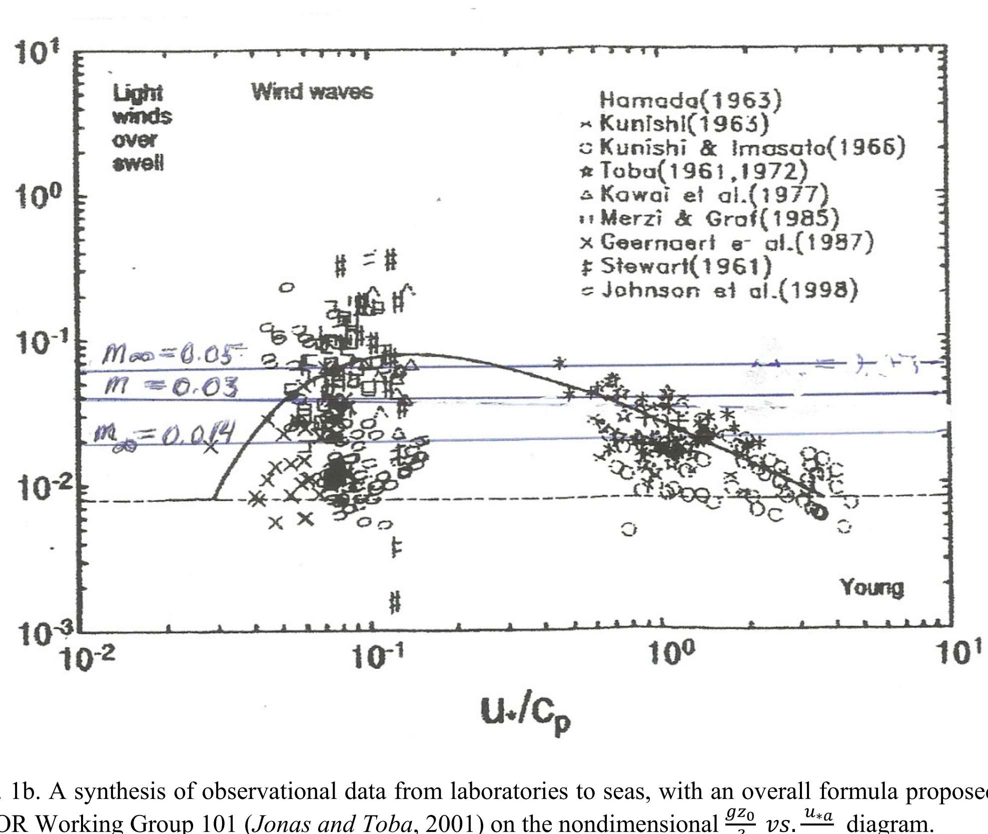

Fig. 1b. A synthesis of observational data from laboratories to seas, with an overall formula proposed by SCOR Working Group 101 (*Jonas and Toba*, 2001) on the nondimensional $\frac{gz_0}{u_{*a}^2}$ $vs.$ $\frac{u_{*a}}{c_p}$ diagram.

In case of wind waves roughness elements distribution in space depends on angular waves distribution. In isotropic waves $X/A \sim 1$, but for narrow angular distribution of waves $X/A << 1$ and varies by an order of magnitude. Formula (4) together with modified Kitaigorodskii roughness length model (*Hansen and Larsen*, 1997) permits to find variation of Charnock constant with wave age c_p/u_{*a} rather close to the observations for wave growth stage (Fig. 1b). In their interesting calculations, Hansen and Larsen used the model of equilibrium wind wave spectra with transition from inertial wind dependent ω^{-4} range of scales (*Kitaigorodskii*, 2004, 2013) to the so-called dissipation subrange as suggested in *Kitaigorodskii* (1983). The transitional frequency ω_g was derived by matching two regimes (*Kitaigorodskii*, 1983) as

$$\frac{\omega_g U_{*a}}{g} = \frac{\alpha_s}{2B} \tag{8}$$

where α_s and B are nondimensional coefficients correspondingly in Kitaigorodskii ω^{-4} and Phillips ω^{-5} forms of the wind wave spectra. For waves they estimate X/A in (7) as the ratio of roughness wavelet height h to the wavelength λ times the fraction of wavelets where flow separation occurs. However, these important improvements to Kitaigorodskii roughness length model (*Hansen and Larsen*, 1997), where wavelets were considered as moving roughness do not yet answer to two important questions which naturally are created by Figures (1a, b) taken by me from a monograph of *Jones and Toba* (2001) and a paper by *Toba et al*. (2006). One is, why the Charnock constant has a maximum value at some intermediate values of wave age, i.e. why sea surface roughness is increasing during first stages of wind waves growth, and decreasing at the latest stages. And the second, but probably the main one, is how we can use (or incorporate) the concept of aerodynamic roughness of the sea surface in the general similarity theory for nonlinear surface wind waves, in particular can we replace Charnock initial idea of localized roughness depending only on wind speed (or friction velocity) with something different.

3 *Roughness parameter of the sea surface as a part of the similarity theory for nonlinear wind waves*

One of the conclusions of Kitaigorodskii's derivation (3) is that the proportionality constant between effective height of roughness elements for the sea surface h_s and the length scale based on u_{*a} and g must be of order one, opposite to the value of Charnock constant m. It shows that only those parts of wave spectra which contribute significantly for the overall height of roughness elements responsible for flow separation behind them can be important in determination of h_s for the sea surface (*Hansen and Larsen*, 1997). We can imagine that under certain conditions of strong steady wind the wave field may be brought to a very high energy level ($\overline{\zeta^2, t}$) with incipient breaking of almost all wave crests and the probability that each wave crest is a roughness element is close to unity. In such circumstances the waves phase speed is much smaller than the wind speed and the effect of moving roughness elements in the Kitaigorodskii roughness length model can be ignored (*Hansen and Larsen*, 1997). This leads us to important conclusion that *Phillips* (1958) subrange of wind spectra, so-called dissipation subrange, can be a main contributor to $\sqrt{\zeta^2}$ for the sea surface, and that the value of h_s can be derived by the integral

$$h_s = [2 \int_{\infty}^{\omega_g} S(\omega) d\omega]^{1/2} \quad (9)$$

where ω_g is a low frequency boundary of Phillips subrange (*Kitaigorodskii*, 1998, 2013) where the role of Kitaigorodskii filter in (4) can be neglected. To illustrate this we can refer to following results. For Burling spectra (*Kitaigorodskii*, 1962) the range of variations of $\omega_g U_{*a}/g$ was (0.25–0.18) with average value 0.22. This leads to $c_g \leq 4.5 U_{*a}$ which indicates that wavelets from dissipation subrange $c < c_g$ can behave as <u>non-moving roughness elements</u> with overall height (9). From the data analysis of Kitaigorodskii (1983) the range of variations of $\frac{\omega_g U_a}{g}$ has been found as

$$\frac{\omega_g U_a}{g} = 3.3 - 1.5 \quad (10)$$

To compare it with *Burling* (1959) results we for simplicity assume as before $U_a \approx 28\, u_{*a}$, where U_a at 10 m height, what will lead to $\frac{\omega_g U_{*a}}{g} = 0.11 - 0.05$ with average value $\frac{\omega_g U_{*a}}{g} \approx 0.08$. This leads to $c_g \leq 12\, u_{*a}$, which again indicates that it is the range of scales, where wavelets from Phillips dissipation subrange still can play a role of slowly moving roughness elements, behind which the separation of mean air flow can occur. All the above shows that for determination of h_s values in (9) we can use the Phillips form of wave spectra

$$S(\omega) = \beta\, g^2\, \omega^{-5}; \quad \omega \geq \omega_g \quad (11)$$

where β is a Phillips constant. An intriguing question arises – how from a wind independent form of wave spectra (11) we can receive the scale of wave heights (as heights of roughness elements) described by Charnock formula (3), which makes this scale strongly <u>wind-dependent</u>. To avoid the answer to this question, most wave modellers made a wrong choice – they were using the variable Phillips constants (β, *B*) instead of accepting Kitaigorodskii theory (*Kitaigorodskii*, 1998, 2013) of variable ω_g. To demonstrate this we can use the following example from *Kitaigorodskii* (2003). If as before for simplicity we assume the relationship

$$U_{*a} \approx \frac{1}{28} U_a \quad (12)$$

where U_a is a wind speed at approximately 10 m level, then for Burling data ($\frac{\omega_g U_{*a}}{g} = 0.22$) (*Kitaigorodskii*, 1962) we will have in terms of wind speed scaling

$$\tilde{\omega}_g = \frac{\omega_g U_a}{g} = 6.15 \quad (13)$$

Substituting (11) we can receive the following formulae

$$h_s = \beta^{1/2} \frac{U_a^2}{g} \tilde{\omega}_g^{-2} \tag{14}$$

It is interesting that in Fig. 2, which we present here, the best fit for measured h_s from the spectra (11) was correspondent also to $\varpi_g = 6$, which is probably the good value for not too large fetches and winds of weak or moderate strength like in *JONSWAP* (1973), *Birling* (1959) and *Kitaigorodskii* (1962). It follows from (14) and (2,7) that

$$Z_0 = \alpha_L \frac{X}{A} \beta^{1/2} \frac{U_a^2}{g} \tilde{\omega}_g^{-2} \tag{15}$$

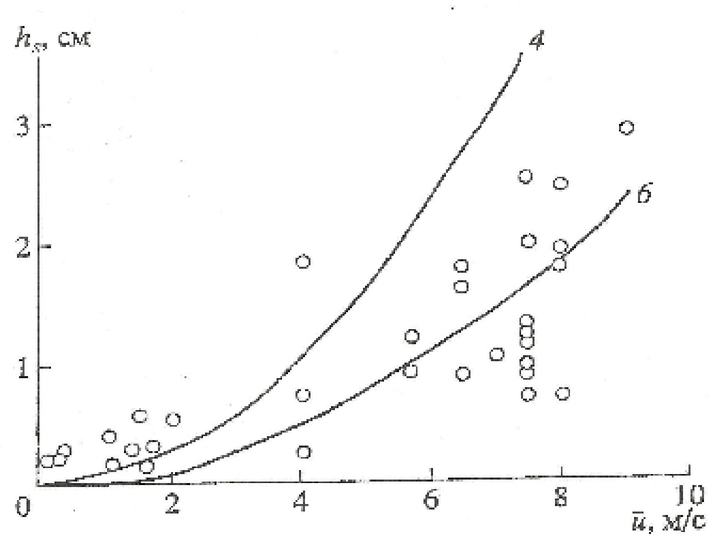

Fig. 2. The effective height of the roughness elements h_s of the sea surface for different wind speeds (from *Kitaigorodskii et al*., 1995). The curves correspond to the formulae (14) for $\omega_g U_a/g = 4, 6$.

From Fig. 2 and formulae (15) we found Z_0 increasing with decrease of ϖ_g which behaves as peak frequency during wind wave growth (*Kitaigorodskii*, 1998, 2004). Again to demonstrate that the latter fact is real reason for the variability of the sea surface roughness, it is very important to stress here the remarkable difference of the properties of dissipation Phillips subrange (5, 11) and the Kolmogorov's type of viscous subrange in the theory of locally isotropic three dimensional turbulence with internal scale $l_\nu = (\frac{\nu^3}{\varepsilon_\nu})^{1/4}$ where ε_ν is a viscous dissipation of kinetic energy of turbulence. Increase of energy supply to turbulence will lead to increase of values of ε_ν and decrease of the value of l_ν cut off length for so called inertial subrange, thus enlarging the region where direct effect of molecular viscosity can be ignored.

In difference with this property of Kolmogorov's turbulence the "cut-off" scale of the dissipation subrange (5, 11) in wind waves moves to larger scales with increase of energy supply, thus producing the tendency for more longer (larger) waves to develop sharp crests and break, which ultimately will lead to increase of wind wave energy dissipation. This effect was discovered and detailed by the author in *Kitaigorodskii* (1983, 1998). It is also very important to notice that this movement of *dissipation subrange* in wind wave fields to larger scales are independent of where the region of energy supply is – is it in the same region as wave dissipation scales, or exists at larger peak waves producing so called direct energy cascade, or in case of so-called inverse energy cascade among wind wave components located at $\omega = \infty$, i.e. at scales less than dissipation scales during wind wave growth. (In the latter case all growth of waves components with $\omega < \omega_g$ must be attributed only to wind forcing, which is possible, but remains to be proved). Now the tendency of growing Z_0 (or Charnock constant) with wind wave growth according to (15) is probably a correct explanation of part of Fig. (1a, b), when during wave growth dissipation subrange enlarges to its maximum values, and when $X/A \approx 0.1$. The sea surface remained sand type roughness when practically majority of waves behave as roughness elements. But at the later stages $X/A << 1$ and decrease of X/A can overcome the tendency of dissipation subrange to move noticeably to low frequencies due to the smallness of the phase speeds of roughness producing wavelets (*Hansen and Larsen*, 1997). Waves around peak are not any more roughness elements themselves, and they can be like a moving platform with smaller scales roughness elements on it. Anyway the diminishing of roughness after their maximum values at the intermediate values of wave age (the second half of Fig. 1a) remains not explained. The author recently (*Kitaigorodskii*, 2003) has tried to find other scales for roughness parameter Z_0 of the sea surface, but finally considered that Z_0 must be part of general similarity theory of wind waves and suggested instead of Charnock formula (1) its empirical generalization

$$m = m\left(\frac{c_p}{u_{*a}}\right) \tag{16}$$

The form of (16) was suggested in *Jones and Toba* (2001) as

$$\frac{g\,Z_0}{U_{*a}^2} = m = 0.03\left(\frac{c_p}{u_{*a}}\right)\exp\left\{-0.14\,\frac{c_p}{u_{*a}}\right\}; \quad 0.35 < \frac{c_p}{u_{*a}} < 35 \tag{17}$$

$$\frac{g\,Z_0}{U_{*a}^2} = m = 0.008; \quad \frac{c_p}{u_{*a}} > 35 \tag{18}$$

The formulae (16–18) show that during wave growth at the early stages sea surface becomes more rough, and at the later stages it becomes smoother with the maximum value of Charnock constant somewhere in the middle of their growth. This is difficult to explain, but on the other hand it is a pleasant fact for estimates of the sensitivity of the wave forecast to the choice of the values of Z_0 (or the measured wind as governing parameter at measured level, see Fig. 4). For example choosing m in (18), i.e. using Charnock formulae for Z_0 we can see the following differences in formulae for peak frequency $f_0 = \frac{g}{c_p}$ for mature waves (*Komen et al.*, 1984) when in terms of 10 m wind U_a:

$$\frac{\omega_p}{2\,\pi} = 0.13\ \frac{g}{U_a} \qquad (19)$$

and in terms of 19.5 wind as in *Pierson and Moskowitz* (1964)

$$\frac{\omega_p}{2\,\pi} = 0.14\ \frac{g}{U_a} \qquad (20)$$

Because of high powers in wind velocity in wind speed scaling for wave energy, the difference in the relationship $u_{*a}(U_a)$ (difference in drag coefficients) can be more significant to wave forecasting, but the applicability and usefulness of traditional wind speed scaling (*Kitaigorodskii*, 1962, 2012) cannot be questioned seriously only on the grounds of the variations in Z_0 during wind wave growth. To avoid uncertainty in the choice of the level for wind speed as the governing parameter in similarity theory for wind wave statistical characteristics (Kitaigorodskii theory), it was suggested recently (*Kabatchenko et al*., 2002) to use geostrophic wind, whose values can be derived from the atmospheric pressure fields. To connect the friction velocity u_{*a} to the geostrophic wind G, variability of geostrophic drag coefficient u_*/G must be taken into account. In the case of neutral stratification

$$\frac{U_{*a}}{G} = \varphi\ (\frac{U_{*a}}{\Omega Z_0}) \qquad (21)$$

and (21) together with (16–18) will close the problem of defining the behaviour of wind block during wind wave growth in the presence of constant atmospheric pressure gradient.

In the near-coastal waters, the atmospheric boundary layer (ABL) is often stratified. In such cases for application of logarithmic boundary layer model for the ABL, empirical information about the thickness of dynamic sublayer is needed. $L_{M0} = -u_{*a}^3/Q$, where Q is buoyancy flux (in marine ABL usually $z_0 \ll L_{M0}$). If a geostrophic wind is chosen as the wind field, then detailization of (21) for a stratified ABL must be taken into account.

4 Fully developed wind waves: hypothesis or idealization?

Now let us return back to so-called wind speed scaling of wave characteristics (*Kitaigorodskii*, 2013). For frequency spectra $S(\omega)$ we have for duration (t) and fetch (X) limited cases

$$\frac{S(\omega)\, g^3}{U_a^5} = F\ (\frac{\omega\, U_a}{g}, \frac{g\, X}{U_a^2}, \frac{g\, t}{U_a}) \qquad (22)$$

To complete the formulation of the model of wind wave growth in presence of atmospheric boundary layer as was described in the sections above, we must add to (22) one more parameter in the set of governing parameters – sea surface roughness Z_0, which together with $U_a(Z)$ can define the velocity scale in ABL friction velocity U_{*a}.

Thus additionally to general hypothesis (22) we must add to the (22) equation for roughness parameter of the sea surface Z_0, which as $S(\omega)$ is part of statistical characteristics of random sea surface elevation $\zeta(x, t)$. Thus

$$\frac{Z_0\, g}{U_a^2} = \Psi\left(\frac{g\, X}{U_a^2}, \frac{g\, t}{U_a}\right) \tag{23}$$

The knowledge of Ψ (23) permits to use together with Z_0 as a governing parameter also friction velocity U_{*a}, which is height independent. Or if we use U_{*a} as the only velocity scale for ABL it permits together with Z_0 to have U_a as velocity scale for ABL for chosen height above the sea surface. Thus in principle the evolution of wave field – wind generated wind waves must be considered by following (22) and (23) together. The further simplification of system (22, 23) can be done as

$$S(\omega)\frac{g^3}{U_a^5} = F_x\left(\frac{\omega\, U_a}{g}, \frac{g\, X}{U_a^2}\right) \tag{24}$$

$$S(\omega)\frac{g^3}{U_a^5} = F_t\left(\frac{\omega\, U_a}{g}, \frac{g\, t}{U_a}\right) \tag{24}$$

This is two-parametric families of solutions for F_x and F_t associated with wind speed scaling (*Kitaigorodskii*, 1962, 2013). There must exist some interrelations between F_x, F_t since X = ½ $c(k)t$, $c(k)$ is group velocity of waves with wave number k (*Kitaigorodskii and Srekalov*, 1962). However, we avoid the discussion of these interrelations and also instead of (23) we use the variation of Z_0 in terms of internal parameters, so that the function Ψ can be replaced by

$$\frac{Z_0\, g}{U_{*a}^2} = m\left(\frac{c_p}{U_a}\right) \tag{26}$$

where c_p – phase peak velocity, whose ratio to wind speed defines so-called wave age.

Now let us consider the possibility to have fully developed (matured) wind wave fields. This can be written as asymptotic regime

$$\tilde{t} = \frac{g\, t}{U_a} \to \infty; \quad \frac{g\, X}{U_a^2} \to \infty; \quad F_t = F_X = F_\infty\left(\frac{\omega\, U_a}{g}\right) \tag{27a}$$

$$\frac{g^2\, E}{U_a^4} = const; \quad \frac{\omega\, U_a}{g} = \frac{2\pi\, U_a}{c_p} = const \tag{27b}$$

$$\frac{Z_0\, g}{U_a^2} = m = const = m_\infty \tag{27c}$$

As was mentioned in our previous paper *Kitaigorodskii* (2013), the existence of independent of fetch and duration stationary wind wave spectra requires fulfilment of at least two conditions (27b). Now we add to them an additional one (27c) for surface roughness parameter. The empirical determination of the value m_∞ in (27c) is an additional requirement for application of similarity theory for wind waves. In this respect Fig. (1a, b) can be considered as this necessary addition to Pierson-Moskowitz case of fully developed waves. The values of constants in (27a, b, c) are not quite independent from each other. Conditions (27a) describe enhancement of energy with wind speed, whereas the next one (27b) movement of the peak towards lower frequency with increase of wind. This type of wind forcing can be interpreted as driving waves to travel quicker (increase their associated phase speed). To distinguish this effect from downshift I suggest to call it wave speeding. It is exactly this aspect of wave growth which leads to diminishing roughness parameter with "wave speeding", because fast moving wavelets, even with sharp crests, will not allow separation of mean air flow behind wavelets thus making the sea surface smoother at last stages of waves' development. (Second half of variation of Charnock constant with wave age on Fig. 1a). In practice Fig. (1a, b) can be interpreted that wavy sea surface can return to its initial roughness given formulae (15). It seems to me, that this actually was observed and corresponds to the situations, when $\underline{\omega}_g$ are noticeably larger than $\underline{\omega}_p$. This fact doesn't allow us to use Pierson-Moskowitz values for $\underline{\omega}_p$ in (15) instead of $\underline{\omega}_g$ but rather support the idea to use directly observed values of $\underline{\tilde{\omega}}_g$ as in *Kitaigorodskii* (2004). The range of the ratio ω_g / ω_p can be easily found from Table 1 in *Kitaigorodskii* (1998). Here we reproduce the corresponding values of this ratio from different sources (Table 1). The average value was 4.37 and we decide to use it together with Pierson-Moskowitz -value $\omega_p U_a/g = 0.140 \cdot 2\pi = 0.88$. This gives for asymptotic regime the value $\tilde{\omega}_g = 4.3 \cdot 0.88 = 3.8$. As we notice it is close to (10) (*Kitaigorodskii*, 1983). If we use as before (12) it will lead to $\omega_g U_{*a}/g = 0.13$ different from the range of $\omega_g U_{*a}/g$ in Burling data (0.25–0.18), because the latter were for too small fetches. Now using average value $\tilde{\omega}_g = 3.8$ we can calculate the roughness parameter Z_0 in (15). It gives with $\beta = 0.0081$

$$Z_0 = \alpha_L \frac{X}{A} \frac{U_a^2}{g} \cdot 6.25 \cdot 10^{-3} \qquad (28)$$

or with (12)

$$\frac{Z_0 g}{U_{*a}^2} = \alpha_L \frac{X}{A} \cdot 4.9 = m \cong m_\infty \qquad (29)$$

So by neglecting the variation of $\tilde{\omega}_g$ when we use its average values from Table 1, we come to the conclusion, that variation of m in (29) is connected only with values of X/A, which for developed waves are of order 10^{-2}.

This gives us the value $m = 0.05$ in general agreement with Figs. 1a-b for developed waves. Thus Charnock formula (1) with m_∞ receives a new interpretation in framework of our above discussion of aerodynamic roughness of the sea surface. It is a measure of roughness parameter in a developed wind wave field, when wavelets responsible for air flow separation contribute to high wave number and frequency tail of wave spectra. The value $m_\infty = 0.05$ gives the first reasonable estimate of m_∞ to use in wave forecasting.

Table 1. Ratio of frequency ω_g as transitional frequency to dissipation subrange in wind wave spectra to the peak frequency ω_p.

ω_g/ω_p	Source
5	Tang- Shemdin frequency spectra
5.57	Slope spectra
5.05	
5.48	
av. 5.27	
6.48	*Banner et al*. (1989)
4.87	Spatial 2-D spectra
4.88	SWOP 2D spatial spectra
2.85	*Hansen et al*. (1992) frequency spectra
3.90	
3.29	
3.27	
av. 4.22	
2.96	*Leykin and Rosenberg* (1984) frequency spectra
2.07	*Lupyan and Sharkov* (1989) spatial 2-D spectra
5.7	
av. 4.22	
4.01	*Jahne and Rimer* (1990) spatial 2-D spectra
4.87	
4.45	
av. 4.22	

Average value $\omega_g/\omega_p = 4.37$

In *Kitaigorodskii* (1962) there was the first indication through the analysis of *Burling* data (1959) of the possibility to have practically fetch-independent wind wave spectra of the form

$$S(\omega) = 6.5 \cdot 10^{-3}\, g^2\, \omega^{-5}; \quad \omega > \omega_g \sim \omega_p \tag{30}$$

$$\lg \frac{S(\omega)\omega^5}{g^2} = a + b\, \frac{u_{*a}\,\omega}{g}; \quad \omega \le \omega_p \tag{31}$$

where *a, b* have been found only slightly varying with fetch (their average values were correspondingly (9, 13) and the range of variation of $\frac{u_{*a}\,\omega_g}{g}$ very small (see Fig. 3 in *Kitaigorodskii*, 1962) being equal 0.25–0.19 with average value

$$\frac{\omega_g\, u_{*a}}{g} \approx 0.22 \tag{32}$$

The only strongly variable with *X* in such parameterization of *S*(ω) was frequency ω_p

$$\frac{\omega_p\, u_{*a}}{g} = \frac{\omega_p\, u_{*a}}{g}\ (\tilde{X}) \tag{33}$$

The formula (33) describes so-called downshift effect and I have found that *Burling* (1959) data on $\tilde{\omega}_p$ does not differ very much from JONSWAP spectra (*Kitaigorodskii*, 2013). Because of (33) in Burling data the author in 1961 refused to accept the concept of fully developed wind waves in spite of the fact that condition of independent of fetch total energy was almost fulfilled for spectra (30, 31).

However a little bit later *Pierson and Moskowitz* (1964) found the data, where both conditions (27b) were satisfied and such spectra was called fully developed wind waves (terminology first introduced by author in *Kitaigorodskii*, 1962). Actually as the main empirical finding for their spectra *Pierson and Moskowitz* (1964) considered the following result

$$\frac{\omega_p\, U_{*a}}{g} = 0.88 \tag{34}$$

which they have used to derive "corrected values of wind speed". So the situation they met was quite different (even opposite) from what I have found in 1962, since they have spectra with one constant value of $\omega_p U_a/g$ without any downshift as in *Kitaigorodskii* (1962). On Fig. 3a I show these spectra with still "uncorrected" values of wind speeds. Some examples from it were not in favour of wind speed scaling. For instance, for U_a = 30 knots the maximum energy level around wave peak was approximately twice smaller than for wind speed 15.47 m/s (35 knots). But after using (34) for "corrected" wind speed values they have found finally the enhancement of energy with wind speed in accordance with prediction of Kitaigorodskii similarity theory (27a, see Fig. 3b). That how the famous *Pierson and Moskowitz* (1964) spectrum for fully developed waves appeared and for long time was (and still is) an important part of wind wave forecasting. Assumption about asymptotic limit for F_∞ (27a) first has been attempted to check in *Komen et al.* (1984). This was done on the basis of so called Hasselmann equation (*Hasselmann*, 1962) for wind wave action $N_K = \frac{g\, F(k)}{\varpi_K}$, which has the form

$$\frac{\partial N_K}{\partial t} + \nabla_K\, \omega_K\, \nabla_r N_K = S_{in}(N_K) + S_{diss}(N_K) + S_{nl}(N_K) \tag{35}$$

where S_{nl} describes nonlinear transfer and is given as an explicit expression through so called collision integral for four resonantly inter acting gravity waves (*Hasselmann*, 1962).

The dissipation and generation terms are introduced in (35) phenomenologically. In their absence and in case of horizontal homogeneity (35) takes the form

$$\frac{\partial N_K}{\partial t} = S_{nl}(N_K) \tag{36}$$

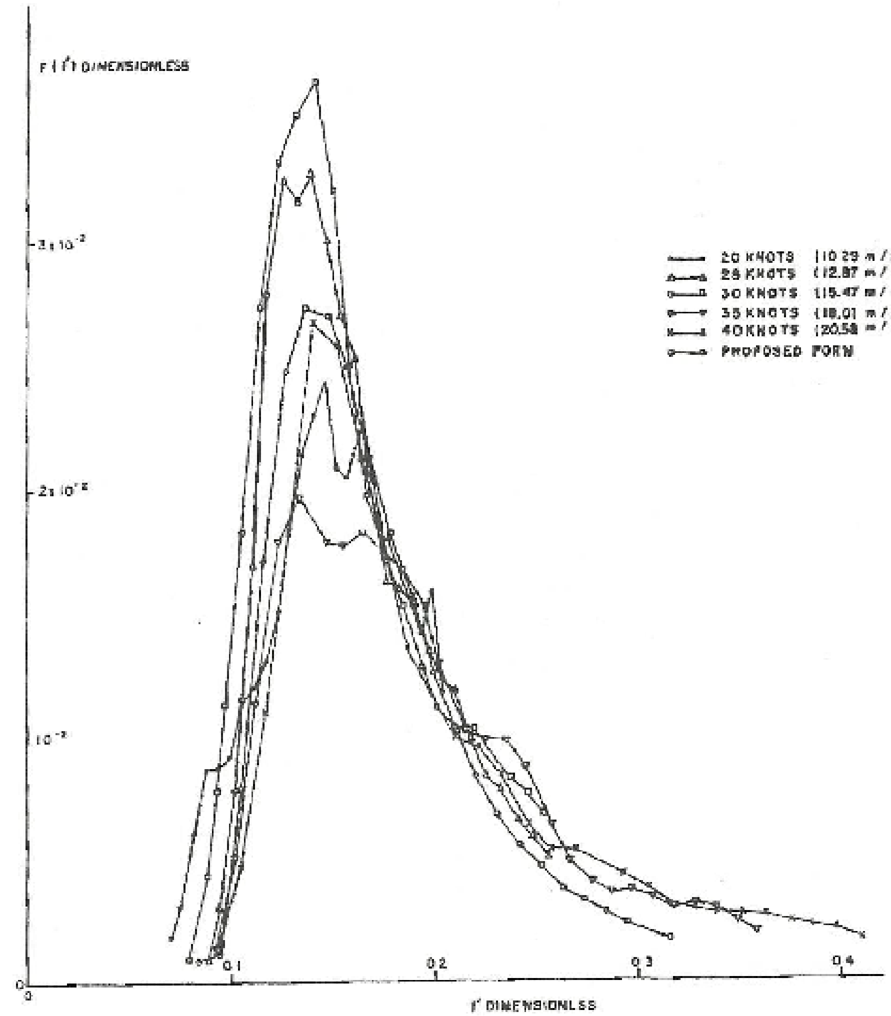

Fig. 3a. The nondimensional wind wave spectra $S(\omega)g^3/u_a^5$ for the five nominal "uncorrected" wind speeds (from *Pierson and Moskovitz,* 1964).

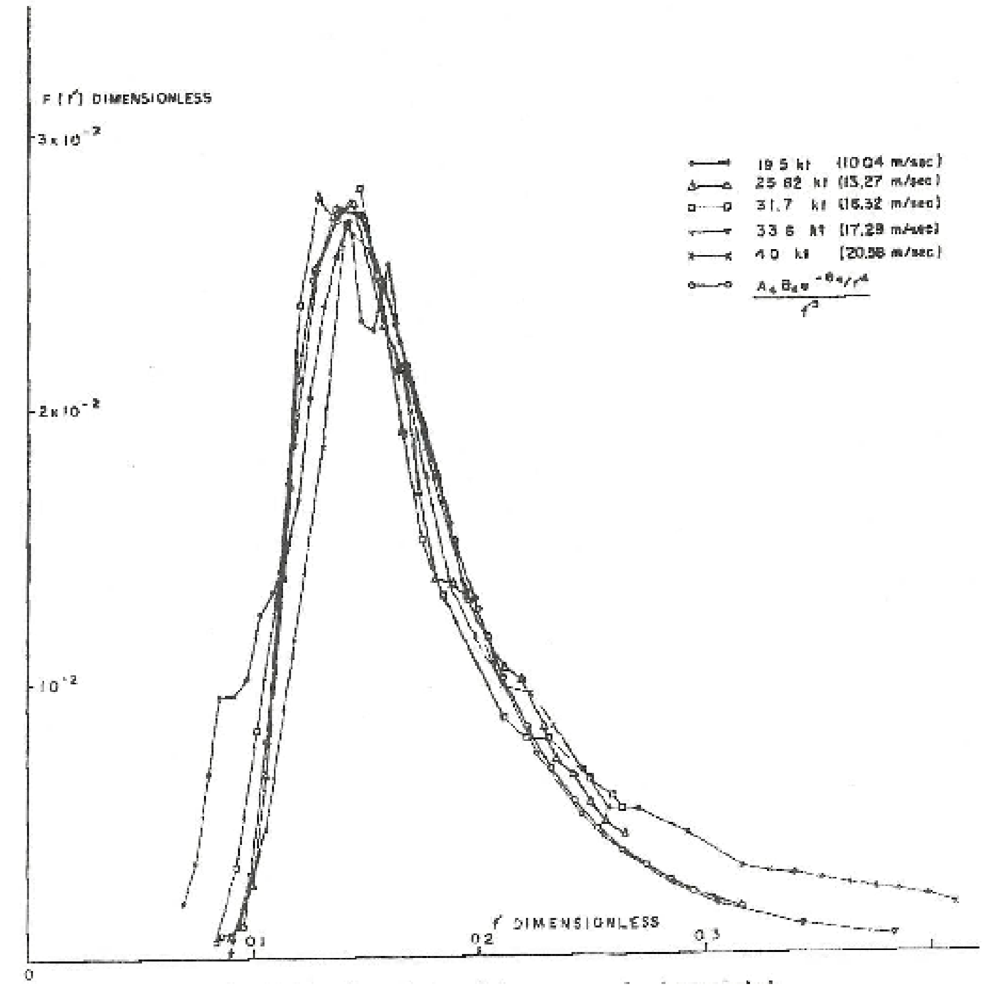

Fig. 3b. The nondimensional wind wave spectra $S(\omega)g^3/u_a^5$ for the five nominal corrected wind speeds (from *Pierson and Moskovitz*, 1964).

In *Komen et al.* (1984) technical and mathematical difficulties of numerical evaluation of S_{nl} were successfully resolved. The solution of (37) require also a knowledge of initial conditions, but for stationary and homogenous case representing the infinitely large fetch and duration (35) is reduced to

$$S_{nl} + S_{diss} + S_{in} = 0 \tag{37}$$

The fulfilment of (37) was also checked in *Komen et al*. (1984), which undoubtedly was the first great achievement of numerical wave modelling.

The result of calculations in *Komen et al.* (1984), though very interesting, do not permit from our point of view to make some final conclusions about energy balance of mature wind wave fields.

The weak points in the interpretations in *Komen et al.* (1984) from my point of view were the roles they attribute to changes of Phillips and Charnock constants in their

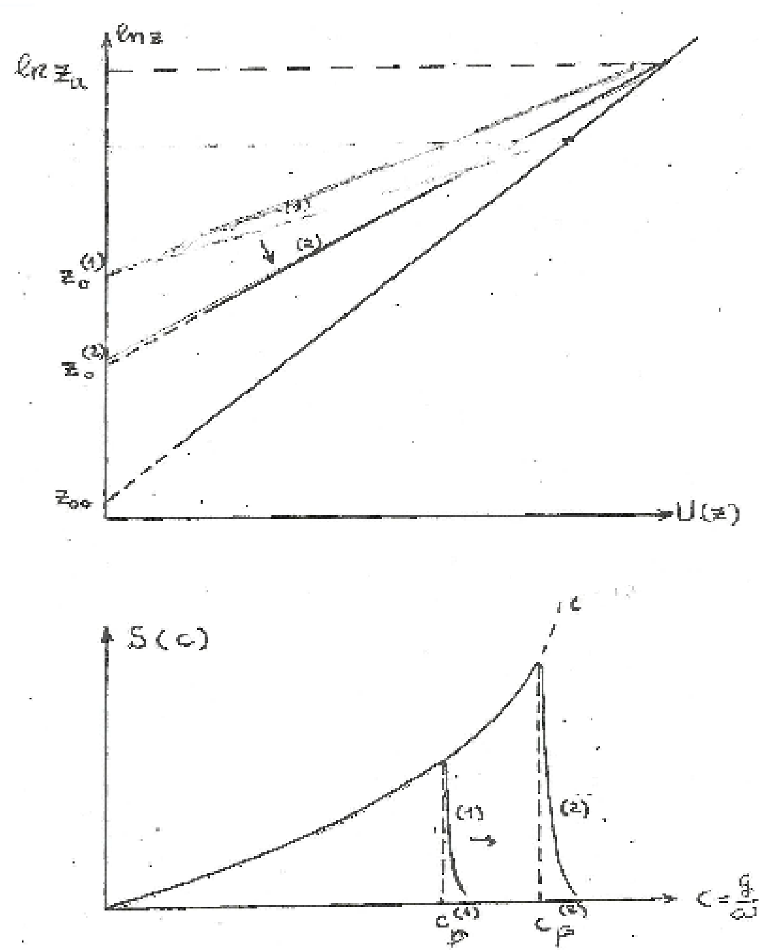

Fig. 4. Qualitative picture of the evolution of mean wind profile $u_a(z)$ in the logarithmic boundary layer during wind wave growth with wave spectra $S(c)$. !!
Z_{00} = the roughness parameter of the sea surface at the initial stage of wind wave growth
Z_{01} = the roughness parameter of the sea surface at the intermediate stages of wave growth
(Z_{01} is close to the Charnock expression with m = 0.05)
Z_{02} = the roughness parameter of the sea surface at the latest stages of wind waves development ($Z_{02} < Z_{01}$ and close to Z_{00})

Calculations of source terms in (35) (Phillips constant from 0.0081 to 0.005, and B_k factor $B_k = \frac{U_a}{U_{*a}} \cdot \frac{1}{28}$ from 0.85 to 1.02). As we have shown above, there are many empirical indications, that with changes not only angular distribution, but also wind speed (not to mention peak frequency) the value of Charnock constant can easily change by more than order of magnitude. This will help to explain the necessity of introducing the variable values of constant B_k exactly in the range, which can be attributed to the changes in Charnock constant reported in this paper. Table 2 gives a clear picture for interrelationships between B_k and m and the range of variations of their values.

Table 2. Values of the numerical factor $\beta_k = \frac{U_5}{u_*}\frac{1}{28}$ for different values of the Charnock constant m and different wind speeds U_{10}; U_5 and U_{10} are the wind speeds at 5 m and 10 m, respectively, height.

m / U_a (m/s)	10^{-1}	10^{-2}	$5 \cdot 10^{-3}$	10^{-3}
10	0.75	0.95	1.01	1.16
20	0.63	0.83	0.89	1.04
25	0.59	0.79	0.85	1.00

My criticism of the work of *Komen et al*. (1984) is basically due to their far going conclusion not about balance of energy of wind wave spectrum of fully developed waves, but about the balance in equilibrium ranges of wind wave spectra. It is only their ignorance of the important role of changes in the positions of the boundaries of dissipation subrange, does not permit authors of such big projects like SWOP and JONSWAP to discover the regions of $k^{-7/2}$ law (in Stereowave project) and ω^{-4} law (in JONSWAP) as parts of equilibrium regime of the wind wave spectrum, which I proposed (*Kitaigorodskii*, 1962, 1983).

5 *The aerodynamic roughness of the sea surface and drag coefficients for very high (hurricane) winds*

The other important aspect of the parameterizations (27a–c) for fully developed waves is the possibility of a limiting state in the aerodynamic roughness of the sea surface. This question can be of critical importance in understanding and modelling the developments of hurricanes and other intense storms. However, the limits for aerodynamic roughness of the sea surface are not necessarily described by assuming the validity of (27c). In contrast, (27c) are usually identified with observed increase of drag coefficient with increase of wind speed. The data supporting these wind speed dependent drag coefficients cover a relatively small range of wind speeds, typically 4–20 m/s, with relatively few data points above 15 m/s. Can such picture be extrapolated to very high winds for modelling severe extra tropical cyclones and hurricanes – the question many people want to know the answer. Fig. 2 from the paper by *Donelan et al*. (2004) shows a remarkable saturation of the drag coefficient once the wind speed exceeds 33 m/s. Beyond this speed the surface simply does not become any rougher in aerodynamic sense. The measurements of *Donelan et al*. (2004) unfortunately concern laboratory waves suggesting the 33 m/s saturation wind speed (altitude 10 m). The saturation level of the drag coefficient was 0.0025. This corresponds to a roughness length of 3.35 mm. Using (27c) we come to the value $m_\infty = 0.012$. The valuable field measurements of *Large and Pond* (1981) would suggest saturation at 0.0028 what with 33 m/s wind would lead to $m_\infty =$ 0.0168, and the wind sounding profiles of *Powell et al*. (2003) showed saturation of the drag coefficient at 0.0026 at about wind of 35 m/s. This leads to $m_\infty = 0.0122$. These three values of m_∞ give the average of 0.014. These results for $m_\infty(u_{*a}/c_p)$ are shown in Fig. (1a, b).

There are still doubts how reliable they are. To explain why during wave growth the surface can become aerodynamically smoother, it was suggested that at high winds the separation of airflows occurs at breaking waves, which can produce a shear layer between the outer flow and the flow trapped in the separation zone. The outer flow, unable to follow the wave surface, does not "see" the troughs of the waves and skips from breaking crest to breaking crest. In our language, it will diminish the value of X/A in (7) thus diminishing the roughness. Thus in conditions of continuous breaking of the largest waves, the aerodynamic roughness of the surface is limited in spite of changes in geometrical roughness of large waves. Unfortunately, this hypothetical picture is based on the results of laboratory experiments, which are unable to reproduce fully developed wind waves in the open ocean. The oceanographic community prefer now to use less ambitious title for fully developed waves calling them mature waves. However, the unresolved question remains what value must be chosen for m_∞ in case of mature waves ($U_a/c_p \sim 1$). The difference between $m_\infty = 0.05$–0.08 and $m_\infty = 0.014$ is still too large to be ignored and also too large to be attributed to molecular viscosity. Even the difference in the values of m_∞ for mature waves ($m_\infty = 0.012$) and $m_\infty = 0.0168$ (extrapolated from *Large and Pond*, 1981) indicate that *Large and Pond* (1981) extrapolated values of drag coefficient for high winds (> 33 m s^{-1}) are not justified. The modelling of hurricanes in the recent interesting paper by *Bell et al*. (2012) supports the latter conclusion. Thus we come to the conclusion that Charnock parameterization of roughness is too simplified (m = constant) and data presented here shows that the range of its variability is described by Fig. (1a, b). Therefore we can turn again attention to generalization of Charnock formula (16).

6 Discussions

It is rather common nowadays to consider the sea surface roughness parameter as part of the models of Atmospheric Boundary Layers above the ocean, which is needed to calculate the fluxes of heat and moisture through the air-sea interface. In this paper, I suggest for the first time to consider it as the part of the general similarity theory for wind waves, when roughness parameter is considered as one of the independent governing parameters in defining the processes of wind wave growth. So the results of such approach must be the forecasting together with wind waves characteristics also a value of roughness parameter of the sea surface and its variability. Dependence of roughness of the sea surface on the characteristics of wind wave field is demonstrated here both theoretically and empirically. All previous results in this direction were analysed and very often revisited. Thus this paper continues the author's effort to explain the unified approach to the description of most difficult parts of ocean-atmosphere interactions – physics of air-sea interaction. Together with calculations of gas transfer between ocean and atmosphere this paper gives some foundations for a serious physical basis to establish the desired relationship between characteristics of wind waves and fluxes of momentum and energy across air-sea interface.

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