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THE ACCURACY OF DAMPING CONSTANTS DETERMINED BY STANDARD CALIBRATION OF AN ELECTROMAGNETIC SEISMOGRAPH*

by

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Abstract

Numerous procedures for measuring the damping constants of both seismometer and galvanometer are applied in routine practice without estimating their errors. Two methods – the first based on the amplitude ratio and the second on the decay of the maximum deflection – are analyzed in more detail in terms of the attainable accuracy. The parameters listed in the tables yield estimates of the maximum error of the damping constant for given measuring conditions.

1. *Introduction*

Determination of the damping constant is one of the basic tasks in seismograph calibration [3, 5, 9]. Damping constants of seismometer and galvanometer are necessary for calculating magnification and phase response, coupling coefficient and critical resistances. Most methods for measuring damping are based on analysis of the free movement of both systems. The free movement of a galvanometer is easy to record directly using the standard recording equipment of the seismograph.

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However, direct recording of magnified seismometer oscillations is much more complicated and requires a special optical device, with exception of certain special cases where it can be achieved using the recording galvanometer of the seismograph [7]. However, these problems will not be discussed here. We shall deal instead with the record of the linear magnified free movement of the seismometer and/or galvanometer.

Both systems are considered ideal linear oscillators. In the amplitude ratio method the free movement excited by a short impulse at time $t = 0$, when the deflection a from the equilibrium position is zero, is given by the well-known formulae

$$a = k_1 e^{-Dr} \sin(r\sqrt{1-D^2}) \qquad \text{for } D < 1, \tag{1a}$$

$$a = k_2 r e^{-r} \qquad \text{for } D = 1, \tag{1b}$$

$$a = k_3 e^{-Dr} \sinh(r\sqrt{D^2-1}) \qquad \text{for } D > 1, \tag{1c}$$

where the scale factors k_i depend on the magnification of the system movement, initial velocity, damping constant D and period of the system T. The reduced time $r = \omega t$, $\omega = 2\pi/T$. A short impulse means here »short» with respect to the free period T of the system. By the release of the system with zero velocity in the decay of max. deflection method, where the initial deflection $a = a_0 \neq 0$ at $t = 0$, the deflection a at reduced time r is given as follows:

$$a = a_0 e^{-Dr} \left[\frac{D}{\sqrt{1-D^2}} \sin(r\sqrt{1-D^2}) + \cos(r\sqrt{1-D^2}) \right] \qquad \text{for } D < 1, \tag{2a}$$

$$a = a_0 e^{-r}(1+r) \qquad \text{for } D = 1, \tag{2b}$$

$$a = a_0 e^{-Dr} \left[\frac{D}{\sqrt{D^2-1}} \sinh(r\sqrt{D^2-1}) + \cosh(r\sqrt{D^2-1}) \right] \qquad \text{for } D > 1. \tag{2c}$$

It is obvious that the movement of the system caused by the impulse excitation starting at the time r_0 of the first maximum a_m is the same as for the release excitation with $a_0 = a_m$ and having the same damping constant D. At time of the first maximum r_0 given by

$$r_0 = \tan^{-1}(\sqrt{1-D^2}/D) / \sqrt{1-D^2} \qquad \text{for } D < 1, \tag{3a}$$

$$r_0 = 1 \qquad \text{for } D = 1, \tag{3b}$$

$$r_0 = \tanh^{-1}(\sqrt{D^2-1}/D) / \sqrt{D^2-1} \qquad \text{for } D > 1, \tag{3c}$$

the velocity of the system is zero as well as the initial velocity for the release excitation. The condition which must satisfy the impulse test for equivalency with the release test is that the duration of external excitation must be smaller than the time r_0 of the first maximum.

All methods of processing the free movement can be divided into 2 groups, according to the parameters measured: (i) only the amplitudes (*i.e.* maximum deflections) are measured, and the ratio of amplitudes is used for the damping determination, (ii) 2 or 3 deflection points and the corresponding times (or the time intervals between these points) on the response curve are measured and the damping determined from the deflection ratio.

No universal method is available for measuring the whole range of damping constants used with electromagnetic seismographs. Measurement of amplitudes is more suitable for a weakly damped system. It has the advantage that the points at which the deflection is measured are well defined without time determination. Measurement of deflection points and time intervals is more suitable when damping values are near critical, and for overdamped systems. Many modifications of these methods are currently in use [6]. In the further estimation of the errors arising during determination of the damping constant we shall examine the methods of the first group for weakly damped systems. For the other cases we shall select a method based on the decay of the deflection. From this point of view the release excitation is more suitable because the moment of release can be marked separately on the record and determined more accurately than the position of the first maximum of the impulse excitation.

The accuracy attainable in measuring the deflections depends largely on the quality of the recorded trace. For the fine and sharp lines on the photographic paper an accuracy of 0.2 mm can be reached. This value will be used later in evaluation of the methods. To keep the relative error of deflections moderate (*e.g.* smaller than 1 %) measurement of amplitudes greater than 20 mm is preferred. The maximum deflections must of course be within the limits of the linear behaviour of the system. The direct linear proportionality between the trace deflection and the angular deflection of the system holds true for the circular scale. The measured straight-line deflection a_m should be corrected if the error exceeds the necessary accuracy of the deflection determination. The correction term $\bar{a}_m$ is

$$\bar{a}_m = a_m - L \tan^{-1}(a_m/L) \tag{4}$$

and the correct deflection from the zero position $a = a_m - \bar{a}_m$. Here L is the distance between the mirror of the galvanometer and the linear scale at the zero position when the light beam is perpendicular to the scale. Some values of $\bar{a}_m$ are

Table 1. Corrections $\bar{a}_m$(mm) of the measured trace deflection a_m at the recording distance L.

L (mm)	a_m (mm)			
	20	50	100	150
1000	0.003	0.042	0.33	1.11
800	0.004	0.065	0.52	1.72
600	0.007	0.115	0.91	3.01
400	0.017	0.258	2.01	6.49

given in Table 1.

The distance $L = 1$ m is usually used for galvanometers and standard drum recorders, in which case corrections for the deflections up to 100 mm do not exceed 0.2 mm. The correction term for the minimum deflections of 20 mm can be neglected even for the shortest recording distance given in Table 1, which are often used in galvanometer oscillographs. If the light beam in the rest position is not perpendicular to the linear scale the correction terms depend on the deflection polarity and they should be estimated for the particular arrangement of the recorder.

2. *Amplitude ratio method*

The general formula for the damping constant D of the linear system determined from the ratio of successive amplitudes $\nu_1 = A_k/A_{k+1}$ is

$$D = \ln \nu_1 / \sqrt{\ln^2 \nu_1 + \pi^2}. \quad (5)$$

Here both A_k and A_{k+1} are either zero-to-peak or peak-to-peak amplitudes, with $A_k > A_{k+1}$ (Fig. 1). Usually the amplitudes are used because their measurement does not require knowledge of the zero line, and because their errors are smaller. The measured zero-to-peak amplitudes should be corrected as mentioned above before they are used in (5). If the peak-to-peak amplitudes are used the correction term should be determined separately for both zero-to-peak amplitudes, which need not be measured with great accuracy. The sum of corrections should then be subtracted from the measured peak-to-peak amplitude. For the estimated maximum relative error $\bar{D}_1$ (%) and the maximum absolute error $\bar{D}_1$ of the damping constant we have

$$\bar{D}_1\,(\%) = \bar{\nu}_1(\%)\,(1 - D^2)/\ln \nu_1\,, \quad (6)$$

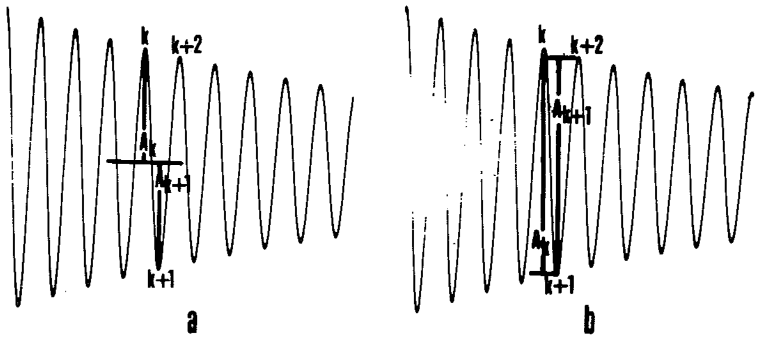


Fig. 1. Amplitudes used for the determination of the damping ratio: (a) zero-to-peak, (b) peak-to-peak.

$$\bar{D}_1 = \bar{\nu}_1(\%)(1-D^2)D/(100 \ln \nu_1), \tag{7}$$

where $\bar{\nu}_1$ (%) is the estimated maximum relative error of the amplitude ratio ν_1.

For the damping ratio $\nu_1 \leqq 1.56$ the approximation of (5) $D = \ln \nu_1/\pi$ gives an error smaller than 1 per cent. Equations (6) and (7) may be simplified to

$$\bar{D}_1(\%) = \bar{\nu}_1(\%)/\ln \nu_1 , \tag{8}$$

$$\bar{D}_1 = \bar{\nu}_1(\%)/(100\,\pi). \tag{9}$$

The relative error $\bar{D}_1$ has its maximum for an undamped system ($\ln \nu_1 \to 0$), and decreases with increasing damping. The maximum absolute error $\bar{D}_1$ is almost constant for the range over which ν_1 is applicable as can be seen from (9). The maximum errors of the damping constant for the relative error of the amplitude ratio $\bar{\nu}_1 = 1$ % are listed in Table 2 and presented in Fig. 2, curves 1 and 3. The maximum error of the amplitude ratio given above can be quaranteed if we take the maximum amplitude A_k = 100 mm and the minimum amplitude $A_{k+1} \geqq 20$ mm and the maximum error of the amplitude determination $\bar{a} \leqq 0.2$ mm. Both amplitudes – maximum 100 mm and minimum 20 mm – are either zero-to-peak or peak-to-peak. In the range of ν_1 from 1.01 to 5.0 the relative errors of the damping constant are from 100.5 % to 0.5 % and the absolute errors from 0.0032 to 0.002.

To reduce the errors of the damping constant determined in (5) the ratio $\nu_n = A_k/A_{k+n}$ ($n > 1$) is used instead of the ratio of successive amplitudes ν_1. For the ratio ν_n it holds that

$$\nu_n = \sqrt[n]{A_k/A_{k+n}} \,. \tag{10}$$

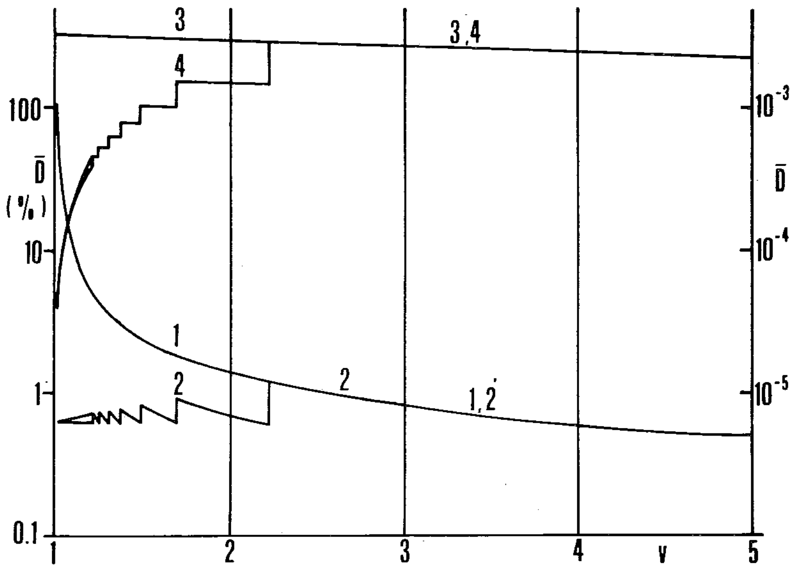


Fig. 2. Relative errors $\bar{D}_1$ (%) curve 1, $\bar{D}_n$(%) curve 2 and the absolute errors $\bar{D}_1$ curve 3, $\bar{D}_n$ curve 4 of the damping constant D for the error in the damping ratio $\nu_1 = 1\,\%$.

The maximum relative error of ν_n, expressed as a percentage in the form $\bar{\nu}_n$ (%) is in the relation with $\bar{\nu}_1$ (%)

$$\bar{\nu}_n = \bar{\nu}_1/n. \tag{11}$$

According to (6) and (7) the errors of the damping constant using ν_n are

$$\bar{D}_n = \bar{D}_1/n. \tag{12}$$

Due to restriction of the minimum amplitude to values equal to or greater than 20 mm the applicable value of n depends on the damping constant. For the particular damping ratio given in Table 2 the calculated values of n are listed for the initial amplitude of 100 mm in the last column. Using these values the relative errors will decrease for $\nu \leqq 2$, and will not exceed 1 % over the whole range (Fig. 2, curve 2). In the same way the absolute errors of the damping constant smaller than 0.28 are smaller than with $n = 1$. For $D \leqq 0.051$ ($\nu \leqq 1.175$) an improvement in accuracy of at least one order of magnitude is achieved since $n \geqq 10$ (Fig. 2, curve 4).

For $n = 1 - 30$ the corresponding amplitude ratio and damping constants are

Table 2. Relative error $\bar{D}_1$ (%) and absolute error $\bar{D}_1$ in the damping constant determined by the ratio of successive amplitudes and the maximum number n for the application of the amplitude ratio.

ν	D	$\bar{D}_1$ (%)	$\bar{D}_1$	n
1.01	0.00316	100	0.0032	162
1.02	0.00630	50.5		81
1.03	0.00941	33.8		54
1.04	0.0125	22.5		41
1.05	0.0155	20.5		33
1.06	0.0185	17.2		27
1.07	0.0215	14.8		23
1.08	0.0245	13.0		20
1.09	0.0274	11.6		18
1.10	0.0303	10.5		16
1.11	0.0332	9.6		15
1.12	0.0360	8.8		14
1.13	0.0389	8.2		13
1.14	0.0417	7.6		12
1.15	0.0445	7.1		11
1.16	0.0472	6.7		10
1.17	0.0500	6.4		10
1.18	0.0526	6.0		9
1.19	0.0553	5.7		9
1.20	0.0580	5.5		8
1.21	0.0606	5.2		8
1.22	0.0632	5.0		8
1.23	0.0657	4.8		7
1.24	0.0683	4.6		7
1.25	0.0710	4.5		7
1.26	0.0734	4.3		6
1.27	0.0759	4.2		6
1.28	0.0783	4.0		6
1.29	0.0808	3.9	0.0032	6
1.30	0.0835	3.8	0.0031	6
1.31	0.0856	3.7		5
1.32	0.0880	3.6		5
1.33	0.0904	3.5		5
1.34	0.0927	3.5		5
1.35	0.0955	3.3		5
1.40	0.1065	2.9		4
1.45	0.1174	2.4		4
1.50	0.1280	2.4		3
1.60	0.1480	2.1		3
1.70	0.1665	1.8	0.0031	3
1.80	0.1839	1.6	0.0030	2
1.90	0.2002	1.5	0.0030	2
2.00	0.2154	1.4	0.0030	2
2.5	0.2800	1.0	0.0028	1
3.0	0.3301	0.8	0.0027	1
3.5	0.3704	0.7	0.0025	1
4.0	0.4037	0.6	0.0024	1
4.5	0.4318	0.5	0.0023	1
5.0	0.4559	0.5	0.0022	1

Table 3. Maximum damping ratio ν and corresponding damping constant D for application of amplitude ratio for a given value of n.

n	ν	D	n	ν	D
1	5.000	0.4559	16	1.106	0.03200
2	2.236	0.2481	17	1.099	0.03012
3	1.710	0.1683	18	1.093	0.02845
4	1.495	0.1270	19	1.088	0.02695
5	1.380	0.1019	20	1.084	0.02561
6	1.308	0.08507	21	1.080	0.02439
7	1.258	0.07299	22	1.076	0.02328
8	1.223	0.06391	23	1.072	0.02227
9	1.196	0.05683	24	1.069	0.02134
10	1.175	0.05116	25	1.066	0.02049
11	1.157	0.04652	26	1.064	0.01970
12	1.143	0.04265	27	1.061	0.01897
13	1.132	0.03938	28	1.059	0.01829
14	1.122	0.03657	29	1.057	0.01766
15	1.113	0.03413	30	1.055	0.01707

given in Table 3. The highest measurable damping constant is 0.456 (n = 1). Use of the amplitude ratio method for measuring higher damping constants needs initial amplitudes $A_k > 100$ mm and/or $A_{k+n} < 20$ mm. The first requirement is limited by the range of linearity of the system the second by the limit of accuracy of linear measurements of amplitudes.

With this method determination of a damping constant smaller than 0.45 is very simple and the maximum attainable error of 2–3 % is quite satisfactory.

One use for damping constants is in determination of the critical resistance of the system transducer. This is necessary for the calculation of resistors values when adjusting the damping constant to the prescribed value [3]. The critical resistance R_c is defined as the total resistance of the circuit which provides electromagnetic damping of system equal to the critical value ($D = 1$) when the other sources of damping are zero. If D_1 and D_2 are the damping constants for the total circuit resistances R_1 and R_2, respectively, the critical resistance is

$$R_c = (D_2 - D_1) R_1 R_2 / (R_1 - R_2) . \quad (13)$$

In the case of a galvanometer the first measurement is usually made for the open circuit ($R_1 = \infty$) with corresponding damping constant D_0 and

$$R_c = (D_2 - D_0) R_2 . \quad (14)$$

The maximum error $\bar{R}_c$ of the critical resistance can be found from

$$\bar{R}_c(\%) = \bar{D}_{21}(\%) + \bar{R}(\%), \tag{15}$$

where $\bar{D}_{21}$ (%) is the relative error of the damping difference $D_2 - D_1$ and $\bar{R}$ (%) is the estimated maximum error of the resistance expression $R_1 R_2/(R_1 - R_2)$ in (13). If we take $\bar{D}_{21}$ as the maximum error with the given errors $\bar{D}_1$ and $\bar{D}_2$ for the amplitude ratio ν, we arrive at the minimum difference between the two damping constants

$$D_2 - D_1 = 100(\bar{D}_1 + \bar{D}_2)/\bar{D}_{21}(\%) \tag{16}$$

According to Table 2 the magnitude of $\bar{D}_1 + \bar{D}_2$ for $\nu_1 = 1\,\%$ is between 0.0044 and 0.0064, which yields a difference in damping constants of $0.44 \leqq D_2 - D_1 \leqq 0.66$ for a maximum error of $\bar{D}_{21} = 1\,\%$. This difference cannot be attained because the maximum measurable damping constant in this case in only 0.456.

Better results can be obtained for the amplitude ratio $\nu_n (n > 1)$, of course. If the open circuit damping constant (or the damping constant in the first measurement) is small, *e.g.* $D_1 \leqq 0.05$; then $n \geqq 10$ may be used, $\bar{D}_1 + \bar{D}_2 \leqq 0.00032 + 0.0022 = 0.0025$ and the difference $D_2 - D_1 = 0.25$ is satisfied for $0.3 \leqq D_2 \leqq 0.45$. If the constant D_1 is bigger, as it is with long-period galvanometers, in which the open circuit damping constant $D_0 = D_1 = 0.2$, n may only be equal to 2. For the maximum damping constant $D_2 = 0.45$ we have in this case $\bar{D}_1 + \bar{D}_2 = 0.0015 + 0.0022 = 0.0037$ and from (16) it follows that $\bar{D}_{21} = 1.5\,\%$. By adding to this the error of the total circuit resistance $\bar{R}$, the maximum error for the particular conditions of measurement can be tested.

3. *Deflection ratio method*

This method is based on the ratio of deflections a_0 and a_1 which are measured at the time of the system release ($t_0 = 0$) and at time t_1. The corresponding reduced times are then $r_0 = 0$ and $r_1 = 2\pi t_1/T$, respectively. The ratio of these two deflections $Q = a_1/a_0 < 1$, and for a_0 equal to unity, *i.e.* $Q = a_1$ (Fig. 3). Unlike the previous method, where the points at which the deflections are measured are determined by the extremes of the decay curve, here the time interval t_1 and the free period T of the system must be known in order to calculate r_1. Further, the deflection cannot be measured without knowledge of the zero line. The time r_1 for the deflection ratio Q_1 from the recorded curve is then compared with the values calculated for the set of damping constants to find the best approximation. The most comprehensive tables of the reduced time, which have been published by Morencos in [4], are extended in Table 4. The listed reduced time $r = r_1 - r_0$

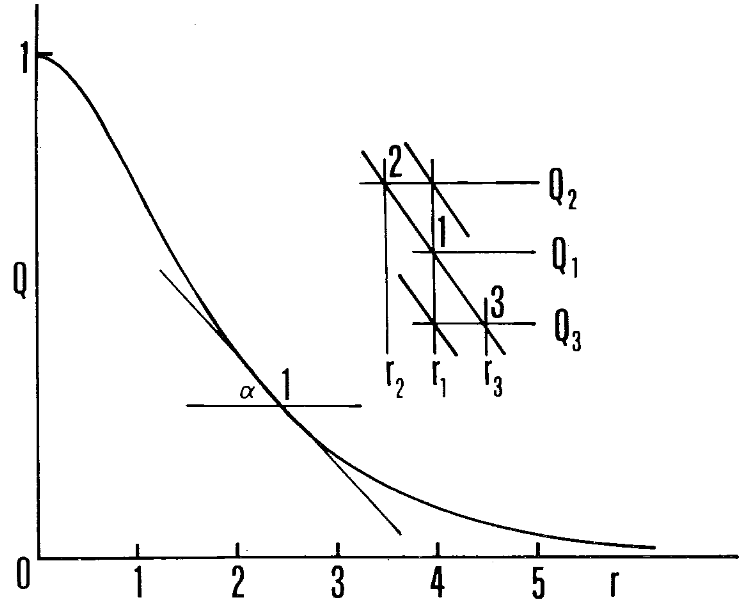


Fig. 3. The deflection ratio Q, with maximum deflection normalized to unity in relation to the reduced time r and the ratio Q for errors of time measurement.

is given with four significant figures for the range of the ratio Q from 0.1 to 0.7 and the damping constants in the range 0.3 – 3.0. The step of both parameters is 0.05. The general course of the reduced time in Fig. 4 shows its monotonous increase for $Q \to 0$ and particular damping constant D. We shall try to find some estimates for the errors in the damping constants determined by this method.

The attainable accuracy of the damping constant is limited by the error of the free period determination, time axis resolution and the precision of the deflection measurement on the record. To get an accurate value of a deflection a_1 corresponding a given ratio Q_1, a sufficient angle α (Fig. 3) should appear between the tangent of the decay curve and the straight line parallel to the time axis at the deflection $a = a_1$. An angle α greater than 10° is suitable for use with fine records. To obtain a sufficient time resolution for the given accuracy of the abscissa length determination (*e.g.* 0.1 – 0.2 mm) the minimum recording speed should be limited. On the other hand, the upper limit of the recording speed is reduced by the angle α, which decreases with increasing recording speed.

If the linear resolution on the abscissa is Δx and the time uncertainty should be smaller or equal to Δt, the recording speed v_r must be

Table 4. Reduced time r for the damping constants D and the deflection ratio Q.

D	Q												
	0.70	0.65	0.60	0.55	0.50	0.45	0.40	0.35	0.30	0.25	0.20	0.15	0.10
0.30	0.8677	0.9499	1.029	1.106	1.182	1.257	1.332	1.406	1.481	1.557	1.634	1.713	1.794
0.35	0.8811	0.9661	1.048	1.129	1.208	1.287	1.366	1.445	1.525	1.607	1.690	1.776	1.865
0.40	0.8949	0.9828	1.068	1.152	1.236	1.319	1.402	1.486	1.572	1.660	1.750	1.845	1.944
0.45	0.9091	1.000	1.089	1.177	1.264	1.352	1.440	1.530	1.622	1.717	1.816	1.920	2.030
0.50	0.9237	1.018	1.111	1.202	1.294	1.386	1.480	1.576	1.675	1.778	1.886	2.001	2.126
0.55	0.9388	1.037	1.133	1.229	1.325	1.423	1.522	1.625	1.732	1.844	1.963	2.091	2.232
0.60	0.9543	1.056	1.156	1.257	1.358	1.461	1.567	1.677	1.792	1.915	2.046	2.189	2.350
0.65	0.9703	1.076	1.180	1.286	1.392	1.501	1.614	1.732	1.857	1.991	2.136	2.298	2.483
0.70	0.9869	1.096	1.205	1.316	1.428	1.544	1.664	1.791	1.926	2.073	2.234	2.416	2.631
0.75	1.004	1.118	1.231	1.347	1.465	1.588	1.717	1.853	2.000	2.161	2.340	2.547	2.797
0.80	1.022	1.140	1.258	1.380	1.505	1.635	1.772	1.919	2.078	2.255	2.454	2.689	2.981
0.85	1.040	1.162	1.286	1.414	1.545	1.684	1.830	1.989	2.162	2.355	2.577	2.844	3.184
0.90	1.058	1.186	1.315	1.449	1.588	1.735	1.892	2.062	2.250	2.462	2.709	3.010	3.405
0.95	1.078	1.210	1.345	1.485	1.632	1.788	1.956	2.139	2.342	2.574	2.848	3.187	3.642
1.00	1.097	1.235	1.376	1.523	1.678	1.844	2.022	2.219	2.439	2.693	2.994	3.372	3.890
1.05	1.118	1.261	1.409	1.563	1.726	1.901	2.092	2.303	2.540	2.816	3.146	3.565	4.145
1.10	1.139	1.287	1.442	1.604	1.776	1.961	2.164	2.389	2.645	2.943	3.303	3.763	4.404
1.15	1.160	1.315	1.476	1.646	1.827	2.023	2.239	2.479	2.754	3.075	3.464	3.964	4.664
1.20	1.182	1.343	1.511	1.689	1.880	2.087	2.315	2.571	2.864	3.208	3.627	4.166	4.922
1.25	1.205	1.372	1.547	1.734	1.934	2.153	2.394	2.666	2.977	3.345	3.793	4.369	5.180
1.30	1.228	1.402	1.585	1.779	1.990	2.220	2.475	2.762	3.093	3.483	3.959	4.572	5.436
1.35	1.252	1.432	1.623	1.826	2.047	2.288	2.557	2.860	3.209	3.621	4.125	4.775	5.690
1.40	1.277	1.464	1.662	1.874	2.105	2.359	2.641	2.960	3.327	3.761	4.293	4.977	5.942
1.45	1.301	1.496	1.702	1.923	2.164	2.430	2.725	3.060	3.445	3.901	4.459	5.178	6.192
1.50	1.327	1.528	1.743	1.973	2.225	2.502	2.811	3.161	3.565	4.042	4.627	5.380	6.441
1.60	1.379	1.595	1.826	2.076	2.348	2.649	2.985	3.365	3.805	4.324	4.960	5.780	6.935
1.70	1.434	1.665	1.913	2.181	2.475	2.799	3.161	3.572	4.046	4.606	5.292	6.177	7.423
1.80	1.491	1.737	2.001	2.289	2.603	2.951	3.339	3.779	4.287	4.888	5.624	6.572	7.909
1.90	1.549	1.810	2.092	2.398	2.733	3.104	3.518	3.987	4.529	5.170	5.955	6.966	8.391
2.0	1.608	1.885	2.184	2.509	2.865	3.258	3.698	4.196	4.772	5.452	6.285	7.358	8.871
2.1	1.669	1.962	2.278	2.621	2.997	3.413	3.878	4.405	5.013	5.733	6.613	7.749	9.349
2.2	1.731	2.040	2.373	2.735	3.131	3.569	4.059	4.615	5.256	6.014	6.942	8.139	9.825
2.3	1.794	2.118	2.468	2.848	3.265	3.725	4.240	4.824	5.498	6.295	7.270	8.527	10.30
2.4	1.858	2.197	2.564	2.963	3.400	3.882	4.422	5.034	5.740	6.575	7.598	8.915	10.77
2.5	1.922	2.277	2.661	3.078	3.534	4.039	4.604	5.243	5.982	6.855	7.924	9.303	11.24
2.6	1.988	2.358	2.758	3.193	3.670	4.197	4.786	5.453	6.224	7.136	8.251	9.689	11.72
2.7	2.053	2.439	2.856	3.309	3.806	4.354	4.968	5.663	6.466	7.415	8.578	10.08	12.19
2.8	2.119	2.521	2.954	3.425	3.942	4.512	5.150	5.873	6.708	7.695	8.903	10.46	12.66
2.9	2.186	2.603	3.053	3.542	4.078	4.670	5.332	6.083	6.950	7.975	9.230	10.85	13.13
3.0	2.253	2.685	3.152	3.659	4.214	4.828	5.515	6.293	7.192	8.254	9.555	11.23	13.59

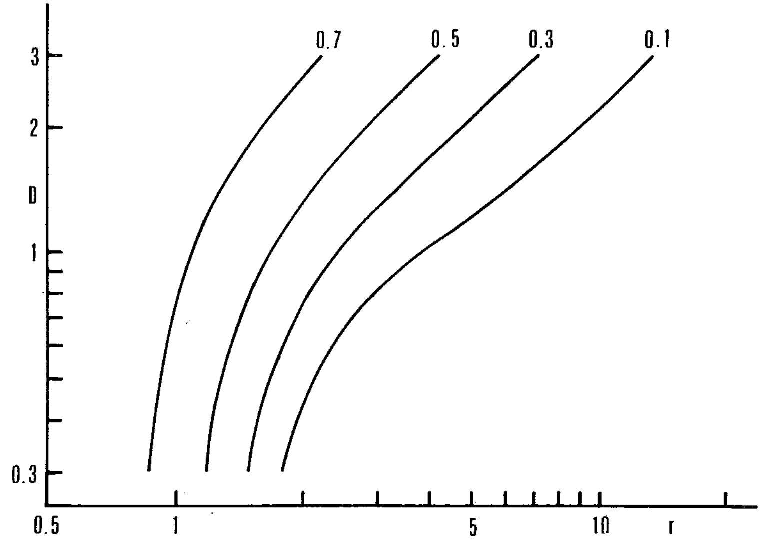


Fig. 4. Reduced time r versus damping constant D for the given deflection ratio Q.

$$\nu_r \geqq \Delta x/\Delta t. \tag{17}$$

For the slope of the tangent of the recorded decay curve

$$\left|\frac{da}{dx}\right| = \left|\frac{da}{dr}\right| \frac{2\pi}{T} \nu_r > \mathrm{tg}\alpha' \tag{18}$$

where α' is the minimum permissible angle α. Using Eqs (17) and (18) we get for the minimum recording speed $\nu_{\min}$ and the maximum speed $\nu_{\max}$

$$\nu_{\min} = \frac{\Delta x}{\Delta t} \leqq \nu_r \leqq \left|\frac{da}{dr}\right| \frac{2\pi}{T\,\mathrm{tg}\alpha'} = \nu_{\max} \tag{19}$$

If the errors Δx and Δt are fixed, the upper limit of the recording speed depends on the constants of system T and D and on the deflection ratio Q. Because $\nu_{\max} \geqq \nu_{\min}$ the application of this method is limited to certain intervals of Q. For a rough estimation of the recording speed the values of $s = |da/dr|$ for particular damping constants and deflection ratio for $a_0 = 100$ mm are given in Table 5 and Fig. 5. Using these values and a period T in seconds, the calculated maximum recording speed $\nu_{\max}$ from (19) is in mm/s.

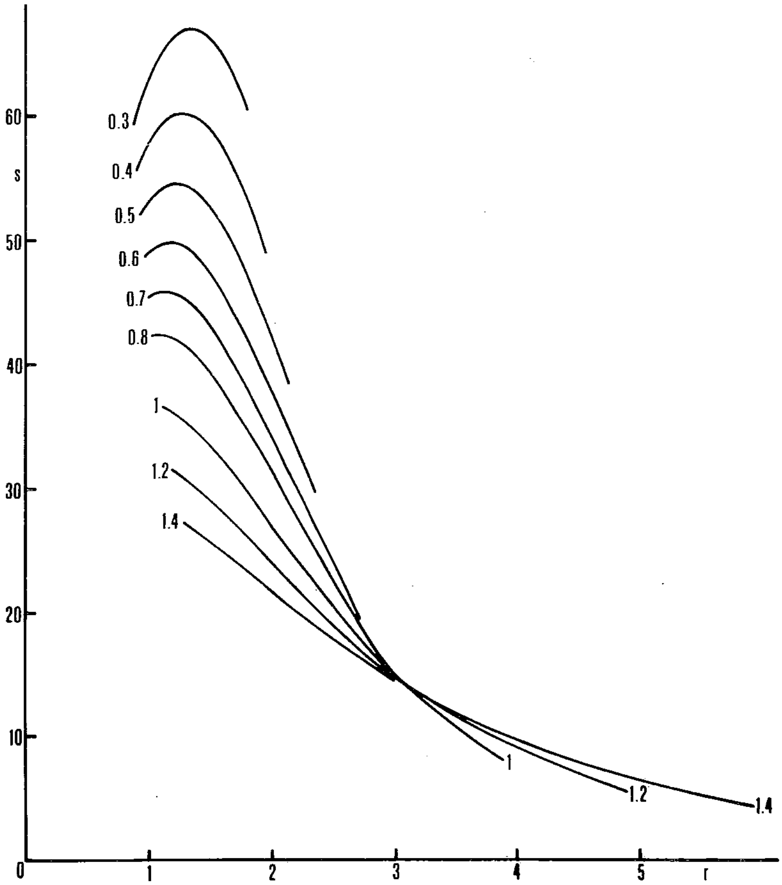
The greater the value of s, the greater the applicable maximum recording speed and the greater the accuracy of the time measurement is reached. For this reason

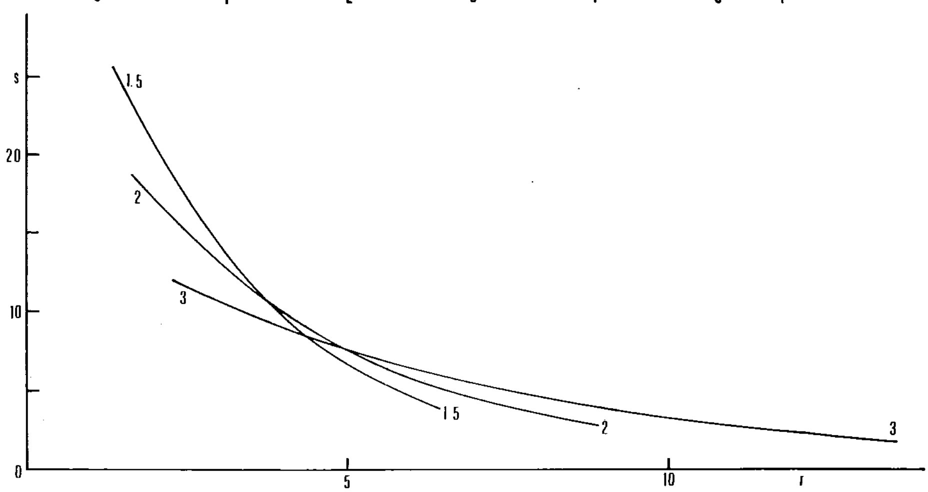
Table 5. Values of s for damping constants D and deflection ratio Q.

D	Q						
	0.7	0.6	0.5	0.4	0.3	0.2	0.1
0.3	59.5	64.0	66.4	67.1	66.4	64.2	60.6
0.4	55.8	59.1	60.3	59.7	57.7	54.1	49.0
0.5	52.2	54.4	54.4	52.8	49.6	44.9	38.4
0.6	48.7	49.9	49.0	46.4	42.2	36.5	29.1
0.7	45.5	45.7	43.9	40.5	35.7	29.3	21.2
0.8	42.3	41.7	39.3	35.3	30.0	23.3	15.0
0.9	39.4	38.1	35.1	30.7	25.2	18.5	10.7
1.0	36.6	34.7	31.3	26.8	21.3	15.0	7.9
1.1	34.0	31.7	28.0	23.5	18.2	12.5	6.3
1.2	31.6	29.0	25.2	20.7	15.8	10.7	5.4
1.3	29.4	26.6	22.8	18.5	14.0	9.4	4.7
1.4	27.4	24.4	20.7	16.7	12.6	8.4	4.2
1.5	25.5	22.5	19.0	15.2	11.4	7.6	3.8
2.0	18.7	16.1	13.4	10.7	8.0	5.4	2.7
2.5	14.6	12.5	10.4	8.3	6.3	4.2	2.1
3.0	12.0	10.3	8.6	6.9	5.1	3.4	1.7

the reduced time r near the maximum s for the particular damping constant is preferred, *i.e.* higher values of the deflection ratio are more suitable. An error $\bar{r}$ in measuring the time interval t_1 causes measurement of the wrong deflection on the decay curve. At time $r_2 = r_1 - \bar{r}$ we get a greater deflection than at the correct time r_1 for fixed Q_1, and the longer time $r_3 = r_1 + \bar{r}$ yields a smaller value of the deflection (Fig. 3). The errors in deflection $\bar{a}$ calculated for errors in the reduced time $\bar{r} = \pm 0.1$ and for the initial deflection $a_0 = 100$ mm are presented in Fig. 6. The greatest errors appear for the maximum value of Q and with small damping constants. These cases, however, give the best accuracy for determination of time t_1, as mentioned above.

The deflections determined due to measurements at erroneous times r_2 and r_3 yield deflection ratios Q_2 and Q_3, respectively (Fig. 3). If we assume that these values are valid at the correct time r_1 we get damping constants D_2 and D_3 instead of D_1. The errors $\bar{D} = D_i - D_1$ in the damping constants derived in this way are presented for $\bar{r} = \pm 0.1$ in Fig. 7. There are very great errors for damping less than critical. The error in the reduced time $\bar{r} = 0.1$, which was used for testing, gives a time error of 0.016 s for a system with free period 1 s. In this case, with an assumed error of linear measurement of time equal to 0.2 mm, a recording speed of 13.5 mm/s is needed.





Figs. 5a and 5b. Values of s versus the reduced time r for the given damping constants D.

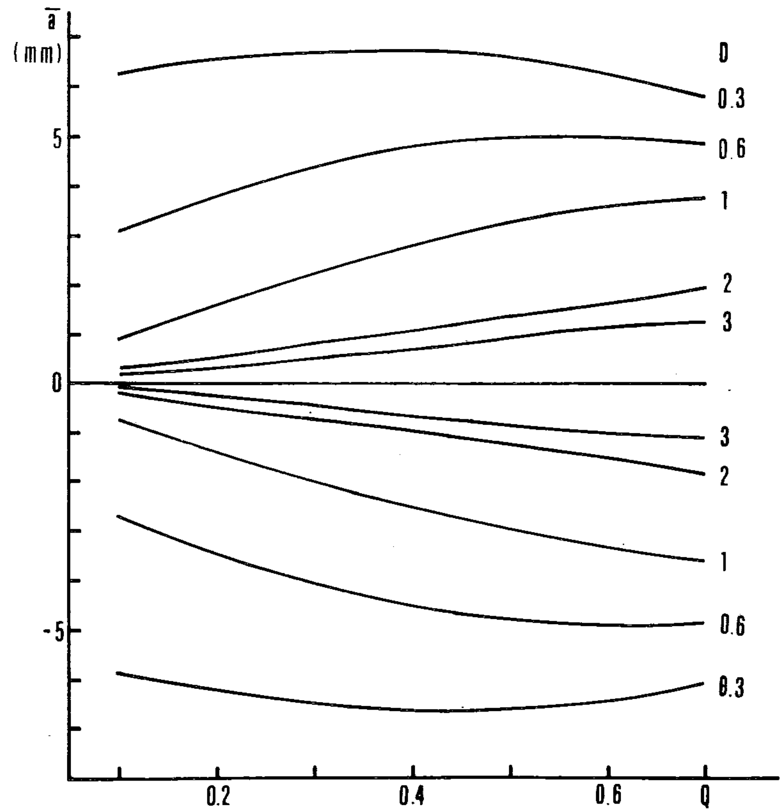


Fig. 6. Deviations $\bar{a}$ (mm) of the deflection for the deviations $\bar{r}$ from the correct time: $\bar{r}$ = −0.1 in the upper part, $\bar{r}$ = 0.1 in the lower part.

The standard recorders of electromagnetic seismographs have a maximum recording speed of only 1 mm/s by the short-period systems. For the long-period galvanometer with period $T = 100$ s the necessary resolution of the time axis is 1.6 s, which is achieved at a recording speed of 0.135 mm/s. The standard recording speed of the long-period electromagnetic seismograph (0.25 mm/s) is therefore sufficient.

From these model tests it follows that the errors in time measurement limit the effectiveness of the damping constant determination, particularly for short-period systems without special recording devices. However, the recording speed cannot be arbitrarily increased for solving this problem because relation (19) is not satisfied and because the point on the decay curve for a given deflection cannot be determined with certainty.

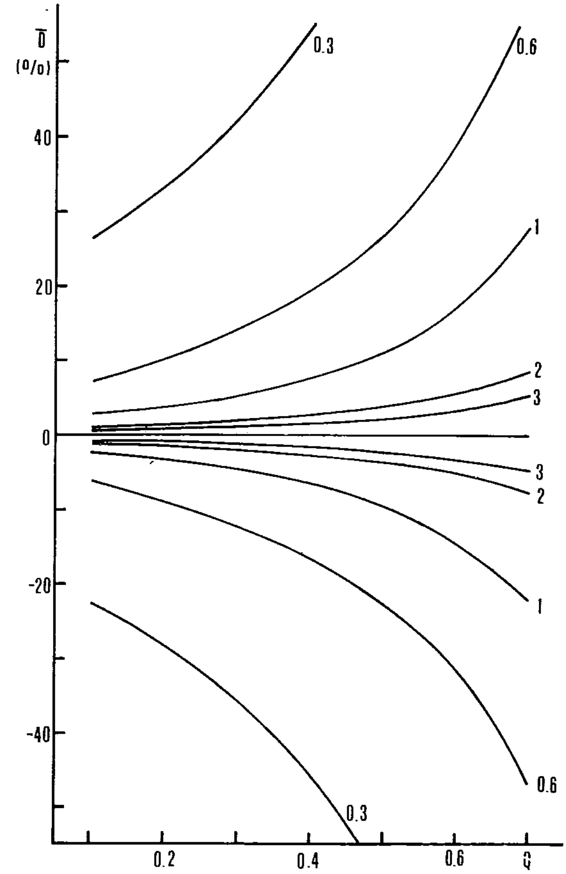


Fig. 7. Errors in the damping constant $\bar{D}$ (%) due to error in the time determination: $\bar{r} = -0.1$ in the upper part, $\bar{r} = 0.1$ in the lower part.

If the time interval t_1 is correctly determined the influence of assumed maximum deflection errors $\bar{a} = \pm 0.2$ mm is small. With an initial deflection a_0 = 100 mm as in the preceding cases the damping constant error is up to 1 % for $Q < 0.4$. The relative errors $\bar{D}$ (%) in Fig. 8 are valid for $\bar{a} > 0$; for $\bar{a} < 0$ the errors have a negative sign.

Deflection ratios smaller than those listed in Table 4 are used for checking the adjustment of long-period galvanometers to critical damping [1, 2]. The light spot for correct damping should be stationary with no further movement at time equal to the free period T after the release of the galvanometer. This time corresponds to the reduced time $r = 2\pi$ and to the deflection ratio Q = 1.36 % of the initial

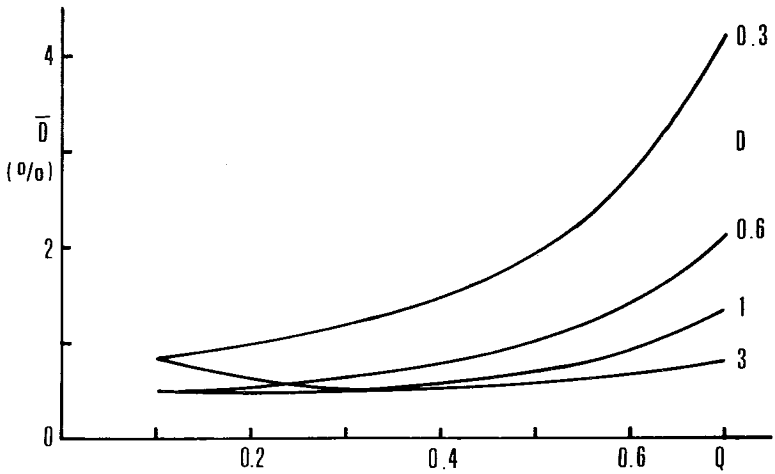


Fig. 8. Errors $\bar{D}$ (%) in the damping constant due to error $\bar{a}$ = 0.2 mm of the deflection.

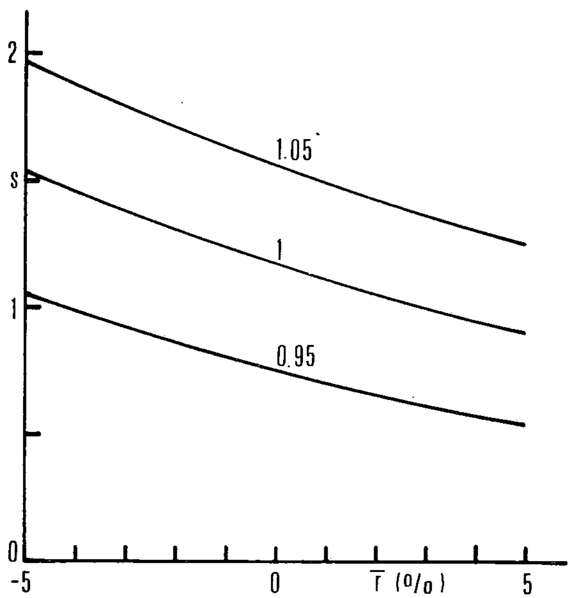


Fig. 9. Values of s versus the reduced time deviations $\bar{r}$ (%) from $r = 2\pi$ for the damping constants in the range 0.95 – 1.05.

deflection. Minimum values of s for 5 % errors in the reduced time and 5 % errors in the damping constant are between 0.54 and 1.25 (Fig. 9). The maximum recording speed 0.19 – 0.44 mm/s for $\alpha' = 10^\circ$ is comparable with the speed of the standard drum recorder (0.25 mm/s). Assuming maximum errors of 0.5 mm in the linear measurement on both axes we get for T = 100 s and a_0 = 100 mm: $\bar{r}$ = 2 % and Q = (1.36 + 0.5) % (Fig. 10). For these maximum errors the range of the damping constants will be from 0.96 to 1.04, *i.e.* the maximum error of the critical damping constant determination will be ±4 % [8].

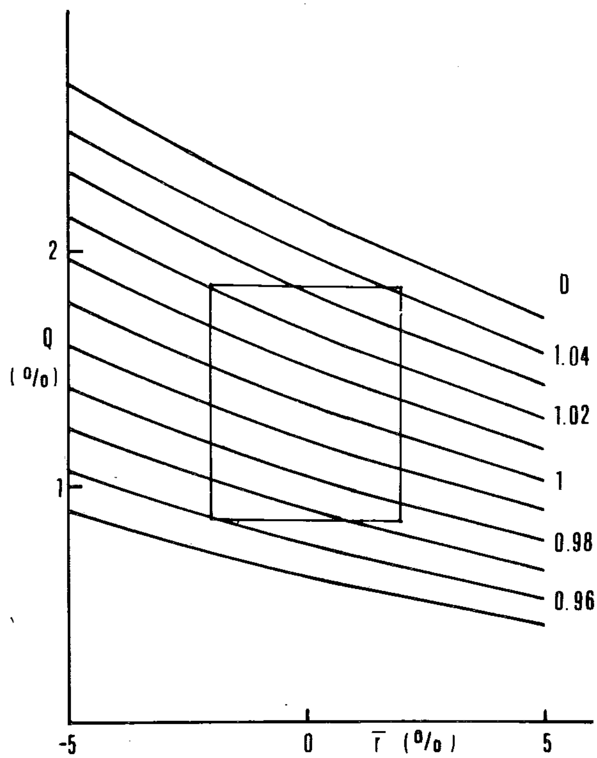


Fig. 10. Deflection ratio Q(%) versus time deviations $\bar{r}$ (%) from $r = 2\pi$ for damping constants near the critical value.

Table 6. Deflection ratio Q (%) for deviations of the reduced time $\bar{r}$ (%) from $r = 2\pi$ for damping constants near the critical value.

D	$\bar{r}$ (%)										
	−5	−4	−3	−2	−1	0	1	2	3	4	5
0.95	0.9042	0.8399	0.7798	0.7235	0.6709	0.6216	0.5757	0.5327	0.4926	0.4552	0.4203
0.96	1.075	1.004	0.9385	0.8765	0.8183	0.7636	0.7124	0.6643	0.6192	0.5769	0.5373
0.97	1.248	1.172	1.100	1.032	0.9686	0.9086	0.8521	0.7989	0.7489	0.7018	0.6575
0.98	1.424	1.342	1.264	1.191	1.122	1.056	0.9947	0.9365	0.8816	0.8297	0.7808
0.99	1.602	1.514	1.431	1.352	1.277	1.207	1.140	1.077	1.017	0.9606	0.9071
1.00	1.782	1.688	1.600	1.516	1.436	1.360	1.288	1.220	1.155	1.094	1.036
1.01	1.964	1.865	1.771	1.681	1.596	1.516	1.439	1.366	1.297	1.231	1.168
1.02	2.148	2.044	1.944	1.850	1.759	1.674	1.592	1.514	1.440	1.370	1.303
1.03	2.334	2.225	2.120	2.053	1.925	1.834	1.747	1.665	1.586	1.511	1.440
1.04	2.522	2.407	2.297	2.192	2.092	1.996	1.905	1.818	1.735	1.655	1.579
1.05	2.712	2.592	2.477	2.367	2.261	2.161	2.065	1.973	1.885	1.801	1.721

4. *Conclusions*

The two methods for determining the damping constant mentioned above are often used at seismic stations because of the simplicity of measurements and calculation procedures. The amplitude ratio method is sufficiently accurate with the standard analog records for weakly damped systems. The deflection decay method, which is applicable to damping near the critical value and for over-damped systems, makes special demands on the time axis resolution and on determination of the point for a given deflection on the decay curve. Due to the recording speed of the standard electromagnetic seismographs the error in the damping constant measurement of short-period systems is considerably higher than with the first method.
To improve the accuracy of the damping constant determination, another procedure should be used. This problem will be treated in a separate paper.

REFERENCES

1. ANONYMOUS, 1962: *Operation and maintenance manual, world-wide seismograph system, model 10700: calibration of the world-wide seismograph system.* Geotechnical corporation, Garland.
2. ANONYMOUS, 1970: *Calibration of the WWSSN and the ESN seismograph systems.* U.S. Coast and Geodetic Survey, Washington, D.C.
3. ARANOVICH, Z.I., KIRNOS, D.P. and V.M. FREMD (ed.), 1974: *Apparatus and methods of observations in the seismic stations of the USSR* (in Russian). Moscow, Nauka.
4. MORENCOS, J.A., 1966: The determination of seismometer damping. *Bull. Seism. Soc. Am.*, 56, 383–392.

5. SAWARENSKI, E.F. and D.P. KIRNOS, 1960: *Elemente der Seismologie und Seismometrie.* Berlin, Akademie-Verlag.
6. TEUPSER, CH., 1962: Die Eichung und Prüfung von elektrodynamischen Seismographen. *Freiberger Forschungshefte* C 130, Berlin, Akademie-Verlag.
7. TOBYÁŠ, V., 1974: Calibration of a seismometer of an electromagnetic seismograph. *Geofysikalni sbornik* 20(1972), 263–286. Praha, Academia.
8. TOBYÁŠ, V., TEIKARI, P. and E. VESANEN, 1976: Constants of long-period seismographs at Nurmijärvi (Finland). *Geophysica,* 14, 99–109.
9. WILLMORE, P.L. (ed.), 1970: *Manual of seismological observatory practice.* International seismological centre, Edinburgh.