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ELECTROMAGNETIC INDUCTION IN THE EARTH BY AN ELECTROJET CURRENT SYSTEM HARMONIC IN TIME AND SPACE

by

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Abstract

This paper deals theoretically and numerically with electromagnetic induction in the earth caused by a current system consisting of a horizontal straight line current (*i.e.* an electrojet) and of vertical currents which start upwards from the electrojet. The time dependence and the space dependence in the direction of the electrojet are harmonic. The latter dependence is described by a so-called longitudinal propagation constant q. The role of the vertical currents is to prevent electric charges from accumulating at the electrojet. The earth is regarded as a half-space with a flat surface, and it is assumed to be electromagnetically homogeneous.

Rigorous formulas for the total electric field and for the total magnetic (variation) field on the earth's surface are given. The main purpose of this paper is to investigate the influence of q on this electromagnetic field. The investigation is based on three numerical examples. As a general conclusion, all field components approach zero as q increases and are already very small when $q \approx 5 \cdot 10^{-5}\ \mathrm{m}^{-1}$ (corresponding to 126 km). At smaller values of q the absolute values of field components have maxima and minima, three of them being zero when $q = 0$.

The validity of the magnetotelluric plane wave assumption is also considered in this paper. Generally, an increase in q decreases the validity, but there exist clear exceptions from this simple behaviour.

1. *Introduction*

The simplest model of auroral and equatorial electrojet currents for theoretical consideration is obviously an infinitely long horizontal straight line current (see *e.g.* HERMANCE and PELTIER, 1970). In studies of electromagnetic induction in the earth a harmonic time dependence is usually assigned to the electrojet, and the earth is treated as an infinite half-space with a flat surface. The latter assumption implies the local nature of the discussion. The half-space outside the earth is normally considered homogeneous (exclusive of the electrojet) and electromagnetically similar to a vacuum.

PELTIER and HERMANCE (1971) and HIBBS and JONES (1973; 1976) have previously considered extensions of line currents to sheet current models, whilst PIRJOLA (1982; 1985) has developed the original time-harmonic line current model in a different manner by also assigning a harmonic space dependence in the direction of the line, *i.e.* a harmonic longitudinal space dependence. The current then behaves as a wave propagating along the line. A generalization of this kind was also included in the discussion by WAIT (1980), although he did not consider a line current explicitly. Due to the equation of continuity necessarily satisfied, the longitudinal space dependence implies the existence of electric charge on the line. This has to be regarded as a disadvantage from the geophysical point of view because such an accumulation of charge is in reality prevented by the high conductivity of the ionosphere (*cf.* PIRJOLA, 1985). Hence, it would seem natural to improve Pirjola's model by assuming that the half-space outside the earth consists of two parts separated by a boundary parallel to the earth's surface: a highly-conducting ionosphere and a poorly-conducting lower atmosphere, and that the line current is situated in the ionosphere.

However, LEHTO (1983; 1984) makes the improvement in a different way; he maintains the electromagnetic homogeneity of the half-space above the earth, but adds vertical currents to the electrojet system which start upwards from the line current and make the divergence of the total current vanish, *i.e.* no charges accumulate. It should be noted that the geomagnetic field is almost vertical in auroral zones. Thus Lehto's model corresponds to an auroral electrojet current system which is completed by including field-aligned currents. In fact, Lehto does not restrict his theoretical formulation to a line current electrojet, but he deals with a sheet current, and even the space and time dependencies are arbitrary, not necessarily harmonic.

In the theoretical discussion of PIRJOLA (1982) the earth is mainly assumed to be composed of homogeneous horizontal layers whose number is arbitrary, but formal extensions to any vertical changes and even to lateral variations perpendicular to the electrojet are also included. Lehto, however, wants to avoid mathematical overload unessential as viewed from the point of the electrojet current system, and confines his treatment to a two-layer-earth model. Nevertheless the final formulas derived by Lehto and expressing the electromagnetic field on the earth's surface are complicated and difficult to be applied in numerical computations. So LEHTO (1983) only considers two simple situations as numerical examples: a time-independent case, and the case of no longitudinal space dependence. In the latter case there are also no vertical currents. The second paper (LEHTO, 1984) is purely theoretical.

In this paper we will, basing on Lehto's equations, discuss cases in which the current system is both time-dependent and varies longitudinally, but these dependencies are assumed to be harmonic. In particular, we will consider the effect of the so-called longitudinal propagation constant, which expresses the longitudinal space dependence, on the values of the electromagnetic field components observed on the earth's surface. For simplicity, the earth is assumed to be homogeneous here. This paper thus corresponds to the treatment in PIRJOLA (1985), but with one important difference: vertical currents are present now.

2. *Theory*

As indicated above, the earth is described as a homogeneous infinite half-space with a flat surface. We adopt a right-handed Cartesian coordinate system with the xy-plane coinciding with the earth's surface and the z-axis pointing downwards into the earth. Let us denote the conductivity, permittivity and permeability of the earth by σ, ϵ and μ, respectively, which are scalars and constant in both time and space.

We further assume that the electrojet current system, which is the primary source in the electromagnetic induction phenomenon, is given by

$$\bar{j}=\bar{j}(r,t) = Je^{i(\omega t-qy)}\,\delta(x)\,(\delta(z+h)\;\hat{e}_y - iq(1-\theta(z+h))\,\hat{e}_z) \tag{1}$$

where J is a complex constant implying the magnitudes and phases of the currents in the system. δ's are Dirac delta functions and θ is a Heaviside step function. The whole current system is thus situated in the plane $x = 0$, and the horizontal part of it lies there on the line $z = -h$ while the vertical currents occupy the half-plane $z \leqslant -h$. The height h is positive. We assume the angular frequency ω positive, and the longitudinal propagation constant q is real and non-negative. (Obviously, however, the treatment of the present problem would be straightforward also for $\omega \leqslant 0$ or $q < 0$, and if a Fourier synthesis with respect to ω and q is performed, a half-plane of the ωq-plane has to be included.) As the unit vector $\hat{e}_y$ expresses the direction of the electrojet, it should most probably point to the east or west, though in the present theoretical discussion it may have any horizontal direction. The parameter q could, of course, also be called a wave number or a space (angular) frequency.

Equation (1) is a special case of formula (8) of LEHTO (1983) or (4) of LEHTO (1984). Lehto also discusses the borders of the current system, but according to his conclusions they need not be taken into account and equation (1) is used directly. As pointed out already above, the divergence of $\bar{j}$ is zero.

Let the conductivity, permittivity and permeability of the upper half-space, called the air, be σ_0, ϵ_0 and μ_0, which are scalars and constant in both time and space. In principle, ϵ_0 and μ_0 need not equal the corresponding vacuum quantities, but of course, it is reasonable to give them these values (*cf.* Section 3.1).

With these assumptions and notations the electric field $\bar{E}$ and the magnetic (variation) field $\bar{B}$ observed on the earth's surface have the following expressions:

$$E_x(x,y,t) = -\frac{\omega\mu_0 q(k^2-k_0^2)\eta^2\eta_1^2 J e^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{\frac{b}{\varkappa_0}e^{-\varkappa_0 h}\sin bx}{A}\,db, \tag{2}$$

$$E_y(x,y,t) = \frac{i\omega\mu_0\eta^2\eta_1^2 J e^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{\left(k^2\varkappa_0\left(1+\frac{\mu k_0^2\varkappa}{\mu_0 k^2\varkappa_0}\right)-\frac{q^2}{\varkappa_0}(k^2-k_0^2)\right)e^{-\varkappa_0 h}\cos bx}{A}\,db, \tag{3}$$

$$E_z(x,y,t) = \frac{\omega\mu_0 q k^2\eta^2\eta_1^2 J e^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{\left(1+\frac{\mu_0\varkappa}{\mu\varkappa_0}\right)e^{-\varkappa_0 h}\cos bx}{A}\,db, \tag{4}$$

$$B_x(x,y,t) = \frac{\mu_0 k_0^2\eta^2\eta_1^2 J e^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{\left(\left(1+\frac{\mu_0\varkappa}{\mu\varkappa_0}\right)k^2-\left(1+\frac{\mu_0 k^2\varkappa}{\mu k_0^2\varkappa_0}\right)b^2\right)e^{-\varkappa_0 h}\cos bx}{A}\,db, \tag{5}$$

$$B_y(x,y,t) = \frac{i\mu_0 q k_0^2\eta^2\eta_1^2 J e^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{b\left(1+\frac{\mu_0 k^2\varkappa}{\mu k_0^2\varkappa_0}\right)e^{-\varkappa_0 h}\sin bx}{A}\,db, \tag{6}$$

and

$$B_z(x,y,t) = \frac{\mu_0 k^2 \eta^2 \eta_1^2 J e^{i(\omega t - qy)}}{\pi} \int_0^\infty \frac{b\varkappa_0 \left(1 + \frac{\mu k_0^2 \varkappa}{\mu_0 k^2 \varkappa_0}\right) e^{-\varkappa_0 h} \sin bx}{A} \, db. \tag{7}$$

These equations can be obtained from formulas (113)–(117), (124), (125), (132)–(137) and (195) of LEHTO (1983) or from formulas (46), (47), (50)–(61) of LEHTO (1984), utilizing also the homogeneity of the earth. The notations involved in equations (2)–(7) are the same as those used by PIRJOLA (1982; 1985), *i.e.*

$$k_0^2 = \omega^2 \mu_0 \varepsilon_0 - i\omega\mu_0\sigma_0, \qquad -\frac{\pi}{4} \leqq \arg k_0 \leqq 0, \tag{8}$$

$$k^2 = \omega^2 \mu\varepsilon - i\omega\mu\sigma, \qquad -\frac{\pi}{4} \leqq \arg k \leqq 0, \tag{9}$$

$$\eta^2 = k_0^2 - q^2, \qquad -\frac{\pi}{2} < \arg\eta \leqq 0 \text{ or } \arg\eta = \frac{\pi}{2}, \tag{10}$$

$$\eta_1^2 = k^2 - q^2, \qquad -\frac{\pi}{2} < \arg\eta_1 \leqq 0 \text{ or } \arg\eta_1 = \frac{\pi}{2}, \tag{11}$$

$$\varkappa_0^2 = b^2 - \eta^2, \qquad 0 \leqq \arg\varkappa_0 \leqq \frac{\pi}{2}, \tag{12}$$

$$\varkappa^2 = b^2 - \eta_1^2, \qquad 0 \leqq \arg\varkappa \leqq \frac{\pi}{2}, \tag{13}$$

and

$$A = q^2 b^2 (k^2 - k_0^2)^2 - \left(\eta_1^2 \varkappa_0 + \frac{\mu}{\mu_0} \eta^2 \varkappa\right)\left(k_0^2 \eta_1^2 \varkappa_0 + \frac{\mu_0}{\mu} k^2 \eta^2 \varkappa\right). \tag{14}$$

Lehto defines the argument ranges of η (or η_0) and η_1 differently, but it does not matter, because only η^2 and η_1^2 occur in the equations for $\bar{E}$ and $\bar{B}$. The subscript M referring originally to the fact that the field is considered on the earth's surface and used in PIRJOLA (1982; 1985) is omitted here because it does not play any role. Equations (2)–(7) are rigorous based on complete Maxwell's equations which include the displacement currents. But in practice, approximations simplifying the formulas are possible. However, in the present numerical computations the rigorous equations can be used as well and so the discussion of approximations is neglected here.

When formulas (2)–(7) are compared to the corresponding equations of PIRJOLA (1982; 1985), which were derived for a time- and space-harmonic line current but without vertical currents, it is seen that B_z is exactly the same in the two cases while the five other components differ. The equality of B_z is not self-evident, for although B_z is affected neither by vertical currents nor by an accumulating charge it is influenced by horizontal earth currents different in the two cases. However, the difference of these earth currents must now be spatially symmetric in such a way that it does not give rise to any B_z component.

3. *Numerical calculations*

3.1 Values of the parameters

The values selected below for the parameters will be the same as used by PIRJOLA (1985), which is a general reference for this whole section and further references can be found there.

The electromagnetic field $\bar{E}$, $\bar{B}$ expressed by equations (2)–(7) with (8)–(14) is a function of many parameters, and an investigation of the effects of all of them would be very laborious in practice. Hence, as mentioned above, we will concentrate upon studying the influence of changes in q on $\bar{E}$ and $\bar{B}$. Thus all other parameters except for q and also for ω and x will be kept constant throughout this paper as follows: J = 100 kA, h = 100 km, $\sigma_0 = 2 \cdot 10^{-14}\,\Omega^{-1}\text{m}^{-1}$, $\epsilon_0 = 8.854 \cdot 10^{-12}$ AsV^{-1}m^{-1}, $\mu_0 = 4\pi \cdot 10^{-7}$ VsA^{-1}m^{-1}, $\sigma = 10^{-2}\,\Omega^{-1}\text{m}^{-1}$, $\epsilon = 5\,\epsilon_0$ and $\mu = \mu_0$.

The current intensity J could also be bigger for an electrojet, but its choice is not very important, because the components of $\bar{E}$ and $\bar{B}$ are linear with respect to J. The assumption that J is real simply means that the phase of the primary horizontal line current is $\omega t - qy$. The value of h probably represents a minimum of realistic electrojet altitudes.

The value of σ_0 is a typical conductivity of the air near the earth's surface. ϵ_0 and μ_0 correspond to vacuum. The value of σ may be regarded as a kind of average conductivity for Scandinavia, and ϵ and μ are usable for »normal» earth. On the other hand, however, ϵ does not have influence on the results in practice.

We will consider three different values for the period T (*i.e.* $2\pi/\omega$) selected from the range of typical geomagnetic variations: 20 s, 3 min and 2 h, and in these cases x will be 100 km, 500 km and 10 km, respectively. The values of q

discussed in this paper are in the range $0...5 \cdot 10^{-5}\,\mathrm{m}^{-1}$, which seems reasonable if we assume that minimum longitudinal scale lengths of electrojets are in the order of a hundred kilometres. It should be noted that ω and q always define a (longitudinal) velocity as ω/q (*cf.* HUTTON, 1972). For *e.g.* $T = 2$ h and $q = 5 \cdot 10^{-6}\,\mathrm{m}^{-1}$ this velocity equals $175\ \mathrm{ms}^{-1}$.

The integrals in equations (2)–(7) are computed using the Fast Hankel Transform after the following formulas are first employed:

$$\sin bx = \sqrt{\frac{\pi bx}{2}}\, J_{1/2}(bx) \tag{15}$$

and

$$\cos bx = \sqrt{\frac{\pi bx}{2}}\, J_{-1/2}(bx) \tag{16}$$

where $J_{\pm 1/2}$ denotes the Bessel function of the first kind (JOHANSEN and SØRENSEN, 1979). This method of computation is much faster and thus more effective than that used in PIRJOLA (1985) and obviously equally accurate (at least) for practically interesting values of the parameters.

3.2 Example 1

Let the period T now equal 20 s and $x = 100$ km. The absolute values (*i.e.* the amplitudes) of the components of the electromagnetic field given by equations (2)–(7) are shown as functions of q in Fig. 1. All components approach zero when q is large, but the behaviours differ at smaller values of q. The absolute values of the components E_x, E_z and B_y, which vanish when $q = 0$, first reach maxima and then decrease with q. The maxima of $|E_x|$ and $|B_y|$ occur at about $q \approx 6 \cdot 10^{-6}\,\mathrm{m}^{-1}$. $|E_z|$ goes to a maximum value already approximately at $q = 10^{-9}\,\mathrm{m}^{-1}$, and the maximum is much higher (≈ 233 V/km), *i.e.* about ten times higher than the top of the frame in the figure. $|B_z|$ seems to diminish monotonically with q, but $|E_y|$ and $|B_x|$ have »weak» minima near $q = 10^{-5}\,\mathrm{m}^{-1}$.

We will compare these results to the discussion of PIRJOLA (1985) in which the primary source does not contain vertical currents. The comparison is made between the present Fig. 1 and Fig. 1 of PIRJOLA (1985), so T and x are the same. The behaviour of the absolute values of the field components as functions of q is in general roughly similar in the two cases; $|B_z|$ is exactly the same (*cf.* Chapter 2). However, significant differences also exist:

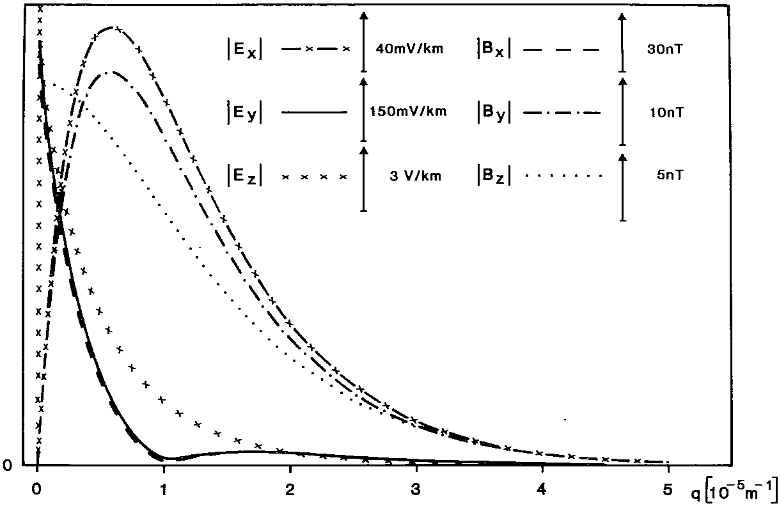

Fig. 1. Amplitudes of the components of the electric ($\bar{E}$) and magnetic ($\bar{B}$) field on the earth's surface as functions of the longitudinal propagation constant q. The period T = 20 s, the distance x = 100 km and the other parameters have the values given in Section 3.1. The maximum of $|E_z|$, which occurs at a very small value of q, is about ten times higher than the top of the figure.

1. In PIRJOLA (1985) approximately the same value of q, *i.e.* 10^{-5}m^{-1}, gives maxima to $|E_x|$, $|E_z|$ and $|B_y|$. This value is larger than the values of q that yield the corresponding maxima in Fig. 1, and expressing it more generally, the changes with q are slower in PIRJOLA (1985) than here.
2. $|E_y|$ and $|B_x|$ do not have any minima in PIRJOLA (1985).
3. The maximum values of $|E_x|$ and $|B_y|$ are considerably bigger while that of $|E_z|$ is very much smaller in Fig. 1 than in PIRJOLA (1985). (The latter conclusion is true also when we take into account the fact mentioned that $|E_z|$ actually goes ten times higher than the top of Fig. 1.) The largest values of the three other components are equal in the two cases because they are achieved at q = 0 where the models are the same.

The observation that $|E_z|$ is reduced to small (and reasonable) values as compared to PIRJOLA (1985) was expectable, since the enormous values obtained there were clearly caused by the ungeophysical primary charge which is now prevented from accumulating by the vertical currents.

The amplitudes of the field components on the earth's surface and their phase shifts with respect to $\omega t - qy$, which is the phase of the primary horizontal electrojet current (see Section 3.1), are given in Table I when $q = 10^{-6}\text{m}^{-1}$. The table

Table I. Amplitudes and phase shifts with respect to the quantity $\omega t - qy$ of the components of the electric ($\bar{E}$) and magnetic ($\bar{B}$) field on the earth's surface when the period $T = 20$ s, the distance $x = 100$ km and the longitudinal propagation constant $q = 10^{-6}\text{m}^{-1}$. The other parameters have the values given in Section 3.1.

Component	Absolute value in V/km or nT	Phase shift in degrees
E_x	0.119	–40.3
E_y	0.738	–134.2
E_z	15.0	180.0
B_x	146.8	1.5
B_y	23.9	–85.9
B_z	28.2	142.3

shows that the phases of the components differ from each other. B_x is the only whose phase is nearly the same as that of the electrojet. E_z has very accurately the opposite phase. According to equation (1) the phase shift of the vertical currents with respect to $\omega t - qy$ is -90°, so B_y oscillates almost in phase with vertical currents. As compared to the phases expressed in the corresponding table of PIRJOLA (1985), the phases of E_y and B_x are approximately and of B_z, of course, exactly the same in these two tables, but the phases of the three other components differ significantly. A comparison between absolute values given in the two tables naturally yields results which support conclusions drawn already above.

3.3 Examples 2 and 3

We now assume that $T = 3$ min and $x = 500$ km. Fig. 2 shows the amplitudes of the electromagnetic field components on the earth's surface as functions of q. Similarly to Fig. 1 $|E_z|$ starts from zero at $q = 0$ and reaches its maximum, which is about 8.6 V/km, *i.e.* almost four times higher than the top of the frame in the figure, already at a very small value of q ($\approx 1.2 \cdot 10^{-10}\text{m}^{-1}$). The behaviour of $|E_y|$ and $|B_x|$ differs from that in Fig. 1: The minima are now deeper and the largest values are obtained at non-zero values of q. The curves for the other components have the same shapes in Figures 1 and 2, but it should be noted that the horizontal scale is bigger in Fig. 2, so the decrease with q is more rapid for $T = 3$ min, $x = 500$ km than for $T = 20$ s, $x = 100$ km. The (vertical) sensitivities of all six components are higher in Fig. 2 than in Fig. 1.

Let us finally set T equal to 2 h and x equal to 10 km. The amplitudes of the field components on the earth's surface are depicted in Fig. 3 as functions of q.

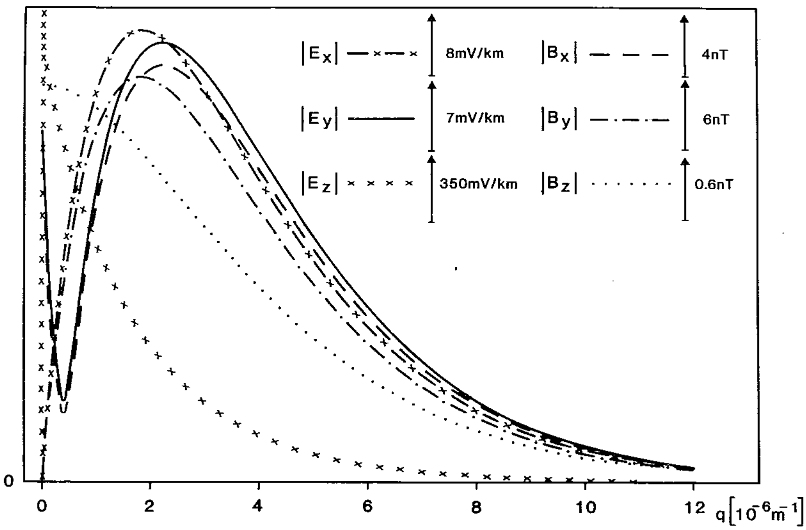

Fig. 2. Amplitudes of the components of the electric ($\bar{E}$) and magnetic ($\bar{B}$) field on the earth's surface as functions of the longitudinal propagation constant q. The period $T = 3$ min, the distance $x = 500$ km, and the other parameters have the values given in Section 3.1. The maximum of $|E_z|$, which occurs at a very small value of q, is almost four times higher than the top of the figure.

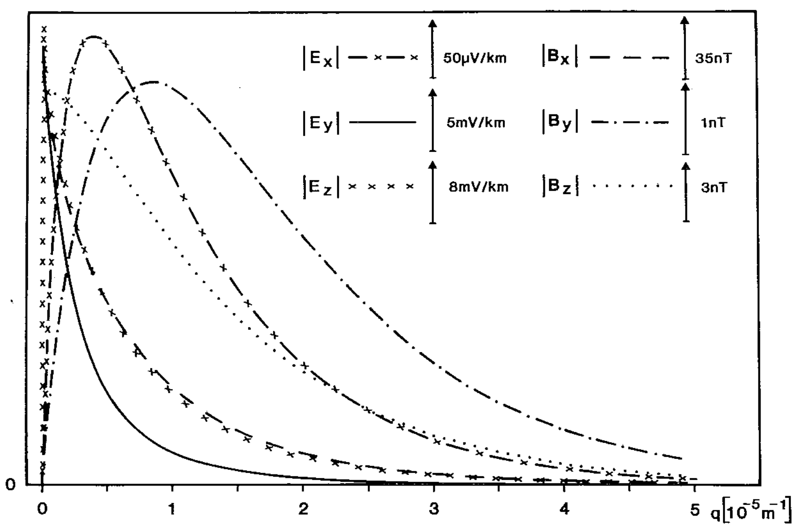

Fig. 3. Amplitudes of the components of the electric ($\bar{E}$) and magnetic ($\bar{B}$) field on the earth's surface as functions of the longitudinal propagation constant q. The period $T = 2$ h, the distance $x = 10$ km, and the other parameters have the values given in Section 3.1.

Also in this case $|E_z|$ is zero when $q = 0$ and gets it maximum value at an extremely small value of q ($\approx 8.1 \cdot 10^{-12} \mathrm{m}^{-1}$), but now the maximum remains inside the frame of the figure. Contrary to Figures 1 and 2, $|E_y|$ and $|B_x|$ do not have any minima in Fig. 3. The horizontal scale is the same in Fig. 3 as in Fig. 1, and the decrease of the components with q is very roughly similar in these two figures. The sensitivities of the electric components and of the magnetic y-component are bigger in Fig. 3 than in Figures 1 and 2. The sensitivity of $|B_x|$ is a little lower and that of $|B_z|$ somewhat higher in Fig. 3 than in Fig. 1.

Similarly to Section 3.2, results shown in Figures 2 and 3 could be compared to the discussion of PIRJOLA (1985), and differences would be found. It is, however, neglected here.

3.4 Plane wave assumption

The »basic equation of magnetotellurics» is expressed as

$$\frac{E_x}{B_y} = -\frac{E_y}{B_x} = \sqrt{\frac{\omega}{\mu_0 \sigma}}\, e^{i\pi/4} \ . \tag{17}$$

(CAGNIARD, 1953, p. 616). The notations are the same as above, the E- and B-quantities being measured on the flat surface of a homogeneous half-space earth characterized by σ and μ_0. (If the conductivity changed spatially, σ should be replaced by a so-called apparent conductivity and the phase angle would depend on the frequency in the basic equation above.) Formula (17) can easily be derived by assuming that the electromagnetic field depends inside the earth solely on the vertical space coordinate z and on time. This is exactly the situation if the primary field of the induction phenomenon is a harmonic plane wave vertically incident on the earth. For some further discussion PIRJOLA (1985) is referred to here.

We will now consider the validity of equations (17), *i.e.* the validity of the »plane wave assumption», when the primary field is not a plane wave but the field produced by the current system (1). The field components that will be discussed are thus given by formulas (2), (3), (5) and (6). It would seem natural that a decrease of the parameter q makes the field components better satisfy formulas (17), since the larger q the more important the horizontal ($y-$) variation of the primary field, *i.e.* the bigger the difference between the behaviour of the primary field and that of a vertical plane wave. However, even for $q = 0$ the primary field is not horizontally constant but varies with x.

Let T = 20 s and x = 100 km. Table II gives the absolute values (amplitudes)

Table II. Amplitudes of the horizontal electric field components E_x and E_y on the earth's surface for different values of the longitudinal propagation constant q. The period $T = 20$ s, the distance $x = 100$ km. The other parameters have the values given in Section 3.1. $|E_x(pw)|$ and $|E_y(pw)|$ denote the amplitudes that are obtained when the plane wave assumption is made. Relative errors occurring if $|E_x(pw)|$ and $|E_y(pw)|$ are substituted for $|E_x|$ and $|E_y|$, respectively, are also given.

q in 10^{-7}m^{-1}	$\|E_x\|$ in V/km	$\|E_x(pw)\|$ in V/km	Error in %	$\|E_y\|$ in V/km	$\|E_y(pw)\|$ in V/km	Error in %
0	0	0	0	$9.995 \cdot 10^{-1}$	$9.959 \cdot 10^{-1}$	0.36
1	$1.431 \cdot 10^{-2}$	$1.432 \cdot 10^{-2}$	0.052	$9.689 \cdot 10^{-1}$	$9.653 \cdot 10^{-1}$	0.37
5	$6.592 \cdot 10^{-2}$	$6.595 \cdot 10^{-2}$	0.052	$8.576 \cdot 10^{-1}$	$8.540 \cdot 10^{-1}$	0.42
10	$1.194 \cdot 10^{-1}$	$1.195 \cdot 10^{-1}$	0.053	$7.378 \cdot 10^{-1}$	$7.342 \cdot 10^{-1}$	0.49
50	$2.856 \cdot 10^{-1}$	$2.858 \cdot 10^{-1}$	0.053	$2.065 \cdot 10^{-1}$	$2.028 \cdot 10^{-1}$	1.8
100	$2.403 \cdot 10^{-1}$	$2.405 \cdot 10^{-1}$	0.060	$1.887 \cdot 10^{-2}$	$1.097 \cdot 10^{-2}$	41.9
150	$1.558 \cdot 10^{-1}$	$1.560 \cdot 10^{-1}$	0.11	$3.145 \cdot 10^{-2}$	$3.156 \cdot 10^{-2}$	0.35
200	$9.121 \cdot 10^{-2}$	$9.141 \cdot 10^{-2}$	0.23	$2.841 \cdot 10^{-2}$	$2.894 \cdot 10^{-2}$	1.9
350	$1.417 \cdot 10^{-2}$	$1.440 \cdot 10^{-2}$	1.6	$6.461 \cdot 10^{-3}$	$6.774 \cdot 10^{-3}$	4.8
500	$1.865 \cdot 10^{-3}$	$1.972 \cdot 10^{-3}$	5.7	$9.618 \cdot 10^{-4}$	$1.065 \cdot 10^{-3}$	10.7

of E_x and E_y calculated using formulas (2) and (3) with ten different values of q. $|E_x(pw)|$ and $|E_y(pw)|$ denote the amplitudes that are obtained from equations (17) when B_y and B_x have their correct values computed with formulas (5) and (6). In formulas (17) σ, of course, then has the same value $10^{-2}\Omega^{-1}\text{m}^{-1}$ as above, and $\mu_0 = 4\pi \cdot 10^{-7}$ VsA^{-1}m^{-1}. (If the permeability of the earth differed from μ_0, the right-hand side of (17) should be multiplied by $\sqrt{\mu/\mu_0}$.) The relative errors which are made if $|E_x|$ and $|E_y|$ are replaced by $|E_x(pw)|$ and $|E_y(pw)|$ are also given in Table II.

According to Table II the plane wave assumption seems to be well acceptable for $|E_x|$ and $|B_y|$ when $q \lesssim 5 \cdot 10^{-5}\text{m}^{-1}$, and as expected the error increases with q. But concerning $|E_y|$ and $|B_x|$ the situation is more complex: The plane wave assumption can be made within error limits of about 10 % if the value of q is not close to 10^{-5}m^{-1}; thus values in the vicinity of the minima of $|E_y|$ and $|B_x|$ do not satisfy equation (17). The error shown in the last column of Table II is not a monotonic function of q, so the straightforward guess made above that a decrease of q improves the validity of the plane wave assumption is not completely true. It should be noted here that the drawing accuracy in Fig. 1 is not sufficient to enable one to draw the conclusions seen in Table II; in Fig. 1 $|E_y|/|B_x|$ seems roughly independent of q making the error also independent of q.

We considered here only absolute values of the quantities occurring in formulas (17). A thorough investigation of the validity of these equations would, of course, require discussion of arguments, too. For brevity, we will do it only for one particular combination of the parameters T, x and q: 20 s, 100 km, $10^{-6}\mathrm{m}^{-1}$. Table I yields then the following arguments to the ratios E_x/B_y and $-E_y/B_x$: 45.6° and 44.3°. Thus taking also into account the results given in Table II, formulas (17) are well satisfied with respect to both absolute values and arguments (at least) for the particular values of the parameters mentioned above.

In the case T = 3 min, x = 500 km the errors are roughly comparable to those expressed in Table II, and a maximum occurs in the error as concerns $|E_y|$ and $|B_x|$. But when T = 2 h and x = 10 km the errors are much bigger and the plane wave assumption is totally invalid. This is understandable because an increase in the period decreases the validity of the plane wave assumption (WAIT, 1954). The large errors are evidently also contributed by the small value of x, since generally the closer the primary source the greater the spatial variation of the primary field (*cf.* PIRJOLA, 1985). Comparing the cases (20 s, 100 km) and (3 min, 500 km) the effects of T and x obviously compensate each other.

4. *Concluding remarks*

This paper deals theoretically and numerically with electromagnetic induction in the earth by an electrojet current oscillating harmonically in time and in the longitudinal space coordinate. To avoid accumulation of charge at the electrojet, the model is completed by including vertical currents which make the divergence of the whole primary current system vanish. Since the »return» currents are vertical, the model is suitable for an auroral rather than for an equatorial electrojet. The earth is assumed to be a half-space with a flat surface. For simplicity the earth is electromagnetically homogeneous, which is a rough approximation and should be neglected in future studies. Also the effect caused by changing the value of the conductivity of a homogeneous earth should be investigated later in detail. It is not clear which value is the most reasonable to be used for a real earth, and the choice, of course, depends on the frequency discussed. The numerical results of this paper, which concern the electromagnetic variation field observed on the earth's surface, are based on formulas (2)–(7) rigorously derivable from Maxwell's equations.

The main purpose of this paper is to demonstrate the influence of longitudinal changes of the electrojet, which are described by the longitudinal propagation constant or wave number q, on the field on the earth's surface. Most of the other parameters are kept constant in the computations, and three different combinations of the period T of the time oscillation and the horizontal distance x from the electrojet are used: (20 s, 100 km), (3 min, 500 km) and (2 h, 10 km). In general, all field components approach zero as q grows and are very small already at $q \approx 5 \cdot 10^{-5} m^{-1}$, which corresponds to a wave length of about 126 km. (This is the largest value of q discussed in this paper.) At smaller values of q maxima and minima occur in the absolute values of field components. The electric components perpendicular to and the magnetic component parallel to the electrojet vanish when $q = 0$.

The validity of the plane wave assumption utilized normally in magnetotelluric studies is also discussed in this paper. As a general conclusion, an increase of q decreases the validity, but there seem to exist special values of q for which the error caused by this assumption is large, and the error again turns small at bigger values of q. If $T = 2$ h and $x = 10$ km the error is very large for all values of q, so the plane wave assumption is completely unacceptable.

Studies of harmonic time and space dependencies can easily be extended to arbitrary dependencies by Fourier synthesis procedures (see Chapter 2). *E.g.* a model consisting of a horizontal electrojet line current of finite length L_e and of vertical currents at its ends could be considered later; π/L_e would probably be a characteristic value of q then.

As a different future study and as an interesting comparison to the present results, the model of this paper could be modified by neglecting the vertical currents but assuming that the horizontal electrojet is situated in a highly-conducting ionosphere that lies above a poorly-conducting air (*cf.* Chapter 1). To keep the theoretical treatment easy enough, it is obviously convenient (at least to begin with) to assume that the conductivity of the ionosphere is isotropic and does not vary in space which are idealizations of reality.

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