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A Note on the Variability of the Heights of Tidal Benthic Boundary Layers

S.A. Kitaigorodskii

Department of Earth and Planetary Sciences
The Johns Hopkins University
Baltimore, MD 21218

Abstract

The benthic boundary layer (BBL) is the fluid layer adjacent to the sea bottom. In the past the BBL has been modelled as an Ekman layer. However, as was recently shown by the author (Kitaigorodskii, 1988, Kitaigorodskii and Joffre, 1988) in presence of imposed background stable stratification the Ekman boundary layer model failed and the height of BBL is determined by the scale $L_N = u_*/N$, where u_* is the friction velocity based on the bottom stress and N is the buoyancy frequency, which is chosen to characterize the background stratification (can be considered also as initial stratification). Here we apply this idea to the vertical mixing in estuarine system in presence of strong tides. Some estimates relevant to the description of the variability of heights of BBL in such conditions are presented also.

The purpose of this note is to show that the benthic boundary layer (BBL) generated by tidal waves can be strongly influenced by the presence of imposed background stable stratifications. Since the thickness of BBL is much less than the wavelength the model for laminar BBL in Prandtl approximation can be written as

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial z^2} = \frac{\partial U_\infty}{\partial t} \tag{1}$$

where U_∞ is free stream velocity which in the case of tidal wave on finite depth can be presented as

$$U_\infty = V_o \cos(kx - \sigma t) \tag{2}$$

$$V_o = \frac{a\sigma}{shkD} \tag{3}$$

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here a is wave amplitude, k and σ are correspondingly the wave number and frequency, ν is kinematic viscosity.

The thickness of laminar benthic boundary layer in such case is given by the usual expression

$$\delta = \left(\frac{2\nu}{\sigma}\right)^{1/2} = \left(\frac{\nu T}{\pi}\right)^{1/2} \qquad (4a,b)$$

where T is wave period.

Let us underline here the fact that this definition of boundary layer is based on the attenuation of the rotational part of the velocity field in $e^{-1.0} \approx 0.36$ from its value at the bottom (*Phillips*, 1977). Such definition of the boundary layer thickness is based on the assumption that the vortical boundary layer flow can be to the first order simply be superimposed on the irrotational motion.

It is well known since the work of *Collins* (1963) that in natural conditions BBL is always turbulent (even with relatively small tidal amplitudes). Since the period of tidal wave T and the characteristic length $\lambda = 2\pi/k$ exceeds the typical time and length scales of turbulence in the BBL, the tidal motion can be considered as a continuous sequence of stationary realizations for the BBL, or, at least the parameters of BBL can be treated as slowly varying functions of time and position (compare with T and $\lambda = 2\pi/k$). This justifies the approach when to estimate the height of a turbulent benthic boundary layer effective value for eddy viscosity K_e can be introduced in such a way that instead of (3) we can write

$$h = \left(\frac{2K_e}{\sigma}\right)^{1/2} = \left(\frac{K_e T}{\pi}\right)^{1/2} \qquad (5a,b)$$

For effective turbulent viscosity K_e in shear driven tidal BBL we can use the simplest expression (based on dimensional considerations).

$$K_e = K_e(u_*,h) = constant \cdot u_* h = a_1 u_* h \qquad (6)$$

where a_1 is a numerical constant, supposedly of order one. With (6) the expression (5a,b) can be rewritten as

$$h = 2a_1 L_\sigma \qquad (7)$$

where the scale L_σ is defined as

$$L_\sigma = \frac{u_*}{\sigma} \tag{8}$$

The scaling length L_σ is a fundamental estimate for the thickness of turbulent BL generated by periodic tidal motion. The application of the expression (6), and therefore the scale L_σ to the estimates of the thicknesses of benthic boundary layers need first of all its comparison with internal Ekman scale $L_e = \frac{U_*}{\Omega}$. Since as a rule, for semidiurnal and diurnal tides

$$\frac{L_e}{L_\sigma} = \frac{\sigma}{\Omega} \cong 1 \tag{9}$$

We can come to the very important conclusion that BBL generated by tidal motion in neutrally stratified environment can in principle reach Ekman height. This means that rotation can prevent further growth of shear generated tidal BBL (but possibly rotation is not important for small scale three dimensional turbulence, which determines the value of K_e).

For semidiurnal tides $\sigma = 0.00014\ s^{-1}$ and $\frac{\sigma}{\Omega} = 1.4$, and according to (7)

$$h = 2a_1 L_\sigma = 2a_1\frac{u_*}{\sigma} = 1.42 a_1 L_e \tag{10}$$

Compare this expression with *Charney's* (1969) prediction of value $\lambda = 0.2$ in the expression $h = \lambda L_e$ we find that

$$a_1 \approx 0.14 \tag{11}$$

This value is very different from the value of universal von Karman constant $\kappa \cong 0.4$ in the expression

$$K = \kappa u_* z \tag{12}$$

which can be applied successfully only for constant flux region of Ekman boundary layer. However it is evident that the average value of K according to (12) in the boundary layer $h (h >> z_0, z_0$ - roughness parameter) is very close to its effective value K_e, since $\frac{\kappa}{2}$ is close to a_1. Thus in neutral conditions it is quite appropriate to neglect the effect of rotation on the value of effective vertical turbulent viscosity K_e for BBL.

A typical range of values of u_* in tidal flows with different bottom roughness conditions are 0.7-4 cm/s (*Kagan*, 1968) which gives the heights h of the tidal BBL according to (10, 11) in the range 14-80 m. The value of $u_* = 2.3$ cm/s (the average between 0.7 and 4 cm/s) gives the height of BBL $h \approx 46$ m, which is comparable with the scale L_e.

However, very recent observations of BBL and turbulence close to sea bottom in the Norwegian Sea (*Nabatov and Ozmidov*, 1987), and also the numerical simulation of the formation of BBL in conditions characteristic for estuaries, including the Baltic Sea (initially stable stratification close to two layer structure) (*Rahm* and *Svensson*, 1989), demonstrate quite convincingly that the thickness of the well mixed turbulent region close to the bottom of the sea is sufficiently less than given by the scales L_σ or L_e (!) (according to cited above references the height of BBL in presence of stable stratification, rarely exceeds 10 m!). Following recent works by the author (*Kitaigorodskii*, 1988) it can be explained in general terms by taking into account the effect of initial stable stratification, which can prevent the effective growth of BBL even before rotation can become important as another concurring stabilizing factor. The most simple way to estimate the effect of initial stable stratification on the height of BBL is to use the natural condition about the existence of the limiting value of flux Richardson number Rf_h in the entrainment zone. The basis for the estimate of the flux Richardson number Rf_h is the following simple formula for the entrainment flux (*Kitaigorodskii*, 1988; *Kitaigorodskii and Joffre*, 1988)

$$Q_h = -K_h N^2 \tag{13}$$

where K_h is the effective turbulent diffusivity responsible for the overall entrainment process (*Kitaigorodskii*, 1988; *Kitaigorodskii and Joffre*, 1988). Thus the influence of background stratification appears as stabilizing buoyancy flux. The above mentioned condition for Rf_h can be written as

$$Rf_h = -\frac{Q_h}{u_*^2 \frac{du}{dz}} \leq Rf_{\lim} \tag{14}$$

To proceed further we need more detailed characteristics of turbulence involved in the growth of BBL. The simplest assumption is still that rotation is not important in determining the shear $\frac{du}{dz}$ in (14), so that we can write

$$\frac{du}{dz} \sim \frac{u_*^2}{K_h} \tag{15}$$

and use for K_h the most reliable value-effective turbulent viscosity K_e in (6)

$$K_h \approx K_e = a_1 u_* h \tag{16}$$

However the formula (15) is written with the accuracy of the unknown factor of proportionality. This does not permit to find the exact expression for h, even when Rf_{lim} is known. Nevertheless formulae (15-16) lead to the following results

$$h = bL_n \ ; \ L_N = \frac{u_*}{N} \ ; \tag{17}$$

where the scaling length L_N corresponds to the scale of the well mixed region developed by upward diffusion of turbulent energy in the presence of background stable stratification and b is a numerical coefficient. Its value can be estimated from the expression

$$b \approx \sqrt{\frac{Rf_{\text{lim}}}{a_1}} \tag{18}$$

For the range of $Rf_{lim} \approx$ 0.1-0.15 and value of $a_1 \simeq$ 0.14 we can get b $\approx$ 2-3. From the other hand, the empirical findings, based on the data from stratified atmospheric boundary layers (*Kitaigorodskii*, 1988) gives about <u>twice</u> larger values of b (see Fig. 2 in *Kitaigorodskii*, 1988, and Fig. 4 in *Kitaigorodskii and Joffre*, 1988), where the value of the function $Y_+(\mu_N)$ introduced in *Kitaigorodskii* (1988)

$$h = L_N Y_+(\mu_N); \ \mu_N = \frac{L_N}{L_*} \tag{19}$$

for $\mu_N \leq$ 5.0 are about $Y_+ (\mu_N) \approx const$ = 5 - 10 (!). So it means that assumptions (17, 18) can give the right values of the height of BBL (comparable with its atmospheric analog-sheer driven ABL in presence of initial stratification), only if in the expression (15) we include a constant of proportionately close to 4-5! It means may be that (15, 16) are oversimplifications, compared with more detailed calculations done in *Kitaigorodskii and Joffre* (1988). The explanation of this was also done recently by the author (*Kitaigorodskii*, 1991). Now let us return to the application of the scale L_N to the estimates of the heights of BBL generated by tidal waves. This first of all needs its comparison with the scale $L_\sigma = u_*/\sigma$. The expression (17) can be rewritten as

$$h_{\max} = bL_\sigma \left(\frac{\sigma}{N}\right) \tag{20}$$

and since as a rule for semidiurnal and diurnal tides $\sigma/N << 1$, we can conclude that in presence of imposed stable stratification the BBL generated by tidal motion never reaches equilibrium on the typical Ekman scale $L_e \approx L_\sigma$, but rather achieves a quasi-equilibrium thickness characterized as $h_N = bu_*/N$(!). This is a very important conclusion showing that the depth of BBL can be much smaller than the few tens of meters. Fig. 7b in *Rahm and Svensson* (1989) demonstrates that for values of N/Ω = 28-88 the quasi equilibrium height of BBL approach the values 4-10 m in about 20 hours. It is interesting that for the same values of N/Ω the atmospheric boundary layer, determined by the scale L_N (the value of b in (17) is only twice smaller for atmospheric data), have the height in the range from 50 to 200 m (see Fig. 4 in *Kitaigorodskii and Joffre*, 1988). This is of course natural because, with not a big differences in values of N, the typical ratio of friction velocities in atmospheric and benthic boundary layers are equal at least 20 (!). So the results, presented in *Kitaigorodskii* (1988), *Kitaigorodskii and Joffre* (1988), seem to be in agreement with *Rahm and Svensson* (1989), as well as with direct observation of h in Norwegian Sea (*Nabatov and Ozmidov*, 1987). Therefore the parameterization of the buoyancy flux Q_h (according to *Kitaigorodskii* (1988) and *Kitaigorodskii and Joffre* (1988)) is relevant and together with conditions for flux Richardson number Rf_h it gives a rather good description of experimental data (but only for $\mu_N < 5$). For larger values of μ_N (smaller values of N and thus higher entrainment rates) the picture described by (17, 18) is probably too simplistic (see *Kitaigorodskii*, 1991).

The other way of using Richardson number criteria was also suggested in *Kitaigorodskii* (1988). It is simply to take in (15),

$$\frac{du}{dz} \propto \frac{U_\infty}{h} \tag{21}$$

The latter expression also contains an unknown numerical factor of proportionality and thus can not be quite representative for the process of growth of BBL due to entrainment.

Nevertheless using (21) instead of (15) we can write (14) by using (2, 3) and (17, 18) as

$$h = bL_N \left(\frac{a\sigma}{u_* shkD}\right)^{1/2} = bL_\sigma \left(\frac{\sigma}{N}\right) \left(\frac{a\sigma}{u_* shkD}\right)^{1/2} \tag{22}$$

or

$$h = bL_N \left(\frac{a}{L_\sigma}\right)^{1/2} (shkD)^{-1/2} \tag{23}$$

The latter expression demonstrates that the ratio of tidal wave amplitudes to the scale of the benthic boundary layer (L_σ or L_e) are important in determination of its thickness, as well as the type of tidal waves propagating in shelf areas, which determine the ratio kD.

Nevertheless, using the classical quadratic law with effective drag coefficient C_f, defined as $C_f^{-1/2} = \frac{U_\infty}{u_*}$ we can rewrite (14) together with (21) as

$$h = bL_N (C_f)^{-1/4} \tag{24}$$

For values of $C_f \approx 0.003$ this leads to

$$h \cong 4.2\, bL_N \tag{25}$$

This indicates that to have boundary layer thickness in agreement with observations of *Kitaigorodskii and Joffre* (1988) $h \approx (4\text{-}10)L_N$ we must use the value of b according to (18). However, since both expressions (15, 21) include an unknown proportionality factor, this demonstrates that the weakest point in the arguments with overall Richardson number criteria in entrainment zone (14) is a determination of K_h and $\frac{du}{dz}$. Nevertheless, if we follow the same line of analysis as in *Kitaigorodskii* (1988) we can simply write that the thickness of an equilibrium BBL in presence of imposed stable stratification is governed only by two scales L_N and L_σ, so that

$$h = L_N Y(\mu_\sigma) \tag{26}$$

where

$$\mu_\sigma = \frac{L_N}{L_\sigma} = \frac{\sigma}{N} \tag{27}$$

The two asymptotic expressions for $Y(\mu_\sigma)$ will correspond to $\mu_\sigma \to \infty$ (neutral BBL) when $h \sim L_\sigma$ and $\mu_\sigma \to 0$ when $h \sim L_N$.

The application of these ideas to the determination of the location of thermal shelf fronts, produced by tidally generating BBL see in *Kitaigorodskii* (1991).

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