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ELECTROMAGNETIC INDUCTION IN THE EARTH BY A LINE CURRENT HARMONIC IN TIME AND SPACE

by

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Abstract

This paper deals theoretically with electromagnetic induction in the earth caused by a horizontal straight line current situated above the flat surface of the earth, which is assumed to be electromagnetically homogeneous. The primary current oscillates harmonically with time, and also has a harmonic space dependence along the line, which is expressed by a longitudinal propagation constant q. The space dependence physically implies the existence of a primary charge, too. Rigorous formulas for the total electric field and for the total magnetic (variation) field on the earth's surface are given.

The purpose of this paper is to investigate numerically the effect of q on the electromagnetic field on the earth's surface. The electric component parallel to the primary current, the horizontal perpendicular magnetic component and the vertical magnetic component are discussed in greater detail. The other three components vanish when $q = 0$, and are probably affected more by the above-mentioned primary charge, whose existence is a drawback from the geophysical point of view. The vertical electric component seems to get very large values, and a rough method to reduce the effect of the primary charge on it is presented. The investigation of the influence of q is made by three numerical examples. As a general conclusion, all field components approach zero as q increases. The farther away the point of observation from the primary source the more rapid this decrease of the field.

The validity of the magnetotelluric plane wave assumption and phase differences between magnetic field components are also discussed.

1. *Introduction*

In a previous paper (PIRJOLA, 1982) three theoretical models for the primary field of electromagnetic induction in the earth were discussed: 1) a harmonic plane wave propagating vertically downwards, 2) the field caused by an infinitely long horizontal straight line current which oscillates harmonically in time and is situated above the earth's surface, 3) the field caused by a similar line current which also has a harmonic space dependence along the line. In the third model the current thus behaves as a harmonic wave propagating along the line. The earth was described as an infinite half-space with a flat surface implying that the treatments are applicable to local induction studies. The electromagnetic properties of the earth were assumed to be isotropic and linear and also mainly laterally constant and piecewise constant in the vertical direction, but formal extensions to arbitrary vertical changes and in the third model even to lateral variations perpendicular to the source current were included. In the latter case it was, however, necessary to assume that all lateral changes vanish at the earth's surface. Formulas for the total electric field and the total magnetic (variation) field on the earth's surface were derived basing rigorously on Maxwell's equations and boundary conditions. In the last two models for the primary field the final formulas are complicated integrals over a horizontal wave number.

This paper is concentrated upon the line current models mentioned above. After a brief discussion of the theory, numerical results obtained using the rigorous formulas are presented. Conclusions about the effects of the longitudinal space dependence of the primary current are drawn from these results. For simplicity the earth is here assumed to be homogeneous. Numerical calculations based on the plane wave model (the first model above) are presented by PIRJOLA (1984).

Due to the equation of continuity, longitudinal changes (*i.e.* a non-zero divergence) of a current are inevitably accompanied by a non-zero time derivative of charge density. Hence a primary line charge also exists as concerns the third model. This is, however, not geophysically acceptable, because the line current (mainly) represents electrojet currents flowing in the ionosphere, the conductivity of which is so high that such an accumulation of net charge seems impossible in practice (*cf. e.g.* RISHBETH and GARRIOTT, 1969, pp. 127 and 234; BOSTRÖM, 1973). The existence of the primary charge has thus to be kept in mind when the results of the present first step in studies of longitudinally-changing source currents are discussed. It may be believed that the effect of the charge is most striking in the horizontal electric field component perpendicular to the source, in the vertical electric field component, and in the parallel magnetic field component, which will therefore be called »ungeophysical», the other three being »geophysical». The ungeophysical ones are the components which vanish if there is no longitudinal space dependence. It will be shown in this paper that the vertical electric component really gets extremely large values; the other ungeophysical components are, however, reasonably small.

The third model in PIRJOLA (1982) was later further developed by adding vertical (*i.e.* almost field-aligned in the auroral zone) currents which start upwards from the present horizontal current and make all primary charges vanish (LEHTO, 1983; 1984). The mathematical formalism used by Lehto was of the same type as that of PIRJOLA (1982). I will present numerical results based on Lehto's model in a subsequent paper. Finally, let us point out that WAIT (1980) deals with a model where the divergence of the primary source is not zero and that thus corresponds to the third model of PIRJOLA (1982). In his discussion Wait does not consider a line current explicitly as he discusses the electromagnetic surface impedance for general excitation.

2. *Theory*

The treatment of this paper concentrates on local induction, and so the earth is described as an infinite half-space with a flat surface. A right-handed Cartesian coordinate system with the xy-plane coinciding with the earth's surface and the z-axis pointing downwards into the earth will be used. Let us assume that the primary source current density is given by

$$\bar{j}=\bar{j}(r,t) = Je^{i(\omega t-qy)}\,\delta(z+h)\,\delta(x)\,\hat{e}_y \tag{1}$$

where J is a complex constant implying the magnitude and the phase of the current. δ's are Dirac delta functions (or distributions). The current is thus horizontal and is situated on the line $x = 0$, $z = -h$. The height h and the angular frequency ω are positive. The longitudinal propagation constant q is also real and assumed to be non-negative. In practice the line should most probably be assumed to have roughly an east-west orientation, though in the present treatment the unit vector $\hat{e}_y$, which implies the direction of the line, can point to any horizontal direction.

Let the conductivity, the permittivity and the permeability of the upper half-space, the air, be σ_0, ϵ_0 and μ_0. They are assumed to be scalars and constants in both time and space, and in principle ϵ_0 and μ_0 need not equal the corresponding free space quantities. It follows from the equation of continuity that a primary charge density

$$\varrho = \varrho(r,t) = \frac{q}{\omega - i\dfrac{\sigma_0}{\varepsilon_0}}\,Je^{i(\omega t-qy)}\,\delta(z+h)\,\delta(x) \tag{2}$$

is associated with the current of formula (1). The charge is situated on the line $x=0$, $z=-h$, too.

We further assume that the earth is also electromagnetically isotropic, linear and homogeneous (see STRATTON, 1941, p. 10) and denote the parameters by σ, ϵ and μ, constant in both time and space. As given on pages 105–106 of PIRJOLA (1982), the electric field $\bar{E}_M$ and the magnetic (variation) field $\bar{B}_M$ then have the following expressions on the earth's surface:

$$E_{Mx}(x,y,t) = -\frac{\omega\mu_0 q\eta^2\eta_1^2 Je^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{b\left(\varkappa_0+\frac{\mu}{\mu_0}\varkappa\right)e^{-\varkappa_0 h}\sin bx}{A}\,db, \tag{3}$$

$$E_{My}(x,y,t) = \frac{i\omega\mu_0\eta^2\eta_1^2 Je^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{\left(\eta_1^2\varkappa_0+\frac{\mu}{\mu_0}\eta^2\varkappa\right)e^{-\varkappa_0 h}\cos bx}{A}\,db, \tag{4}$$

$$E_{Mz}(x,y,t) = -\frac{\omega\mu_0 qk^2\eta^2 Je^{i(\omega t-qy)}}{\pi k_0^2}\int_0^\infty \frac{\left(b^2(k^2-k_0^2)+\frac{\mu_0}{\mu}\varkappa\left(\eta_1^2\varkappa_0+\frac{\mu}{\mu_0}\eta^2\varkappa\right)\right)e^{-\varkappa_0 h}\cos bx}{A}\,db, \tag{5}$$

$$B_{Mx}(x,y,t) = -\frac{\mu_0\eta^2 Je^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{\left(q^2b^2(k^2-k_0^2)+\frac{\mu_0}{\mu}k^2\varkappa\left(\eta_1^2\varkappa_0+\frac{\mu}{\mu_0}\eta^2\varkappa\right)\right)e^{-\varkappa_0 h}\cos bx}{A}\,db, \tag{6}$$

$$B_{My}(x,y,t) = -\frac{i\mu_0 q(k^2-k_0^2)\eta^2\eta_1^2 Je^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{be^{-\varkappa_0 h}\sin bx}{A}\,db, \tag{7}$$

and

$$B_{Mz}(x,y,t) = \frac{\mu_0\eta^2\eta_1^2 Je^{i(\omega t-qy)}}{\pi}\int_0^\infty \frac{b\left(k^2\varkappa_0+\frac{\mu}{\mu_0}k_0^2\varkappa\right)e^{-\varkappa_0 h}\sin bx}{A}\,db, \tag{8}$$

where

$$A = q^2b^2(k^2-k_0^2)^2-\left(\eta_1^2\varkappa_0+\frac{\mu}{\mu_0}\eta^2\varkappa\right)\left(k_0^2\eta_1^2\varkappa_0+\frac{\mu_0}{\mu}k^2\eta^2\varkappa\right), \tag{9}$$

$$k_0^2 = \omega^2\mu_0\varepsilon_0 - i\omega\mu_0\sigma_0, \qquad -\frac{\pi}{4}\leqq \arg k_0 \leqq 0, \tag{10}$$

$$k^2 = \omega^2\mu\varepsilon - i\omega\mu\sigma, \qquad -\frac{\pi}{4} \leqq \arg k \leqq 0, \tag{11}$$

$$\eta^2 = k_0^2 - q^2, \qquad -\frac{\pi}{2} < \arg\eta \leqq 0 \text{ or } \arg\eta = \frac{\pi}{2}, \tag{12}$$

$$\eta_1^2 = k^2 - q^2, \qquad -\frac{\pi}{2} < \arg\eta_1 \leqq 0 \text{ or } \arg\eta_1 = \frac{\pi}{2}, \tag{13}$$

$$\varkappa_0^2 = b^2 - \eta^2, \qquad 0 \leqq \arg\varkappa_0 \leqq \frac{\pi}{2}, \tag{14}$$

and

$$\varkappa^2 = b^2 - \eta_1^2, \qquad 0 \leqq \arg\eta \leqq \frac{\pi}{2}, \tag{15}$$

(There are some misprints in the formulas of $\bar{E}_M$ and $\bar{B}_M$ in PIRJOLA (1982).) It should be noted that equations (3)–(8) are rigorous based on complete Maxwell's equations (*i.e. e.g.* displacement currents were not neglected). In practice, however, certain approximations are permissible, which will simplify the formulas and make the physical contents thus probably more comprehensible. But in the numerical computations of this paper we can use the more complex rigorous equations as well. This accuracy may also increase the possibility to use the results in other electromagnetic studies not connected with geomagnetism.

3. *Numerical calculations*

3.1 Values of the parameters

The electromagnetic field $\bar{E}_M$, $\bar{B}_M$ given by equations (3)–(15) depends on several parameters. Investigations of the effects of all of them would hence require an enormous amount of calculations. The purpose of this paper is, however, to study the influence of the longitudinal space dependence of the primary source on $\bar{E}_M$ and $\bar{B}_M$, *i.e.* the effects of q. Thus the other parameters will not have many different values, and the following values are always assumed: J = 100 kA, h = 100 km, $\sigma_0 = 2 \cdot 10^{-14}\ \Omega^{-1}\mathrm{m}^{-1}$, $\epsilon_0 = 8.854 \cdot 10^{-12}$ As/Vm, $\mu_0 = 4\pi \cdot 10^{-7}$ Vs/Am, $\sigma = 10^{-2}\ \Omega^{-1}\mathrm{m}^{-1}$, $\epsilon = 5\,\epsilon_0$ and $\mu = \mu_0$.

The values of J and h were selected regarding an auroral or equatorial electrojet current (*cf.* *e.g.* ALBERTSON and VAN BAELEN, 1970; HERMANGE and PELTIER, 1970; DUCRUIX *et al.*, 1977). The value of h probably represents a minimum of realistic altitudes. The current strength J could also be bigger in practice, but its choice is not very important, because the components of $\bar{E}_M$ and $\bar{B}_M$ increase and decrease in ratio to J. The assumption that J is real means that the phase of the primary current is $\omega t - qy$. In other words, the source current has no phase shift, and phase shifts of the field components discussed below are phase differences to the phase of the current.

The value of σ_0 represents a typical conductivity of the air near the earth's surface (ISRAËL, 1971, pp. 95 and 248). ϵ_0 and μ_0 are free space quantities. The distribution of the conductivity of the earth in Scandinavia given by JONES (1980) indicates that the value of σ is a kind of an average conductivity for Scandinavia. For the choice of ϵ *e.g.* SARAOJA (1946, p. 123) can be referred to, but on the other hand, ϵ does not have influence on the results in practice. The relative permeability of the earth is set equal to unity basing on statements by RIKITAKE (1966, p. 221) and SCHMUCKER (1970, p. 3).

The periods T (*i.e.* $2\pi/\omega$) can be assumed to be greater than about a second in connection with geomagnetic phenomena (*cf.* *e.g.* PATRA and MALLICK, 1980, pp. 224–230). Reasonable values for x giving the distance between the point of observation and the line on the earth's surface below the primary source roughly range from -2000 km to 2000 km. The components E_{Mx}, B_{My} and B_{Mz} are odd with respect to x, while E_{My}, E_{Mz} and B_{Mx} are even. The values of y and t, which occur only in the sinusoidal term $e^{i(\omega t - qy)}$, need not be specified in the present calculations; changes of y and t merely alter the phases of the components.

The most reasonable values for q are obviously in the range $0 \ldots 5 \cdot 10^{-5}\,\mathrm{m}^{-1}$ if minimum longitudinal scale lengths (*i.e.* $2\pi/q$) of electrojet currents are in the order of a hundred kilometres. (CAMPBELL (1978, p. 1161) even mentions the value 10 km.)

The integrals in equations (3)–(8) are calculated using the »Extended Simpson's Rule» (ABRAMOWITZ and STEGUN, 1970, p. 886).

3.2 Example 1

Let the period T be 20 s and x = 100 km. Fig. 1 shows the absolute values (*i.e.* the amplitudes) of the components of the fields $\bar{E}_M$ and $\bar{B}_M$ as functions of q. The components which do not vanish when q is zero diminish with increasing q. After reaching their maxima the other three components also decrease with q. The peaks occur approximately when $q = 10^{-5}\mathrm{m}^{-1}$.

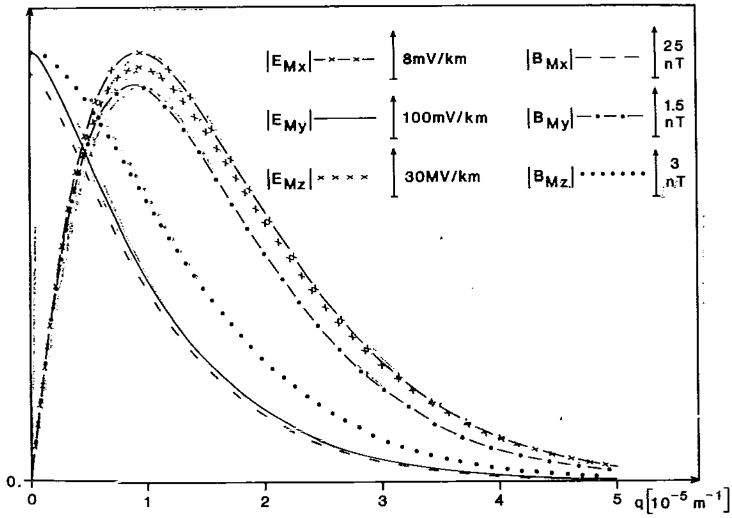

Fig. 1. Amplitudes of the components of the electric ($\bar{E}_M$) and magnetic ($\bar{B}_M$) field on the earth's surface as functions of the longitudinal propagation constant q. The period T = 20 s, the distance x = 100 km and the other parameters have the values given in Section 3.1.

The amplitudes and the phase shifts with respect to $\omega t - qy$ of the field components are given in Table I for $q = 10^{-6}\mathrm{m}^{-1}$.

We observe that the value of $|E_{Mz}|$ is enormous and seems geophysically quite unacceptable, which, as indicated in Chapter 1, is evidently a result of the existence of the primary charge. In reality, accumulation of charge in the ionosphere is prevented by the high conductivity (*cf.* formula (2)). But in the present model the conductivity is constant everywhere above the earth's surface. These arguments

Table I. Amplitudes and phase shifts with respect to the quantity $\omega t - qy$ of the components of the electric ($\bar{E}_M$) and magnetic ($\bar{B}_M$) field on the earth's surface when the period T = 20 s, the distance x = 100 km and the longitudinal propagation constant $q = 10^{-6}\mathrm{m}^{-1}$. The other parameters have the values given in Section 3.1.

Component	Absolute value in V/km or nT	Phase shift in degrees
E_{Mx}	0.0155	90.0
E_{My}	0.976	−134.4
E_{Mz}	$5.57 \cdot 10^{7}$	0.4
B_{Mx}	194.5	1.1
B_{My}	2.8	51.6
B_{Mz}	28.2	142.3

suggest that the effect of the primary charge can roughly be reduced by multiplying $|E_{Mz}|$ of Table I by the ratio of the conductivity $2 \cdot 10^{-14} \Omega^{-1} m^{-1}$ used in the calculations and a realistic value of the ionospheric conductivity, *e.g.* about $10^{-3} \Omega^{-1} m^{-1}$ (BOSTRÖM, 1973). We have to emphasize here that the conductivity of the ionosphere cannot be described by a scalar number but by a tensor. The given value represents the conductivity parallel with the geomagnetic field, *i.e.* the accumulated charges are removed by field-aligned currents. A reduced value in the order of 0.001 V/km is now obtained for $|E_{Mz}|$.

On the other hand, the results given in Table I do not change practically if σ_0 is replaced by zero in the computations, but the coefficient of the reduction would then become zero. A more careful consideration of equation (2) now indicates that the quantity $\omega\epsilon_0$ should rather be used in the numerator of the coefficient of the reduction instead of the assumed conductivity of the air whenever the former has a much larger value than the latter. In the present example $\omega\epsilon_0 = 2.8 \cdot 10^{-12} \Omega^{-1} m^{-1} \gg 2 \cdot 10^{-14} \Omega^{-1} m^{-1}$. The reduction thus gives a value in the order of 0.1 V/km to $|E_{Mz}|$.

It is worth noting here that the vertical fair-weather atmospheric electric field is about 100 V/m and points downwards at the earth's surface (*e.g.* FEYNMAN *et al.*, 1964, p. 9–1). In magnetotelluric studies the vertical electric component is not measured.

The primary electromagnetic field caused by the source expressed in equations (1) and (2) does not have any magnetic y-component (PIRJOLA, 1982, equation (4.60)). Thus B_{My} is totally caused by secondary sources induced in the earth (and in principle slightly also in the conducting air; see PIRJOLA, 1982, p. 5). In this induction the electric z-component plays an important role. The component E_{Mx} is affected by the primary charge and by charges occurring at the earth's surface as a result of vertical currents which are driven by the electric z-component. These conclusions support the assumption indicated in Chapter 1 that the values of E_{Mx} and B_{My} may also be unacceptable from the practical point of view, though their values at least in Table I are reasonable. It seems clear that the other three components E_{My}, B_{Mx} and B_{Mz} are also affected by the existence of the ungeophysical primary change. But they also have an important influence of the geophysical primary current. Thus the ungeophysicality involved in E_{My}, B_{Mx} and B_{Mz} certainly has a minor effect as compared to that included in E_{Mx}, E_{Mz} and B_{My}. (In fact, it will be shown in a subsequent paper that the formula for B_{Mz} (8) is exactly the same if the accumulating primary charge is replaced by vertical currents.)

Table I shows that the phases of the field components differ; however, E_{Mz} and B_{Mx} have nearly the same phase which also roughly equals the phase of the primary current. Because $\omega\epsilon_0 \gg \sigma_0$ the primary charge approximately has the same phase, too. The phase shifts φ given in degrees are often convenient to express in units of time ($= \varphi_t$) which can be made by the formula: $\varphi_t = T\varphi/360°$. We will discuss phase differences more in Section 3.5.

3.3 Examples 2 and 3

We now assume that $T = 3$ min and $x = 500$ km. Fig. 2 shows the amplitudes of the components E_{My}, B_{Mx} and B_{Mz} as functions of q. The other three components are disregarded because of their ungeophysical nature.

The value of $|E_{Mz}|$ would again be very large. The curves of Fig. 2 decrease with q much more rapidly than those of Fig. 1, and also the maxima of the neglected components would occur at smaller values of q.

Let us finally set T equal to 2 h and x equal to 10 km. The amplitudes of E_{My}, B_{Mx} and B_{Mz} are depicted in Fig. 3 as functions of q. Contrary to the above examples $|E_{My}|$ and $|B_{Mx}|$ have maxima, which occur approximately when $q = 1.5 \cdot 10^{-5}\mathrm{m}^{-1}$ and $q = 3.0 \cdot 10^{-6}\mathrm{m}^{-1}$, respectively. The decrease as q increases is slower in Fig. 3 than in Figures 1 and 2 (with some exception as concerns $|B_{Mz}|$

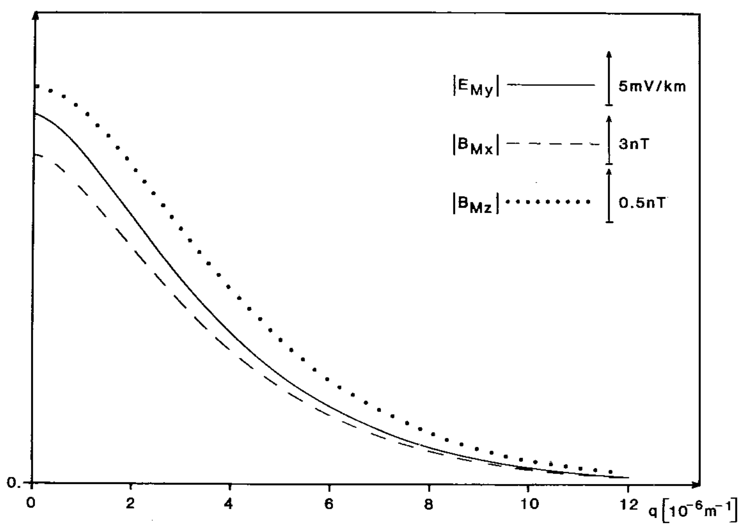

Fig. 2. Amplitudes of the electric field component E_{My} and of the magnetic field components B_{Mx} and B_{Mz} on the earth's surface as functions of the longitudinal propagation constant q. The period $T = 3$ min, the distance $x = 500$ km and the other parameters have the values given in Section 3.1.

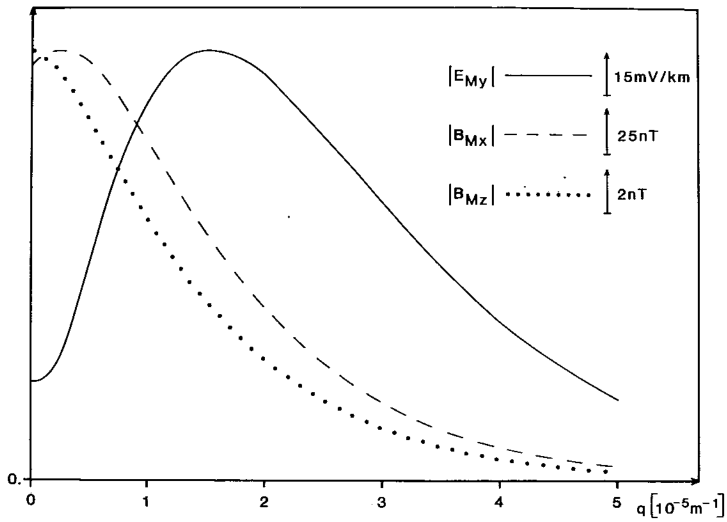

Fig. 3. Amplitudes of the electric field component E_{My} and of the magnetic field components B_{Mx} and B_{Mz} on the earth's surface as functions of the longitudinal propagation constant q. The period $T = 2$ h, the distance $x = 10$ km and the other parameters have the values given in Section 3.1.

in Figures 1 and 3). This indicates that the speed of the decrease depends on the distance from the primary source and grows as the distance increases, and that the period has a minor effect in this respect.

3.4 Plane wave assumption

If it is assumed that the primary field of electromagnetic induction in the earth is a harmonic plane wave (with an angular frequency ω) propagating vertically downwards, and as above, the earth is homogeneous with σ, ϵ and μ_0, the ratio of perpendicular horizontal electric and magnetic field components on the earth's surface is

$$\frac{E_{Mx}}{B_{My}} = -\frac{E_{My}}{B_{Mx}} = \frac{\omega}{k} \tag{16}$$

where k is defined by formula (11) (PIRJOLA, 1982, equation (2.51)). To be precise, the primary field does not occur explicitly in the derivation of equation (16); the only assumption used is that the field inside the earth depends solely on the vertical space coordinate z and is harmonic in time. For geomagnetic frequencies it is reasonable to assume that $\sigma \gg \omega\epsilon$ (Section 3.1), so

$$\frac{E_{Mx}}{B_{My}} = -\frac{E_{My}}{B_{Mx}} = \sqrt{\frac{\omega}{\mu_0 \sigma}}\, e^{i\pi/4} \,. \tag{17}$$

This formula is the »basic equation of magnetotellurics» (CAGNIARD, 1953, p. 616). We can express the right-hand sides of formulas (16) and (17) as Z/μ_0 where Z has the dimension of resistance and is called a plane wave surface impedance at the earth's surface.

If the conductivity of the earth is not uniform, σ must be replaced by a so-called apparent conductivity and the phase angle depends on the frequency in formula (17), but the plane wave surface impedance still reflects properties of the earth merely (CAGNIARD, 1953, pp. 617–618). However, in practice the primary field need not be a vertical plane wave and can have significant horizontal variation. The simple formula (17) then gets invalid as a result of this variation (WAIT, 1954; PRICE, 1962; MADDEN and NELSON, 1964; HERMANCE and PELTIER, 1970; see also equations (3), (4), (6) and (7) of this paper). On the other hand, DMITRIEV and BERDICHEVSKY (1979) have shown that equation (17) is always applicable if the horizontal spatial variations of the horizontal magnetic field on the earth's surface are linear (see also WAIT, 1980).

In this section we will discuss the effect of the longitudinal propagation constant q on the assumption of the validity of the vertical plane wave formulation. It would seem natural that a decrease of q improves the admissibility of the assumption, since the larger q the more significant the horizontal variation of the primary field. It should be noted that even when $q = 0$ the field of the line current treated in this paper is not horizontally uniform but varies in the x-direction.

Let $T = 20$ s and $x = 100$ km. Table II expresses the amplitudes of E_{Mx} and E_{My} for ten different values of q (equations (3) and (4)). $|E_x(pw)|$ and $|E_y(pw)|$ denote the absolute values which are obtained from formula (17) when B_{Mx} and B_{My} have their correct values computed using equations (6) and (7). (As expected, the values of $|E_x(pw)|$ and $|E_y(pw)|$ do not differ practically at all from those which could be calculated with the aid of the rigorous formula (16).) The relative errors which are made if $|E_{Mx}|$ and $|E_{My}|$ are replaced by $|E_x(pw)|$ and $|E_y(pw)|$ are also given in Table II.

Table II shows that the plane wave assumption is well acceptable for $|E_{My}|$ and $|B_{Mx}|$ when $q \lesssim 5 \cdot 10^{-5}\,\mathrm{m}^{-1}$, and even at $q = 5 \cdot 10^{-5}\,\mathrm{m}^{-1}$ the error is only 11.9 %. In the electric x-component the errors are bigger, but anyway $|E_{Mx}|$ and $|E_x(pw)|$ have roughly the same order of magnitude in Table II.

It can be seen *e.g.* from WAIT (1954) that an increase in the period T decreases the validity of the plane wave assumption. Thus it is natural that when $T = 2$ h

Table II. Amplitudes of the horizontal electric field components E_{Mx} and E_{My} on the earth's surface for different values of the longitudinal propagation constant q. The period $T = 20$ s, the distance $x = 100$ km. The other parameters have the values given in Section 3.1. $|E_x(pw)|$ and $|E_y(pw)|$ denote the absolute values which are calculated with the aid of a plane wave assumption. Relative errors between $|E_x(pw)|$ and $|E_{Mx}|$ and between $|E_y(pw)|$ and $|E_{My}|$ are also given.

q in 10^{-7}m^{-1}	$\|E_{Mx}\|$ in V/km	$\|E_x(pw)\|$ in V/km	Error in %	$\|E_{My}\|$ in V/km	$\|E_y(pw)\|$ in V/km	Error in %
0	0	0	0	$9.995 \cdot 10^{-1}$	$9.959 \cdot 10^{-1}$	0.36
1	$1.591 \cdot 10^{-3}$	$1.420 \cdot 10^{-3}$	10.7	$9.991 \cdot 10^{-1}$	$9.955 \cdot 10^{-1}$	0.36
5	$7.893 \cdot 10^{-3}$	$7.039 \cdot 10^{-3}$	10.8	$9.921 \cdot 10^{-1}$	$9.885 \cdot 10^{-1}$	0.37
10	$1.550 \cdot 10^{-2}$	$1.381 \cdot 10^{-2}$	11.0	$9.761 \cdot 10^{-1}$	$9.724 \cdot 10^{-1}$	0.37
50	$5.824 \cdot 10^{-2}$	$5.068 \cdot 10^{-2}$	13.0	$7.498 \cdot 10^{-1}$	$7.461 \cdot 10^{-1}$	0.49
100	$7.072 \cdot 10^{-2}$	$5.934 \cdot 10^{-2}$	16.1	$4.712 \cdot 10^{-1}$	$4.676 \cdot 10^{-1}$	0.76
150	$6.047 \cdot 10^{-2}$	$4.879 \cdot 10^{-2}$	19.3	$2.786 \cdot 10^{-1}$	$2.751 \cdot 10^{-1}$	1.2
200	$4.446 \cdot 10^{-2}$	$3.444 \cdot 10^{-2}$	22.5	$1.594 \cdot 10^{-1}$	$1.563 \cdot 10^{-1}$	2.0
350	$1.179 \cdot 10^{-2}$	$8.021 \cdot 10^{-3}$	32.0	$2.710 \cdot 10^{-2}$	$2.556 \cdot 10^{-2}$	5.7
500	$2.367 \cdot 10^{-3}$	$1.399 \cdot 10^{-3}$	40.9	$4.301 \cdot 10^{-3}$	$3.788 \cdot 10^{-3}$	11.9

and $x = 10$ km (*cf.* Fig. 3) the relative errors are much bigger than in Table II. With these values we could also see that the error in the electric y-component does not increase with q but has a minimum at a non-zero value of q, so the above reasoning of the improvement of the admissibility of the plane wave assumption is not quite right. Besides the long period, the small value of x also enhances the errors. However, the dependence on x is not quite straightforward because the horizontal variation of the primary field may be smaller just below the primary source than a little apart (*cf.* also the above reference to DMITRIEV and BERDICHEVSKY, 1979). *E.g.* when $T = 1$ s and $x = 1$ km the errors will remain very small, especially in the electric y-component. This is partly caused by the small value of T but probably partly also by the localization below the source. With the values used in Fig. 2 the magnitudes of the errors are roughly comparable to those in Table II, which shows that the effects of the changes of T and x tend to compensate each other here.

3.5 Phase differences

It was pointed out in Section 3.2 that the components of the electromagnetic field $\bar{E}_M$, $\bar{B}_M$ do not have the same phase. In this section we will deal with phase differences between the magnetic components B_{Mx} and B_{Mz}.

Let us first discuss the effect of the period T on the phase differences and assume that $x = 100$ km and $q = 0$. Table III expresses the differences between

Table III. Phase differences between the magnetic components B_{Mx} and B_{Mz} on the earth's surface for different values of the period T when the distance x = 100 km and the longitudinal propagation constant q = 0. The other parameters have the values given in Section 3.1.

T	Phase differences between B_{Mx} and B_{Mz}	
in seconds	in degrees	in seconds
1	−136.4	−0.4
20	−141.0	−7.8
60	−145.1	−24.2
180	−150.7	−75.4
600	−157.1	−261.9
7200	−168.5	−3369.6
86400	−175.6	−42136.4

Table IV. Phase differences between the magnetic components B_{Mx} and B_{Mz} on the earth's surface for different values of the longitudinal propagation constant q when the period T = 20 s and the distance x = 100 km. The other parameters have the values given in Section 3.1.

q	Phase differences between B_{Mx} and B_{Mz}	
in $10^{-7}m^{-1}$	in degrees	in seconds
0	−141.0	−7.8
1	−141.0	−7.8
10	−141.2	−7.8
50	−142.8	−7.9
100	−145.5	−8.0
500	−164.1	−9.1
1000	−174.0	−9.7

the phases of B_{Mx} and B_{Mz} both in degrees and in seconds (formula at the end of Section 3.2) for seven values of T. (In fact, the largest value of T, *i.e.* one day, is not reasonable in principle, because the flat-earth model is not acceptable for such a long period.) The component B_{Mz} oscillates in advance of B_{Mx} with time because the differences are negative. It can be concluded from Table III that the phase of B_{Mz} approaches that of the opposite of B_{Mx} (*i.e.* the difference becomes $-180°$) as T increases. However, the quantity $T/2$ + »difference in seconds», which expresses the difference in time between the phase of the opposite of B_{Mx} and the phase of B_{Mz}, grows with T.

Table IV shows the effect of q on the phase differences. It is assumed there that T = 20 s and x = 100 km. The phase difference seems to increase with q, but practically no change occurs in the range $0 \leqslant q \lesssim 5 \cdot 10^{-6} m^{-1}$. An approach towards the value -180° as q grows is indicated. It must be noted here that for $q = 10^{-4} m^{-1}$ the absolute values of the field components are very small (*cf.* Fig. 1).

Formula (4.60) of PIRJOLA (1982) shows that the phase difference between the x- and z-components of the primary magnetic field is $\pm 180^\circ$ at all points on the earth's surface whose x-coordinates are positive, hence especially at $x = 100$ km. Consequently deviations from the value -180° are caused by the effects of the earth, which was even assumed to be homogeneous in the present treatment. The approach of the phase difference towards -180° with increasing T is thus comprehensible since the larger T the less important the induction in the earth.

If ω and q are so small that the Hankel function may be replaced by the expression valid for small arguments in equation (4.60) of PIRJOLA (1982), the x-component of the primary magnetic field has the phase of the primary current and the z-component has (for $x > 0$) the opposite phase (*cf.* formula (3.8) of PIRJOLA, 1982).

4. *Concluding remarks*

This paper deals theoretically with electromagnetic induction in the earth, which is described as an infinite half-space with a flat surface implying the applicability solely to local induction studies. The electromagnetic properties of the earth are assumed to be isotropic, linear and homogeneous. The primary source of the induction is a horizontal straight line current above the earth's surface (*i.e.* in the ionosphere). It oscillates harmonically with time and may also have a harmonic space dependence along the line. This space dependence is expressed by a longitudinal propagation constant q; if $q = 0$ the space dependence vanishes. The longitudinal changes of the current cause accumulation of charge along the line, and so a primary line charge also exists. Rigorous expressions for the total magnetic (variation) and associated electric field observed on the earth's surface and produced by the primary source and induction are given in this paper. The main purpose of this paper is to complete the theoretical discussion in PIRJOLA (1982) by numerical computations and thus take the first step in investigations of the influence of longitudinal variations of the primary current on the electromagnetic field on the earth's surface.

The study of the effects of q is accomplished by numerical examples. In the main example (Example 1) the period of time oscillation $T = 20$ s and the distance of the point of observation from the line below the primary source $x = 100$ km. In Example 2 $T = 3$ min and $x = 500$ km, and in Example 3 $T = 2$ h and $x = 10$ km.

The field also depends on several other parameters, which have fixed values in this paper. One important of them is the conductivity of the earth denoted by σ. In this paper $\sigma = 10^{-2}\Omega^{-1}\mathrm{m}^{-1}$. It is expectable that an increase of σ tends to turn the electric field vertical and the magnetic field horizontal on the earth's surface (*cf.* PIRJOLA, 1982, p. 54). Referring to the treatment of induction by a vertical plane wave it is probable that the decrease of the horizontal electric components is proportional to the inverse of the square root of σ (KAUFMAN and KELLER, 1981, pp. 45 and 50; PIRJOLA, 1982, p. 14; PIRJOLA, 1984).

In Example 1 the amplitudes of the components E_{My}, B_{Mx} and B_{Mz} (x perpendicular and y parallel to the primary source and z vertical) decrease with increasing q, and the amplitudes of the three other components reach maxima at approximately the same value of q ($\approx 10^{-5}\mathrm{m}^{-1}$) and then diminish with q. In Example 2 the amplitudes of E_{My}, B_{Mx} and B_{Mz} also decrease with q and more rapidly than in Example 1. In Example 3 the decrease is the slowest, and the amplitudes of E_{My} and B_{Mx} do not reach their largest values at $q = 0$ but roughly when $q = 1.5 \cdot 10^{-5}\mathrm{m}^{-1}$ and $3.0 \cdot 10^{-6}\mathrm{m}^{-1}$, respectively.

The plane wave assumption basic in the theory of magnetotellurics is also studied in this paper. It is shown that when $T = 20$ s, $x = 100$ km and $q = 0...5 \cdot 10^{-5}\mathrm{m}^{-1}$ the assumption can be made for $|E_{My}|$ and $|B_{Mx}|$. For $|E_{Mx}|$ and $|B_{My}|$ the assumption is correct in the order of magnitude.

The components of the electric and magnetic fields on the earth's surface do not have the same or the opposite phase. Phase differences between B_{Mx} and B_{Mz} are discussed in greater detail. Using first fixed values of x and q (100 km and 0) and then fixed values of T and x (20 s and 100 km) it is seen that the difference approaches (–) 180° as T or q grows. The x- and z-components of the primary magnetic field have a phase difference of 180° on the earth's surface if $x > 0$. Thus deviations from this value are caused by induction in the earth, and the important point is that such deviations occur even in the present case of a homogeneous earth.

As indicated above several times, the primary charge necessary from the physical point of view is ungeophysical. It can be assumed that it has the greatest effect on E_{Mx}, E_{Mz} and B_{My}, which are therefore called ungeophysical in this paper, the other being geophysical. One way to extend the present model to a more geophysical direction is to add vertical currents which prevent the accumulation of the charge. This was done by LEHTO (1983; 1984), and numerical results based on his theory will be presented in a paper that can be considered as a continuation of this paper.

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