556.131.1

ESTIMATING THE EFFECT OF ATMOSPHERIC STABILITY ON LAKE EVAPORATION WITH THE WATER BUDGET METHOD

by

J. VIRTA

University of Helsinki
Department of Geophysics

Abstract

This study presents a method for using the water budget method in the estimation of the effect of atmospheric stability on lake evaporation. Oviously, the evaporation cannot be determined directly with the water budget method, because the groundwater inflow to the lake is usually unknown. However, if the water budget and bulk aerodynamic methods are combined, it is possible to estimate both the net groundwater inflow and the bulk aerodynamic coefficient. It is also possible to estimate the dependence of the bulk aerodynamic coefficient on the stability parameter. This combined method is useful for lakes with simple inflow and outflow conditions. There must also be observation data from rainless periods so that errors in precipitation data can be avoided and errors in the runoff of ungauged catchments minimised.

1. *Introduction*

The simplest method of estimating the real evaporation of a lake is perhaps that based on the water budget. This method, however, includes weak points in the determination of several components of the water budget, and it is not suitable for all lakes for all periods.

The following conditions must be fulfilled in order to obtain reliable results with water budget method:

— The ouflow and a significant part of the inflow must be measurable.

— There must be rainless periods during which inflow and outflow are small.

— The effect of bank strorage must be small.

It is not easy to test the last of these conditions, but the effect of the bank storage is small, at least for periods with little change in water level height.

In the following the water budget method will be applied to two lakes situated in south Finland: Lake Pääjärvi (surface area 13.6 km^2) and Lake Valkea-Mustajärvi (surface area 0.139 km^2).

2. *The method of computation*

The water budget equation of a lake is given by

$$P - E + (I_s + I_g - O_s - O_g)/A_l = dH/dt \tag{1}$$

where P is precipitation, E evaporation, O outflow and I inflow. Subscripts s and g indicate surface and ground water flow, respectively, A_l is the lake surface area and H the water level height.

P, H, O_s and a part of I_s can be measured easily. The remaining part of I_s can be estimated by comparing ungauged catchments with gauged catchments. Very often E, I_g and O_g are unknown.

To close the problem there must be two other conditions. The following method has been developed by HARBECK [3], and was later recommended by WMO, [7]. The method has been applied in Finland to Lake Pääjärvi (VIRTA, [6]), using a constant bulk aerodynamic coefficient.

If the net ground water flow is small, it may be considered constant, *i.e.*

$$(I_g - O_g)/A_l = a \tag{2a}$$

This equation has proved to be suitable for Lake Pääjärvi. A simple time-dependent function

$$(I_g - O_g)/A_l = a + b\,t \tag{2b}$$

has used for Lake Valkea-Mustajärvi. The basis for the form of Eq. 2b is that study period in Lake Valkea-Mustajärvi was rather dry and the ground water inflow to the lake was decreasing.

Evaporation may be presented in an approximate form

$$E = C\varrho_a u_z (q_o - q_z)/\varrho_w \text{ or} \tag{3a}$$

$$E = C' u_z (e_o - e_z), \tag{3b}$$

where C is the nondimensional Dalton number, q_o and q_z the specific humidity at the water surface and at the height of z, e_o and e_z are vapour pressure at the surface and at the height z, and u_z is the wind velocity at the height z. ϱ_a and ϱ_w are the densities of air and water.

Let us define

$$E_w = P + (I_s - O_s)/A_l - dH/dt, \tag{4}$$

E_w may be computed from the measurements. The following equation may now be obtained from equations 1, 2 and 3b

$$E_w = C' u_z (e_o - e_z) - a - b\,t \tag{5}$$

If C' can be considered a constant, then both C', a and b may be computed from the observation data of E_w and $u_z\,(e_o - e_z)$.

Atmospherie stability also has an effect on evaporation when considering longer periods, for example months. This has also been observed in results obtained at Pääjärvi (Elomaa, [2]). In the following a method will be used with which the dependence of the bulk aerodynamic coefficient on the stability parameter can be computed by means of the water budget calculation.

The ratio z/L, where L is the Monin-Obukhov length, can be used as the stability parameter. The ratio is dependent on the flux of sensible heat and other factors. The use of this parameter is difficult because the computation becomes iterative (Launiainen, [5]).

Another parameter that can be used is the bulk Richardson number, which can be computed from measurements carried out at two levels. The form for the parameter presented by Deardorff [2] takes into account the correction due to water vapour using the virtual temperature. The formula for the parameter is

$$R_{iv} = \frac{g\,z}{T_v\,u_z^2}\left[\Theta_z - \Theta_o + 0.61\,T_o \frac{C}{C_H}(q_z - q_o)\right], \tag{6}$$

where T_v is the virtual absolute temperature, which may be replaced by the absolute temperature T_o without significant error. Θ_z and Θ_o are the potential temperatures at the height z and at the surface. C_H is the bulk coefficient of sensible heat (in the unstable region $C \approx C_H$). There exists a dependence between parameters z/L and R_{iv} (Launiainen, [5]), and for this reason parameter R_{iv} will be used as the stability parameter in the following.

The water budget computation cannot be carried out for short periods in the present case. A two-day period has proved suitable for Pääjärvi and a 24 h period for Valkea-Mustajärvi. Stability conditions during long periods of this kind have been evened out, so that it is not possible to get a detailed experimental $C - R_{iv}$ dependence. When considering the observational data, a simple linear dependence is accurate enought, *i.e.*

$$C = C_n(1 + cR_{iv}) \tag{7a}$$

$$C' = C'_n\,(1 + c\,R_{iv}) \tag{7b}$$

where C_n and C'_n both represent the coefficient of neutral stability.

Using equation 5 the following is obtained

$$E_w = \alpha\,\overline{u_z(e_0 - e_z)} + \beta\,\overline{u_z(e_o - e_z)\,R_{iv}} + \gamma + \delta\,t \tag{8}$$

where $\alpha = C'_n$, $\beta = C'_n\,c$, $\gamma = -a$ and $\delta = -b$. Overbar (—) means averaging in time from day and night mean values for Pääjärvi and from hourly means for Valkea-Mustajärvi. Coefficients α, β, γ and δ can be determined by regression analysis.

3. *Results for Lake Pääjärvi*

Details of measurement and earlier computations for Lake Pääjärvi have been presented earlier (Virta, [6]). This analysis was carried out with the data measured in 1969—1970 at a height of 2 m. The values of the constants C' and a were computed from 32 two-day periods with precipitation less than 1 mm/day and inflow less than 5 mm/day. The years 1969 and 1970 were very suitable for this analysis, because the summers were exceptionally dry. The results were

$C' = 0.127$ mm (day mb m/s)$^{-1}$

$a = 1.0$ mm/day

In the present analysis 28 two-day periods were selected from the observational data for the summers of 1969 and 1970. The selection was based on conditions $P = 0$ and $I_s < 5$ mm/day. This condition is somewhat stricter than that in the earlier analysis (Virta, [6]). The results of the computation are shown in Table 1. The height of measurements is 2 m.

Table 1. Results of computations for Lake Pääjärvi using the water budget. t refers to the t-test. R is the multiple correlation coefficient and s_e the standard deviation of residuals.

α	$= 0.119$ mm day^{-1} (mb m/s)$^{-1}$	$t = 8.8$
β	$= -0.437$ mm day^{-1} (mb m/s)$^{-1}$	$t = 2.2$
γ	$= -1.2$ mm/day	
R	$= 0.89$	
s_e	$= 0.84$ mm/day	

The significance level of coefficients α and β was 5 %.

The value 0.119 mm day^{-1} (mb m/s)$^{-1}$ for coefficient α corresponds to the value $1.84 \cdot 10^{-3}$ for coefficient C'_{n}, which is near the values obtained in earlier studies. Launiainen [5] used profile measurements to obtain the value $1.74 \cdot 10^{-3}$ for this coefficient for a bay in the Gulf of Finland with a measuring height of 2 m.

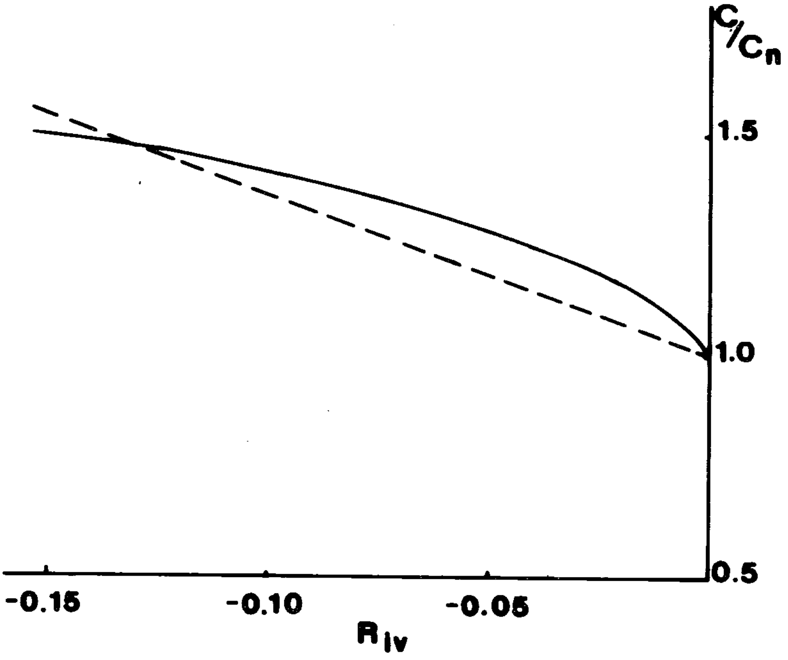

Fig. 1. The dependence of the ratio C/C_n on R_{iv} given by Eq. 9
(--------) for Lake Pääjärvi and given by Launiainen [5]
(———) for a bay in the Gulf of Finland. Height of measurements 2 m.

The order of magnitude of the dependence of the bulk coefficient on the stability parameter also corresponds with earlier values. Using coefficients α and β the following formula may be obtained

$$C/C_n = 1 - 3.67\,R_{iv} \qquad (9)$$

The analysis was performed using data in which the bulk Richardson number varied from 0.00 to —0.14. Fig. 1 gives the ratio C/C_n accorfing to equation 9 at his interval. The ratio computed by LAUNIAINEN [5] for the bay mentioned above has also been drawn into this figure. It can be seen that the present and earlier dependences agree with each other.

Table 2 gives monthly means for Lake Pääjärvi computed in different ways. Colum (a) gives the present calculations, (b) earlier calculations (VIRTA, [6]) with the constant bulk coefficient, (c) ELOMAA'S [2] computation with KONDO'S [4] stability correction method and (d) Elomaa's computation with the constant bulk coefficient. n is the number of days.

Table 2. Monthly averages of evaporation from Lake Pääjärvi (mm/day) obtained with different methods: (a) present study, (b) earlier computation with the constant bulk coefficient (VIRTA, [6]), (c) ELOMAA'S [2] computation with KONDO'S method [4] and ELOMAA'S [2] computation with the constant bulk coefficient. n is the number of days.

		a	b	n	c	d	n
1969	June	4.73	3.99	7			
	July	3.86	3.85	31			
	Aug.	3.96	3.80	23			
	Sept.	2.96	2.90	6			
	Oct.	1.30	1.22	30			
1970	June	4.77	4.59	14	4.43	4.48	13
	July	2.95	3.05	31	3.04	3.09	31
	Aug.	3.18	3.14	28	3.18	3.15	31
	Sept.	2.48	2.50	30	2.74	2.86	27
	Oct.	1.51	1.49	20	1.61	1.63	20

The present study shows that the correction due to stability in the monthly means is at most 10 % during June and October. The differences between the values obtained in the present study and Elomaa's values are considerable. The reason for this is not clear. The treatment of the basic data is different. Elomaa computed evaporation using hourly means, and the present values were computed with 12 h averages. However, calibration of the present method was done using the same averages, and the calibration takes into account systematic instrumental errors and errors due to possible weaknesses in the profile method theory.

4. *Results for Valkea-Mustajärvi*

Hydrologically, Lake Valkea-Mustajärvi (surface area 0.139 km^2, greatest depth 11 m, mean depth 3.4 m) is very simple, because this lake has no surface inflow and its surface outflow is retarded during summer with a dam.

The height for the measurement of wind velocity was 2 m, and that of air temperature and humidity 1.7 m. Pt-1000 resistors were used as temperature sensors and a hair hygrometer as a humidity sensor. The recording took place with an AANDERAA data logger. All sensors were fixed into a mast located near the centre of the lake.

From the observational data from May, June and July 1980 it was possible to select 61 rainless 12 h period and 31 rainless 24 h periods. The stability conditions during these periods were quite different from that in Lake Pääjärvi. The average value of R_{iv} for almost every night was less than —0.15. When considering the form of the curve $C(R_{iv})$ it is not resonable to use linear relationship (Eq. 7) in this low range of R_{iv}. For this reason the following restiriction will be made:

If $\overline{u \Delta e} R_{iv}/\overline{(u \Delta e)} < -0.15$ then $C = C_n (1 - 0.15\, c)$
else $C = C_n (1 + c\, R_{iv})$

Table 3 gives the results of computations.

Table 3. Results of computations for Lake Valkea-Mustajärvi. R is the multiple correlation coefficient and s_e the standard deviation of residuals.

	12 h periods	*t-test*	*24 h periods*	*t-test*
α/mm day^{-1} $(mb\ m/s)^{-1}$	0.0746	6.0	0.143	8.7
β/mm day^{-1} $(mb\ m/s)^{-1}$	—0.602	2.9	—	—
γ/mm day^{-1}	0.48	—	—0.22	—
δ/mm day^{-2}	0.034	5.3	0.035	6.4
R	0.69	—	0.88	—
s_e/mm day^{-1}	1.14	—	0.69	—

The difference between results of two computations may be explained. The dependence of 24 h evaporation on the stability parameter in the low range of avarage R_{iv} is so weak, that this parameter is not a significant variable in the analysis. On the other hand for 12 h periods day time evaporation is more strongly influenced by stability conditions.

The value 0.0746 mm day^{-1} $(\text{mb m/s})^{-1}$ for coefficient α corresponds to the value $1.15 \cdot 10^{-3}$ for coefficient C_n. This may be compared to the value $1.84 \cdot 10^{-3}$ for Pääjärvi, which is a much larger lake. The reason for this difference may be caused by different wind velocity and turbulence conditions above lakes of different size.

For the dependence of the bulk aerodynamic coefficient on the bulk Richardson number R_{iv} the following was obtained

$$C/C_n = 1 - 8.08\, R_{iv} \qquad (R_{iv} < -0.15) \tag{10}$$

Comparison with Eq. 9 again shows the difference between lakes of different size.

5. *Discussion of the results*

As mentioned above both the value of the neutral bulk coefficient and the dependence of the bulk coefficient on the bulk Richarson number agree well with the earlier values. Thus it may be concluded that the method based on the water budget is correct.

There are some advantags in the use of the water budget method, the most important being:

- The instrumentation is very simple. The most important instrument is the water level gauge. The average height of the lake should be measured with an accuracy of at least 0.1 mm.
- There are no assumptions about the wind profile and moisture profile. This is particularly important in small lakes, where the effect of the shore may disturbe the profiles.
- The method gives results which are representative of the whole lake. This may be compared *f.ex.* to the profile method, which gives results only for one point.

The defects of the method are as follows:

- The inflow and outflow of the lake in question must be measurable. This considerably limits the usefulness of the method.
- The calibration is based on rainless periods. It is sometimes necessary to wait for quite a long time for periods for this kind to occur.

- — The method is not very easily adapted for shorter periods, *e.g.* periods of one hour, because there are difficulties in measuring the water level height with the necessary accuracy.
- — The effect of bank storage is unknown. It depends on the type of soil on the shore and the length of the shoreline. The effect may be small, because the water level change is small during the periods in question.

Acknowledgement: The author appreciate very much having received financial support from the Finnish IHD committee (Lake Pääjärvi) and from the Finnish IHP committee (Lake Valkea-Mustajärvi) of the National Research Council for Sciences.

REFERENCES

1. DEARDORFF, J. W., 1968: Dependence of air-sea transfer coefficients on bulk stability. *J. of Geoph. Res.* **73**, 2549—2557.
2. ELOMAA, E., 1977: Pääjärvi representative basin in Finland: Heat balance of a lake. *Fennica* 149, 33 pp, Helsinki.
3. HARBECK, G. E. Jr, 1962: A practical field technique for measuring reservoir evaporation utilizing mass-transfer theory. Studies of evaporation, *U.S. Geol. Survey prof. paper* 272-E, 101—105, Washington.
4. KONDO, J., 1975: Air-sea bulk transfer coefficients in diabatic conditions. *Boundary-Layer meteorology* **9**, 91—112.
5. LAUNIAINEN, J., 1979: Studies of energy exchange between the air and the sea surface on the coastal area of the Gulf of Finland. *Finnish Marine Research* No 246, 3—110, Helsinki.
6. VIRTA, J., 1971: Water and energy budgets of the Lake Pääjärvi particularly for the determination of evaporation. *Finnish IHD report* No 1, 59—66, Helsinki.
7. WMO, 1966: Measurement and estimation of evaporation and evapotranspiration. *Technical note* No 83, 81—83, Geneva.