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SNOWMELT GENERATED RUN-OFF FROM SMALL AREAS AS A DAILY TRANSIENT PROCESS

by

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Abstract

The daily variation of snowmelt generated run-off is discussed. The heat conduction in the snowcover and refreezing during cold nights are discussed. The refreezing phenemena significantly influences the saturation conditions in the top layer of the snowcover. The delay of the run-off as compared to the time of energy input at the snow surface is found to be due in the first place to the water holding capacity of the snowpack and the horizontal travel time of the meltwater along the ground. Since vertical drains develop in the snowcover the vertical transport time of meltwater is short. The horizontal flow in the basal saturated layer of the snowpack is described by Darcy's law and also by the classical kinematic theory for overland flow. Only for rather large areas is the peak run-off reduced compared to the peak melt rate.

1. *Introduction*

When calculating run-off generated from snowmelt the degree-day method is frequently used, and the snowmelt is treated as a quasi stationary process. However, since the meltrate varies over the day, the run-off from small areas can not be computed without regarding the snowmelt as a transient process. The energy input to the snowpack varies over the day, negative heat is accumulated in the snowpack during the night, and the snow does not lose any water until it has reached its water holding capacity. Also the hydraulic conditions of the snowpack changes so that the snow becomes more and more permeable, and gradually channels are formed both in the vertical an in the horizontal direction. A typical example of how the snowmelt generated run-off varies over the day is shown in Figure 1 for some consecutive days.

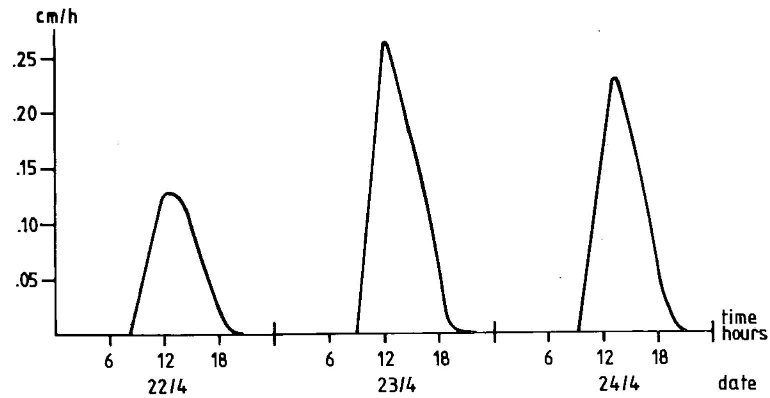

Figure 1. Snowmelt generated run-off from a 8 × 25 m impermeable surface, WREL, Sweden, April 1979.

2. *Energy balance of the snowpack*

The solar radiation intensity varies over the day and so does the air temperature. Whence, the snowmelt rate must vary over the day. In Figure 2 the measured radiation balance and from profile measurements the calculated sensible and latent heat flows during a day of snowmelt in Luleå are shown. Apart from showing that non-steady state conditions prevail the figure shows that the radiation balance is the dominating part of the energy balance.

From Figure 2 it is clear that the energy balance of the snowpack was positive from 7 o'clock in the morning. However, during the night ''negative heat'' was stored in the snowpack, and it took more than 2 hours until the temperature of the entire snowpack rose to 0°C.

The amount of heat that can be lost from the snow to the atmosphere, *i.e* the ''negative heat'' stored in the snowpack, is limited because or the low conductivity of the snow. The heat loss, Φ, from a snowpack of height h having a homogenous temperature of 0°C, which also is the fixed temperature at the base of the snowpack, is

$$\Phi = \varrho_s c_s k_s \frac{-T_a}{h} \left[1 + 2 \sum_{n=1}^{\infty} \exp \frac{-n^2 \pi^2 k_s t}{h} \right] \qquad (1)$$

where ϱ_s = desity of the snow, c_s = specific heat of ice, k_s = thermal diffusivity of snow, T_a = air temperature and t = time.

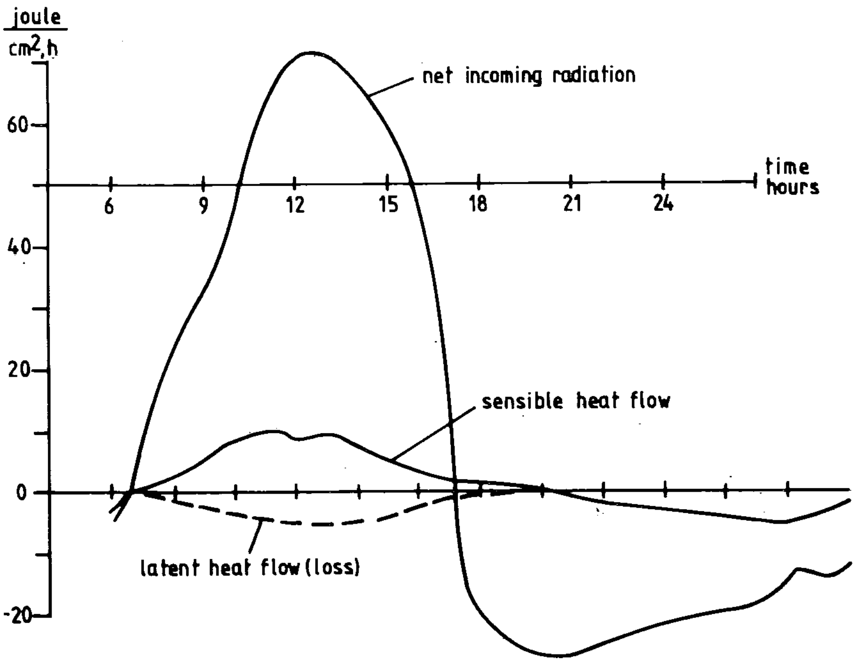

Figure 2. Measured net incoming radiation, latent and sensible heat flows calculated with the profile method to a snow covered surface study plot, WREL, Sweden, April 24, 1979.

For old snow k_s may be about 0.004 cm²/sec. Then for h = 50 cm the heat loss over a period of 12 hours is 2 cal/cm², °C (8 joule/cm², °C). If the air temperature during the night is —5°C, the heat loss is 10 cal/cm² (40 joule/cm²), which correponds to 0.13 cm water equivalents of snowmelt and a delay of the snowmelt of 1—2 hours.

In the previous analysis was not accounted for refreezing of free water in the snow. Heat is released when water freezes to ice, but the heat can only be transported away at a rate determined by the conductivity of the snow and the thermal gradient. Consequently the thermal gradient is very high just below the snow surface. When refreezing is considered the heat flow in the snowpack must be calculated by numerical methods. For a thin layer at the top of the snowpack continuity requires

$$\varrho_s c_s k_s \frac{-2T_a}{\Delta z} \Delta t = \varrho F S \Delta z - \varrho_s c_s T_a \Delta z \tag{2}$$

where Δt = time step and Δz thickness of layer chosen so that there is no heat flow from below into the layer, ϱ = density of water, F = latent heat of fusion, S = liquid water content per unit volume. The temperature distribution is assumed to be linear. The second term on the right hand side is about 50 times smaller than the first term and can therefore be neglected. Eq (2) gives for the top layer

$$\Delta t_1 = A\ \Delta z^2 \tag{3}$$

where

$$A = \frac{\varrho\ F\ S}{\varrho_s\ c_s\ 2\ k_s}\ (-T_a)^{-1} \tag{4}$$

If, still requiring no heat flow from below into the layer under consideration, the required time step for layers below the top layer is found as

$$\Delta t_n = (2n-1)\Delta t_1 \tag{5}$$

where n = number of layer.

The (accumulative) time for the freezing front to reach depth $h_f = m\,\Delta z$ is for small values of Δz

$$\mathrm{t} = 2Ah_f^2 \tag{6}$$

and the heat loss to the atmosphere is approximately

$$\Phi = \pi F S \frac{dh_f}{dt} = 0.5\ \varrho F S A^{-1/2} t^{-1/2} \tag{7}$$

where dh_f/dt = rate of propagation of the freezing front. The total heat loss over the period t is

$$\text{total heat loss} = (2\varrho F S \varrho_s c_s k_s (-T_a))^{1/2} t^{1/2} \tag{8}$$

Using as before $k_s = 0.004$ cm²/sec, $T_a = -5°C$ and using a value of $S = 0.04$ corresponding to a reasonable value of the water holding capacity per unit volume the freezing front is calculated to be 8 cm below the surface after 12 hours, and the total heat loss to the atmosphere is 26 cal/cm² (110 joule/cm²). This value corresponds to 0.33 cm water equivalents of snowmelt.

It is evident that the refreezing phenomenon significantly affects the heat balance of a melting snowcover. It should be emphasized that water hygroscopically or capillary bound below the top layer of the snowpack is not refrozen during the night. The stored "negative heat" to be overcome before melting can start again is not dependent on the refrozen water, which influences the time necessary to reach the water holding capacity of the snow, but only on the temperature distribution within the snowcover. If a linear temperature distribution within the thin refrozen top layer, h_f, is assumed, the stored "negative heat" is

$$H(\text{stored}) = \varrho_s c_s \frac{-T_a}{2} h_f \tag{9}$$

where h_f = the position of the freezing front when the net heat flow to the snow surface becomes positive. When $T_a = -5°C$ and $h_f = 5$—10 cm the "negative heat" is about 2—4 cal/cm² (8—16 joule/cm²). For an energy input corresponding to a melt rate of 0.2 cm/h the run-off is only delayed about 10 minutes after the energy input to the surface has become positive. This is a much lower value than the time of delay of run-off, which was calculated, when refreezing was not taken into account. However, the top layer gains heat at the same time as it is being saturated to its water holding capacity, so the actual time for the temperature to rise 0°C in the entire snowpack depends on when the water holding capacity is reached.

3. *Water holding capacity of the snowpack*

In a snowpack water can exist as hygroscopically bound water adsorbed on the surface of the snow grains, as capillary water held in particular between the small grains and as free water moving downwards through the snowpack. The hygroscopical water and the capillary water can not leave the snowpack until the snow grains melt or change their form. The water holding capacity of a snowpack, *i.e.* the ability of snow to retain water after free drainage, is of primary importance for the run-off process.

In the very early phase of the snowmelt the snow grains are small and the capillary attraction very significant. Soon, however, small ice grains are eliminated by metamorphism and rapid growth continues until diameters of 1—2 mm are achieved. As the small grains disappear the capillary potential is reduced, and the vertical water movement is dominated by gravity force. The capillary fringe is only about 2 cm. Since small grains can retain much more liquid water than large grains, water is suddenly released when the grains grow in size. However, this important phase of the whole snowmelt process does not affect the daily run-off cycle during the peak of the melt period in the spring.

The water holding capacity of ripe snow is reported by U.S. Corps of Engineers [6] to be about 4 % by volume for snow of relative density 0.4, which is 10 % by weight. From measurements in the field and in the laboratory the author has found that the water holding capacity of initially dry snow of relative density 0.2—0.25 is 20—30 % by weight (5—6 % by volume) and that of ripe snow is 10—15 % (4—5 % by volume).

When the snow cover melts at the surface and there is no liquid water in the snowpack, the time needed for the water content to reach the water holding capacity is

$$t = S_i h/m \tag{10}$$

where t = time, S_i= water holding capacity by volume, h = depth of snow cover, m = melt rate at the snow surface. For $S_i = 0.04$, $h = 100$ cm and $m = 0.2$ cm/h, t is 20 hours. This means that if the temperature drops below 0°C during the night and refreezing starts, no run-off takes place.

Now, consider the situation when snowmelt has occurred during the day, and the snowpack is at its full water holding capacity when refreezing starts. As discussed previously the freezing front propagates downwards, but the total distance travelled during a night is only about 5—10 cm. For a liquid water content of 4 % (by volume) the amount of refrozen water is 0.2—0.4 cm^3/cm^2. If the melt rate of the proceeding day is 0.2 cm/h, it takes 1—2 hours for the top layer to once again reach its water holding capacity.

Heating of the top refrozen layer of the snowpack and the increasing of the liquid water content of this layer take part simultaneously, and therefore only the delay to reach the water holding capacity has to be considered.

Inserting the depth of the freezing front found from eq (6) into eq (10) the delay relative to the initiating of snowmelt at the snow surface before any water can leave the top layer of the snowcover is found as

$$t_w = \frac{C_w}{m}\left(\int_0^t - T_a dt\right)^{1/2} = C_w(-\bar{T}_a)^{1/2} t^{1/2} m^{-1} \tag{11}$$

where the integration is performed over the part of the day, when the temperature is below the freezing point, and where the bar indicates the mean value over this time. The coefficient is

$$C_w = \left(\frac{\varrho_s}{\varrho}\frac{c_s}{F} k_s S_i\right)^{1/2} \tag{12}$$

As before m = melt rate at the surface, S_i = water holding capacity per unit volume. The notation is further explained under eq (2). The coefficient is for ripe snow about 0.033 cm, (°C, h)$^{-1/2}$.

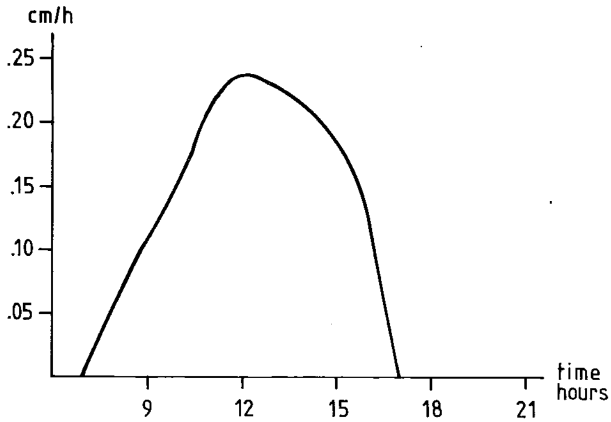

Figure 3. Meltrate at the snow surface, April 24, 1979, the surface study plot, WREL.

From the measurements of April 24 shown in Figure 1, the total energy input has been converted to equivalent melt rate in Figure 3. During 14 hours of the preceeding night the air temperature was below freezing, and the accumulative negative degree-hours were about 60. The melt rate at the surface increased almost linearly with time for the first hours. From eq (11) using C_w = 0.033 cm, (°C, h)$^{-1/2}$ the time for the snowpack to reach its water holding capacity was estimated at 2h/10 min. Thus at 9.10 the meltwater should start to percolate through the snowcover. When the meltwater reaches the base of the snowpack, the run-off from the surface can start. The run-off from the investigated area started on this day at 9.20. Thus the rough calculations agree suprisingly well with the observations.

4. *Vertical transport through the snow cover*

At least in the early phase of the melting period snow can be regarded as a porous medium, in which the flow conditions can be calculated using Darcy's law. The permeability available to the liquid phase depends on the water content. For this reason the first meltwater percolates downwards through the snowpack at a very low rate. As the water content of the snow increases the downward transport velocity increases, and meltwater of later times overtakes the first meltwater of the day. In the very first days of the snowmelt period the vertical permeability may be very small, and the downward travel velocity so low that, if refreezing does not occur, it takes days for the melt flux to reach the base of the snowpack, COLBECK [4]. However, the small ice grains are soon eliminated by metamorphism. By combining Darcy's law and the continuity equatin COLBECK [3,5] showed that the vertical travel velocity is constant for a specific flux of melt water being

$$V_{vert} = B\, m^{2/3} \tag{13}$$

where V_{vert} = vertical travel velocity, m = melt rate at the surface and

$$B = \frac{3}{\Phi_e} \left(\frac{\varrho g k_i}{\mu}\right)^{1/3} \tag{14}$$

where Φ_e = effective porosity of the snowpack, ϱ = density of water, g = acceleration of gravity, k_i = intrinsic premeability, μ = viscosity of water.

The intrinsic permeability is for melting snow with a mean grain diameter of 1—2 mm about $1—3 \cdot 10^{-4}$ cm^2 ($1—3 \cdot 10^{-8}$ m^2). The factor B is then about 2—3 (m/sec)$^{1/3}$, and for m = 0.2 cm/h the vertical transport velocity is 0.8—1.2 cm/min. In a snowpack of thickness 60 cm the horizontal run-off is thus delayed about 1 hour.

After some time, vertical flow channels are developed, and repeated freeze-thaw cycles lead to the formation of ice layers. The vertical transport rate can no more be calculated in a straightforward way. From measurements BENGTSSON and WESTERSTRÖM [1]did not find any delay of run-off due to the vertical transport through the snowpack.

Interflow takes place within the snowpack along the ice layers towards the vertical drains. Vertical drains are of diameter about 0.5—1 m and may be situated 5—10 m apart from each other. The influence area of a drain relative to its area is thus about 100. If the melt water travels fairly fast along the horizontal ice layers ''the effective meltrate'' for the drain is 100 times the melt rate at the snow surface. From eq (13) the vertical travel velocity in the drains assuming a grain diameter of 2 mm and a melt rate at the surface of 0.2 cm/h is calculated to be 25 cm/min. Thus the transport time in the drains is so short that it can be disregarded when estimating the run-off hydrograph.

Horizontal flow velocities are discussed in detail in the next section. The flow velocity along an ice layer can be estimated from Darcy's law. Since the layer just above the ice layer must be saturated, the flow velocity is simply

$$V_{hor} = k_p I_i \tag{15}$$

where the permeability (as velocity) is

$$k_p = \varrho g k_i / \mu \tag{16}$$

and I_i = the slope of the ice layer. For an intrinsic permeability of $3 \cdot 10^{-8}$ m^2 and a slope of 5 % the effective horizontal flow velocity is 1 m/min. It is clear that the horizontal floating time to a drain is of the order of minutes.

5. *Horizontal flow in the snowpack*

When the meltwater reaches the base of the snowpack (the ground) run-off takes place, unless the infiltration rate exceeds the melt rate. Measurements in Luleå have snown that no run-off takes place in the beginning of the melting period, Bengtsson [2].

During the peak of the snowmelt period the soil is saturated to the field capacity and almost all the meltwater runs off along the ground. This is strictly true for the entire melt period, if the ground is impermeable.

The flow in the saturated layer at the base of the snowpack is described by the equation of continuity and a friction equation. For flow of fluids through porous media Darcy's law can be used as the friction equation. This means that the flow velocity is independent of the rate, *s,* at which the meltwater reaches the saturated layer. The run-off is

$$q = k_p I \int_{t-t'}^{t} s/\phi \, dt \tag{17}$$

where q = discharge per unit width, I = horizontal slope of the ground, t = time, s = vertical meltwater input rate to the saturated basal layer, ϕ = porosity of the basal snow layer, and

$$t' = \begin{cases} t_c \text{ if } t \geq t_c \\ t \text{ if } t \leq t_c \end{cases} \tag{18}$$

where the time of concentration is

$$t_c = \frac{L\phi}{k_p I} \tag{19}$$

where L = the run-off lenght of the snow covered area.

At the Division of Water Resources Engineering, Luleå (WREL) the run-off from an impermeable surface study plot of length 25 m and slope 0.02 has been studied. The observed run-off hydrograph of April 24, 1979 is shown in Figure 4.

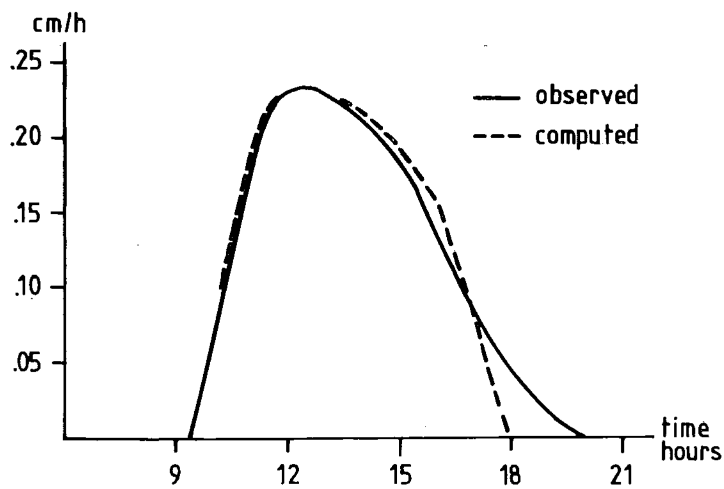

Figure 4. Observed and calculated snowmelt run-off hydrograph assuming Darcian flow, April 24, 1979, study plot at WREL.

For an intrinsic permeability of $3 \cdot 10^{-8}$ m^2corresponding to a ''velocity permeability'' of 0.3 m/sec the time of concentration is from eq (19) estimated at 50 minutes. The run-off hydrograph of April 24, 1979 using the snow surface melt rate shown in Figure 3 was computed assuming that the snowpack had reached its water holding capacity at time 9.20, when the run-off was observed to start. The computations were performed by a straightforward application of Darcian flow. Also the computed hydrograph is shown in Figure 4. The agreement between the calculated and the observed hydrograph is good except for the last part of the recession.

In the classical kinematic theory for overland flow the Manning equation and the continuity equation are combined. When horizontal channels develop in the basal layer of the snowpack, this theory should be applicable also for snowmelt run-off. If the flow channels are assumed to be relatively wide, only the friction at the top and at the bottom of the channels has to be taken into account. The time of concentration is then determined from the implicit expression

$$L = \left(\frac{5}{3}\right) \left(\frac{1}{2}\right)^{2/3} \frac{d\ I^{1/2}}{\phi^{5/3} n} \int_0^{t_c} \left[\int_0^t s\,dt\right]^{2/3} dt \qquad (20)$$

which for constant meltwater input to the basal layer gives

$$t_c = \left(\frac{\text{Ln } 2^{2/3}}{I^{1/2}\ s^{2/3}}\right)^{3/5} \frac{\phi}{d^{3/5}} \qquad (21)$$

where ϕ = porosity of snow when also the channels are considered, d = density of horizontal channels, n = a Manning number. Typical values may be $n = 0.05$, $\phi = 0.6$, $d = 0.1$. The time of concentration for the study plot is then

$$t_c(\text{minutes}) = 30\ s^{-2/5}\ (\text{cm/h}) \qquad (22)$$

which for $s = 0.2$ cm/h is 60 minutes.

The recession of the observed hydrograph is well described by this theory. However, since the run-off during the last part of the recession is very small, no further conclusions should be drawn from the results.

The delay of the peak run-off relative to the peak snowmelt depends on the travel time of the meltwater and thus on the length of the flow path. For a surface area for which the time of concentration is only about 1 hour as is the case with the surface study plot at WREL, Sweden, the maximum run-off rate corresponds to the maximum melt rate. For a surface area of length 200 m and slope 1 % the situation is different. The time of concentration is about 8 hours, and therefore the maximum run-off corresponds to the mean melt rate over 8 hours. The theoretical run-off hydrograph from this area and the run-off from the surface study plot (L = 25 m, I = 2 %) are compared in Figure 5. The maximum run-off from the large area is per unit area only 0.17 cm/h compared to 0.23 cm/h for the small study plot.

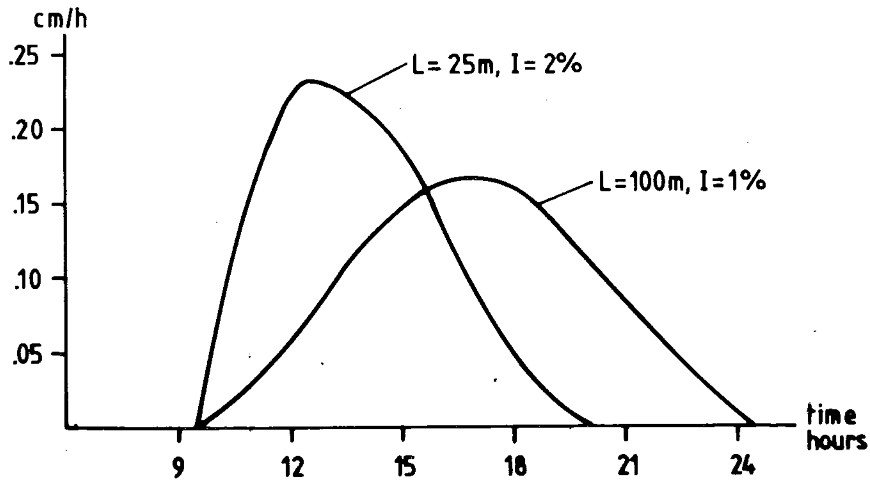

Figure 5. Computed run-off hydrographs using a Darcian approach for two different areas. The melt rate at the surface is given in Figure 3.

It should be noticed that the run-off from the large area continues long after the melting at the surface has stopped. The freezing front proceedes from the top of the snow cover only at a very low speed, and therefore the conditions at the base of the snowpack are not directely affected.

6. *Delay of run-off*

After the heat flow to the snow surface has become positive after a cold night, no run-off is observed downstream of an area until

1) the temperature of the entire snowpack has risen to 0°C.
2) the liquid water content has reached a value corresponding to the water holding capacity of the snowpack.
3) the meltwater has percolated vertically through the snowcover down to the saturated basal layer at the ground.
4) the meltwater input to the base of the snowpack exceeds the infiltration rate into the soil.
5) the meltwater has run along the ground to the outlet position of the area under consideration.

The foregoing analysis has shown that the delay of the run-off as compared to the time of energy input at the snow surface is due in the first place to 2) and 5), and in the beginning of the melting period and for very small slopes also to 4).

The time necessary for rerising the water content of the snowpack to the water holding capacity is approximately proportional to the square root of the accumulative negative degree-hours during the preceeding night and inversely proportional to the melt rate at the snow surface. It may take a few hours before the snowpack regains its waterholding capacity and run-off begins.

The peak of the run-off is, depending on the extension of the snow-covered area, delayed several hours relative to the peak of the melt rate at the snow surface. However, only for rather large surface areas with lengths of several hundred metres is the run-off peak value considerably reduced compared to the peak melt rate.

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