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DERIVATION OF CONSTANTS OF THE SEISMOGRAPH GALVANOMETER BY THE LEAST-SQUARES METHOD

by

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Abstract

The accuracy of determination of the period and damping constant of the seismograph galvanometer was tested using the records of (i) the steady-state harmonic motion driven by the sine-wave current and (ii) the free motion excited by the release of the initial deflection of the galvanometer. The accuracy of constants derived by the least-squares method depends in the first case on the quality of the generator output, *i.e.* the accuracy of the frequency scale and the stability of the current amplitude at different frequencies. In the second case the accuracy is affected mainly by the sampling errors of the time coordinate. Without special equipment for the digitization of records, standard errors of one measurement are greater by one order than in the first case. Both methods are suitable for the very accurate analysis of the long-term stability of galvanometers with the non-negligible open circuit damping and the other linear oscillators which yield analog records of the forced and free motion.

1. *Introduction*

Derivation of the damping constant and its accuracy with the standard calibration methods using records of the free movement of the galvanometer was analysed by TOBYÁŠ and TEIKARI (1980). The amplitude ratio method is sufficiently accurate with the standard recording speed of short-period as well as long-period seismographs and the low damping constants up to $D_g = 0.45$. The deflection ratio method is more suitable for the damping constants near critical and for overdamped systems. Its application is limited mainly in the long-period systems due to the requirements of the high accuracy time coordinate measurements on the record.

An effective tool for the constants derivation from the analog records is the least-squares adjustment of the galvanometer response. It can be applied with the use of small digital computers and yields estimates of errors. Both basic constants, *i.e.* the free period and the damping constant, are determined simultaneously. With respect to the conditions required for routine calibration at seismic stations (evaluation of the analog records received with the standard recording speed without any special digitizing equipment) we shall deal with the processing of the following galvanometer responses:

(i) the forced steady-state motion of the galvanometer excited by the harmonic current with constant amplitude at different frequencies,

(ii) the free motion of the galvanometer excited by its release from the initial deflection from the equilibrium position.

In the present paper the accuracy of both methods is tested for the long-period galvanometer, which has non-negligible open circuit damping, and the results are compared with the standard calibration method.

2. *Theoretical background of the methods*

2.1 Forced harmonic motion

We excite the galvanometer system using a harmonic current with constant amplitude i_{gs} and period T. After the transient motion disappears the trace amplitude is (see MEYER & MOERDER, 1957)

$$Y = Y^\star[(1 - T_g^2/T^2)^2 + 4D_g^2T_g^2/T^2]^{-1/2} = Y^\star U, \tag{1}$$

where T_g and D_g are the free period and the damping constant, respectively. The parameter $Y^\star$ corresponds to the trace amplitude Y for the direct current with amplitude i_{gs} because the amplitude response U is equal to 1 for $T = \infty$. The first estimates of the unknown parameters $Y^\star$, T_g, D_g in (1) are denoted as $Y_0^\star$, T_{g0}, D_{g0} respectively. By developing (1) into a series the observational equation for period $T = T_i$ reads

$$v_i = a_i dY^\star + b_i dT_g + c_i dD_g + l_i \tag{2}$$

with coefficients defined by the following relations:

$$a_i = U_{i0},\tag{3a}$$

$$b_i = Y_0^\star U_{i0}[2(1 - T_{g0}^2/T_i^2) - 4D_{g0}^2]\, T_{g0}/T_i^2,\tag{3b}$$

$$c_i = -4\, Y_0^\star U_{i0}^3 D_{g0}^2\, T_{g0}^2/T_i^2,\tag{3c}$$

$$l_i = Y_0^\star U_{i0} - Y_i.\tag{3d}$$

Here U_{i0} is the amplitude response U calculated for the estimated constants T_{g0}, D_{g0} at period $T = T_i$. Y_i is the measured amplitude on the record for the constant current amplitude in the whole interval periods of T. In practice the current amplitude is adjusted according to periods to get Y_i greater than 20 mm to measure the trace deflection with sufficient accuracy with millimetre scale. The trace amplitudes at different periods are then transformed to the same current amplitude and the corrected values are used for derivation of parameters by the least-squares method. Measurements of at minimum 3 periods yield the basic constants T_g, D_g and the amplitude $Y^\star$ for the determination of the current sensitivity.

It follows from (1) that the linear transformation of excitation period T, *i.e.* $T' = k\,T\ (k \neq 1)$, leads to the same amplitude response U for the galvanometer with period $T_g' = k\,T_g$ and the same damping constant $D_g' = D_g$. The least-squares determination of the damping constant is independent of the systematic shift of the frequency scale. Such generator errors affect only the adjusted galvanometer period which is shifted in the same way. This inaccuracy cannot of course be found from calculations.

2.2 Free motion

The transient motion of the galvanometer after release from its initial deflection $Y^\star$ at time $t = 0$ is given by the formulas (see MEYER & MOERDER, 1957)

$$Y = Y^\star \exp(-rD_g)[D_g/\sqrt{1-D_g^2}\sin(r\sqrt{1-D_g^2}) + \cos(r\sqrt{1-D_g^2})] \quad \text{for } D_g < 1 \tag{4a}$$

$$Y = Y^\star \exp(-r)\,(1 + r) \quad \text{for } D_g = 1,\tag{4b}$$

$$Y = Y^\star \exp(-rD_g)\,[D_g/\sqrt{D_g^2 - 1}\,\sinh(r\sqrt{D_g^2-1}) + \cosh(r\sqrt{D_g^2 - 1})] \text{ for } D_g > 1,\tag{4c}$$

where the reduced time $r = 2\pi t/T_g$.

If we suppose that the correct time of the release is not at $t = 0$ and is shifted by $t^\star$ then we have $r = 2\pi(t + t^\star)/T_g$ and the observational equation for the unknown parameters $Y^\star$, T_g, D_g and $t^\star$ is

$$\nu_i = a_i dY^\star + b_i dT_g + c_i dD_g + d_i dt^\star + l_i. \tag{5}$$

Here $l_i = Y_{i0} - Y_i$, Y_i is the measured trace deflection at time $t = t_i$ and Y_{i0} is the calculated deflection at the same time for the estimated constants T_{g0}, D_{g0} and amplitude $Y^\star$ and the time origin shift $t_0^\star$.

Using the relations $d_0 = \sqrt{1 - D_{g0}^2}$, $r_{i0} = 2\pi(t_i + t_0^\star)/T_{g0}$ it holds for $D_{g0} < 1$ that

$$a_i = \exp(-r_{i0} D_{g0})\,[D_{g0}/d_0 \sin(d_0 r_{i0}) + \cos(d_0 r_{i0})], \tag{6a}$$

$$b_i = Y_0^\star r_{i0} \exp(-r_{i0} D_{g0}) \sin(d_0 r_{i0})/(T_{g0} d_0), \tag{6b}$$

$$c_i = Y_0^\star \exp(-r_{i0} D_{g0})\,[\sin(d_0 r_{i0})/d_0 - r_{i0} \cos(d_0 r_{i0})]/d_0^2, \tag{6c}$$

$$d_i = -2\pi Y_0^\star \exp(-r_{i0} D_{g0}) \sin(d_0 r_{i0})/(T_{g0} d_0), \tag{6d}$$

for $D_{g0} = 1$

$$a_i = \exp(-r_{i0})\,(1 + r_{i0}), \tag{7a}$$

$$b_i = Y_0^\star \exp(-r_{i0})\, r_{i0}^2/T_{g0}, \tag{7b}$$

$$c_i = Y_0^\star \exp(-r_{i0})\, r_{i0}^3/3, \tag{7c}$$

$$d_i = -2\pi Y_0^\star \exp(-r_{i0})\, r_{i0}/T_{g0} \tag{7d}$$

and for $D_{g0} > 1$ with $d_0 = \sqrt{D_{g0}^2 - 1}$

$$a_i = \exp(-r_{i0} D_{g0})\,[D_{g0}/d_0 \sinh(d_0 r_{i0}) + \cosh(d_0 r_{i0})], \tag{8a}$$

$$b_i = Y_0^\star r_{i0} \exp(-r_{i0} D_{g0}) \sinh(d_0 r_{i0})/(T_{g0} d_0), \tag{8b}$$

$$c_i = Y_0^\star \exp(-r_{i0} D_{g0})\,[-\sinh(d_0 r_{i0})/d_0 + r_{i0} \cosh(d_0 r_{i0})]/d_0^2, \tag{8c}$$

$$d_i = -2\pi Y_0^\star \exp(-r_{i0} D_{g0}) \sinh(d_0 r_{i0})/(T_{g0} d_0). \tag{8d}$$

Four different points of the release record at least are necessary for calculating the basic constants T_g, D_g and the initial deflection $Y^\star$ for derivation of the current sensitivity.

The formulas (4a–c) show that the linear distortion of the time scale $t' = kt$ ($k \neq 1$) produces the same distortion of the galvanometer period $T_g' = kT_g$ to fit the galvanometer response with the same damping constant $D'_g = D_g$. The errors of the time coordinate sampling are responsible for the inaccuracy of the galvanometer period.

3. *Experimental results*

Both methods were tested with the Lehner and Griffith Model GL-261 long-period galvanometer, serial No. 591. It was placed on a concrete pillar in a cellar under environmental conditions similar to those in the seismic vault. The temperature changes were within few centigrades during the measurements performed in January – March 1979 and completed in April 1981. The magnetic shunt was adjusted to the minimum magnetic field intensity and tests were made for 5 different dampings below the critical value. The standard drum recorder with recording speed 15 and/or 30 mm/min was used for the galvanometer response records. The recorder was modified so that the drum did not move to the side.

The arrangement of the electric scheme for the galvanometer excitation is as in Fig. 1. The value of R_s was constant in all cases and equal to 93847 Ω. The damping constant of the electromagnetic part of damping was controlled using the resistor R_d. The internal resistance of the galvanometer was in the first term 480.7 Ω in the second 476.4 Ω. Due to the high internal resistance of the digital voltmeter (10 MΩ) and the additional 1 MΩ resistor in series with the power supply, the external resistance of the galvanometer is equal to R_d with a maximum relative error less than 0.001 (R_d maximum value about 1 kΩ). The amplitude of the

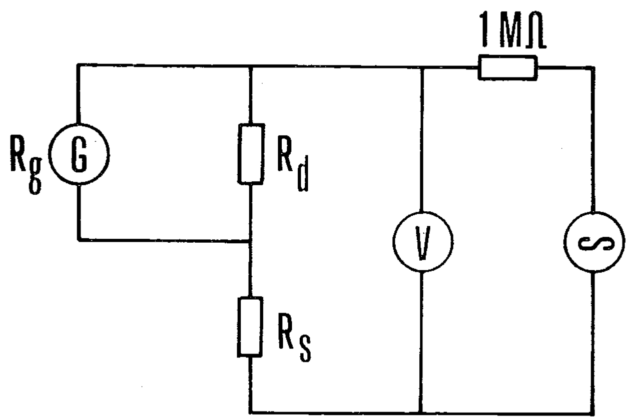

Fig. 1. Scheme of the network for galvanometer excitation.

current flowing through the galvanometer is

$$i_g = \frac{R_d U_0}{R_g R_d + R_s(R_g + R_d)} \tag{9}$$

where U_0 is the voltage amplitude measured by the voltmeter V (KNICK NG20 microvoltmeter). For the harmonic excitation a function generator Airmec type 422 was used, for the release tests a direct current source was applied and the trace for switching on and off was recorded. The accuracy of the voltage amplitude was better than 1 % and the accuracy of the frequency adjustment was better than 0.4 %.

3.1 Steady-state harmonic motion

The distribution of periods at which the galvanometer is driven should be chosen with respect to the free period of the galvanometer and its damping. The comparison with the theoretical amplitude response of the underdamped galvanometer shows that the suitable range of periods is between 0.3 T_g and 2 T_g to get differences with particular responses (Fig. 2). Shorter periods yield no more information, while longer

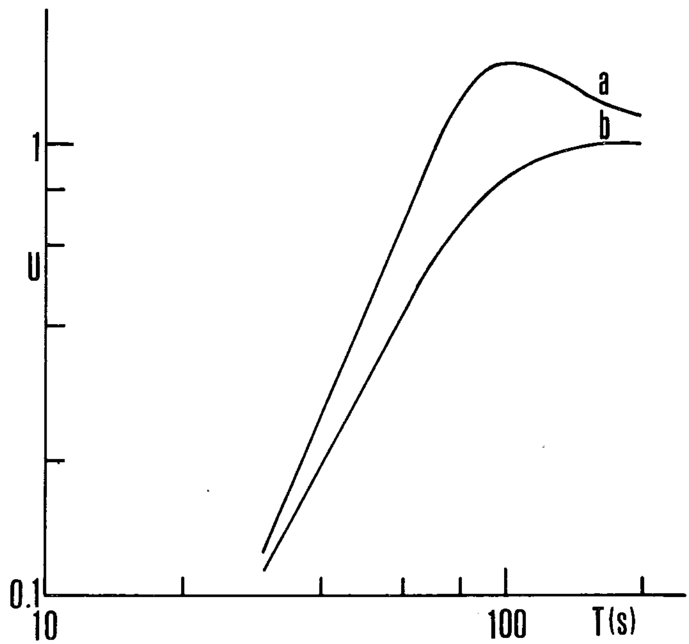

Fig. 2. Amplitude response of the galvanometer with T_g = 90 s and damping constants equal to 0.35 (a) and 0.7 (b).

Table 1. Constants derived by the least-squares method from the steady-state and transient record and the standard constant determination. m_0 is the standard error of one measurement.

Case	Steady-state			Transient			Standard	
	T_g(s)	D_g	m_0	T_g(s)	D_g	m_0	T_g(s)	D_g
1	88.38±0.04	0.3461±0.0006	0.11	85.09±0.64	0.3562±0.0049	1.2	88.6	0.346
2	88.70±0.12	0.4152±0.0015	0.23	86.57±0.41	0.4221±0.0027	0.9	90.5	0.420
3	88.95±0.06	0.4863±0.0009	0.09	85.37±0.59	0.4920±0.0035	1.0	89.3	0.485
4	88.68±0.05	0.5956±0.0008	0.06	81.63±0.61	0.5919±0.0034	0.5	89.1	0.584
5	89.56±0.14	0.6600±0.0023	0.08	89.13±1.00	0.6984±0.0051	0.6	89.4	0.687
Mean	88.85±0.44			85.5±2.7			89.44± 0.62	

periods are useful only for the correct derivation of the direct current sensitivity. Taking the free period of the galvanometer as approx. 90 s, the shortest period applied was 30 s and the longest period was limited by the generator output to 200 s. Measurements for 8 periods (at 30, 50, 70, 90, 110, 130, 150 and 200 seconds) are sufficient. Increasing the number of periods does not substantially affect the accuracy of the least-squares adjustment of the constants. Double amplitudes (not smaller than 20 mm) were measured, using a linear scale with an error less than 0.1–0.2 mm, the frequency was taken to six significant figures without checking its absolute value in any other way.

Results of the least-squares approximation for 5 damping constants are given in Table 1. For the first estimates of parameters with deviations up to 20 % from the real values the final parameters were reached after 3–4 approximation steps. The accuracy of the constants was very good: the standard errors of the free period were not greater than 0.1 % and the standard errors of the damping constants were between 0.2 and 0.4 %. The differences between the observation and the adjusted amplitude responses were mostly below 0.1 mm and only during the 2 shortest periods of the second test were the differences greater than 0.3 and 0.4 mm. The galvanometer period was very stable in cases 1–4. Test 5, carried out 2 years after the start of the experiment indicates an increase of 0.9 s (*i.e.* 1 %). Small standard errors demonstrate accurate period adjustment of the generator, the stability of current amplitudes, small errors of trace amplitude measurements on record and excellent behaviour of the galvanometer as a linear oscillator.

3.2 Transient motion by release

Records of transient motion were directly copied on the millimetre paper and the origin of motion was adjusted so that both ordinates could be directly read off the scale. The sampling rate was limited by the recording speed. With speeds of 30 mm and 15 mm per minute the trace deflections were measured at 2 or 4 seconds. The sampling was stopped at time t_1 after which the deflection was smaller than the threshold deflection $Y_t = 0.5$ mm, *i.e.* for $t > t_1$ it holds that $|Y| \leqslant Y_t$. The value of t_1 depends on the initial deflection $Y^\star$ and the damping (Fig. 3). In our tests with $Y^\star$ between 80 and 150 mm, t_1 is between 100 and 150 seconds. Increasing the number of samples does not yield better results due to errors of the measurement of time and noise at small deviations. Usually 32–40 samples were used for calculations with 3–4 approximation steps as in the preceding method. The digitization procedure was also performed using semiautomatic digitizing devices but the final results were not better than the simple method described above. Shifting of the adjusted origin time $t^\star$ was in the range –0.8 up to 0.9 s. The standard errors of constants were much greater than with the steady-state method; 0.5–1.1 % for the

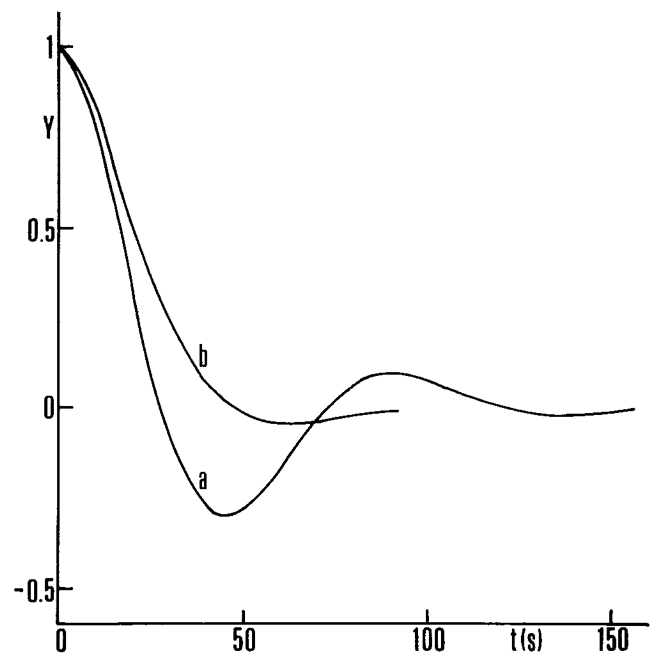

Fig. 3. Normalized transient movement of the galvanometer with damping constants equal to 0.35 (a) and 0.7 (b).

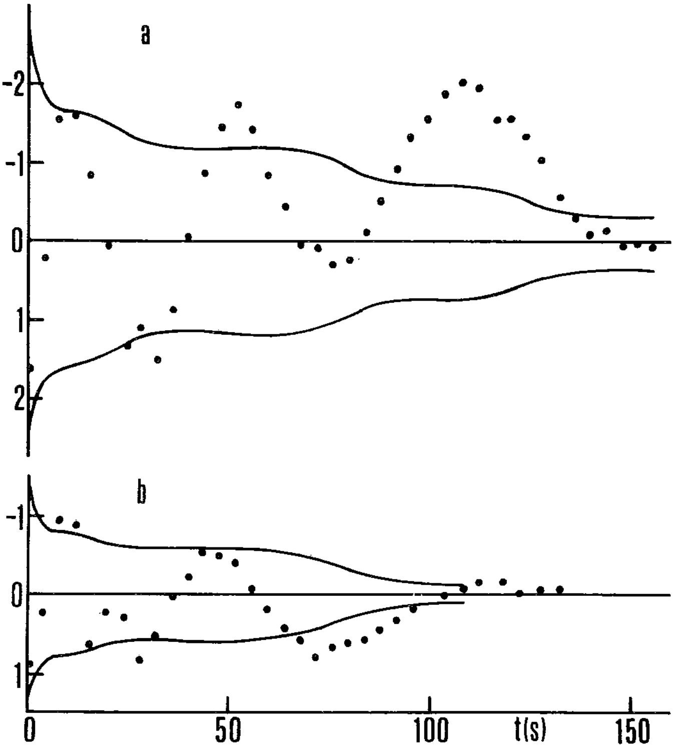

Fig. 4. Differences l_i (mm) and the limits of 3 standard errors of the transient response in cases 1 (a) and 5 (b)..

period and 0.6–1.3 % for the damping constant (Table 1). Such errors are acceptable but the reliability of the derived free period is problematic. It should be constant and the differences in certain tests are too large – 7.6 s being the greatest. The main reason for this discrepancy is an error of the time scale which cannot be reduced without using higher recording speeds or the application of a special digitizing device. In spite of this phenomenon the damping constants correspond well with those gained from the steady-state method.

The greatest deviations of the adjusted release response from the observations were about 2 mm (Fig. 4). The deviations are randomly distributed at the start, and are followed by pronounced systematic deviations due to the phase displacement between both responses. In general the deviations are in the range 3 sigma tolerance limits. Similar results were also obtained for the other tests.

4. *Conclusion*

The standard methods (see TOBYÁŠ and TEIKARI, 1980) of free period and damping constant determination were performed for comparison with the same cases (Table 1). Free periods as well as damping constants (with the exception of case 5) correspond well with the steady-state tests. It is clear that the direct determination of the damping ratio for the largest damping constant in case 5 has the maximum error due to the measuring of small amplitudes.

The specific errors of different methods are applied when calculating the other parameters of the galvanometer that are sometimes used for adjustment of constants and seismograph calibration.

The adjusted deflections $Y^\star$ correspond to the direct current deviations of the galvanometer. We denote them by $Y_s^\star$ and $Y_t^\star$ for the steady-state and the transient methods, respectively, and the corresponding current amplitudes by i_{gs} and i_{gt}. The current sensitivities are defined as $C_{is} = Y_s^\star/i_{gs}$ and $C_{it} = Y_t^\star/i_{gt}$ (Table 2). The standard

Table 2. Deflection Y, current amplitude i_g and current sensitivity C_i for particular cases and methods.

Case	$Y_s^\star$(mm)	i_{gs} (nA)	C_{is} (mm/nA)	$Y_t^\star$ (mm)	i_{gt} (nA)	C_{it} (mm/nA)	Y_d (mm)	C_{id} (mm/nA)
1	83.1±0.1	10.8	7.68	87.0±0.9	10.7	8.09	84.9	7.90
2	87.3±0.2	11.3	7.72	146.1±0.7	18.8	7.79	146.5	7.81
3	84.3±0.1	10.9	7.76	149.6±0.9	18.9	7.90	148.1	7.83
4	89.8±0.1	11.5	7.80	95.1±0.4	12.3	7.74	95.0	7.73
5	98.8±0.2	12.6	7.82	97.6±0.5	12.3	7.92	97.6	7.91
Mean			7.76±0.06			7.89±0.14		7.84±0.07

determination of the current sensitivity is defined by the deflection Y_d measured directly for the same current i_{gt} as in the transient method, *i.e.* $C_{id} = Y_d/i_{gt}$. C_{id} should be equal to C_{it} but due to differences between adjusted and observed amplitudes there are also differences in the sensitivities. The adjusted values of $Y_s^\star$ have standard errors of only 0.1–0.2 %, and $Y_t^\star$ from 0.5 up to 1 %. Standard errors of the average current sensitivity are 0.8–0.9 % for the steady-state and the standard method and 1.8 % for the transient method, but the differences between particular sensitivity determinations are up to 5 %.

Damping constants derived for several circuit resistances C_r are used for the calculation of the critical damping resistance a_g and the open circuit damping constant D_{g0} of the galvanometer. It holds that

$$D_g = D_{g0} + a_g/C_r. \tag{10}$$

We can take $C_r = R_g + R_d$ for the scheme given in Fig. 1 due to the high internal resistance of a voltmeter and a resistor in series. If (10) is used all values D_g must correspond to the same period of the galvanometer. If measurements of D_g are available at different periods they should be transformed to one fixed period for which the calculated critical resistance and D_{g0} are correct. The linear relation between D_g and T_g is valid. The critical resistance and open circuit damping derived by the least-squares method from the above mentioned measurements are listed in Table 3. It was assumed that the correct period of the galvanometer was constant (corresponding to the mean value for the steady state method) and no corrections of damping constants were carried out. The steady-state method yields data with the smallest standard errors. Deviations of adjusted linear approximation (10) from observations are in the range −0.4 %, 0.6 % for the steady-state method, −1.3 %, 1.9 % for the standard and −2.5 %, 3.2 % for the transient method. The largest deviations for both constants are 6–7 % with steady-state and standard method data.

Parameters T_g, a_g and current sensitivity C_i are used for calculating the galvanometric moment of inertia K_g (Aranovich *et al.*, 1974). The following relation is valid

$$K_g = 32.3 \times 10^3 \, a_g T_g^3 / C_i^2. \tag{11}$$

When the moment of inertia is in kg m^2, the critical resistance is in ohms, current sensitivity in millimetres per ampère per metre and period in seconds. Because the recording distance of the galvanometer was 1.034 m, the current sensitivity listed in Table 2 was corrected using Eq. (11). The moment of inertia for the mean parameters of particular methods is given in Table 3. The steady-state and standard method parameters give comparable estimates of errors (3–4 %), the transient method is mainly influenced by great errors of period and the error of K_g is about 13 %.

Table 3. Parameters of the galvanometer calculated for the damping constants and derived by three methods.

Method	D_{g0}	a_g (Ω)	K_g (kg m^2) x 10^{-8}
Steady-state	0.192±9 x 10^{-6}	226.0±0.006	9.1±0.3
Transient	0.187±6 x 10^{-4}	238.2±0.39	8.3±1.1
Standard	0.179±2 x 10^{-4}	240.4±0.12	9.7±0.4

Magnification of the seismograph is proportional to $K_g^{1/2}$ and therefore the standard error of the seismograph scaling factor was in the best case 2 %. The greatest difference in the moment of inertia derived by the methods described was 17 %.

It is obvious that the steady-state method yields the least errors in constants and other parameters of the galvanometer. The results are free of time measurements and the periods given by the generator frequency scale were very accurate. However, in routine practice this method is more suitable for short and intermediate period systems because tests with long-period systems are rather long. On the other hand, evaluation of records is easy and fast. The transient method is more suitable with long-period systems, which yield better conditions for digitization with a sufficient sampling rate. This method is in principle the deflection ratio method having the same problems of time coordinate measurements. It does not need special instrumentation for galvanometer motion excitation but the evaluation of the records without special digitizing equipment is time consuming.

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