551.511.2
551.511.32
551.513.1

A DIAGNOSTIC STUDY ON THE KINETIC ENERGY BALANCE OF THE LONG-TERM MEAN FLOW AND THE ASSOCIATED TRANSIENT FLUCTUATIONS IN THE ATMOSPHERE

by

E.O. HOLOPAINEN

Department of Meteorology
University of Helsinki

Abstract

Some aspects of the geographical distribution of the budget of kinetic energy are discussed on the basis of two sets of upper-air statistics (one roughly covering the period 1950–54, the other 1958–62) for the Northern Hemisphere. Direct estimates have been obtained for those terms which depend only upon the horizontal velocity. The dissipation has been assumed to be proportional to the kinetic energy at the 850 mb level.

There are considerable differences in the amount of kinetic energy between the two sets of statistics over the ocean areas where there is a paucity of data. These differences reflect the uncertainty that still exists concerning some basic climatological characteristics of the atmosphere even over the Northern Hemisphere.

The results show that the horizontal flux divergence is very important in the local balance of total kinetic energy and the local balance of the energy of the mean motion, and that the horizontal (large-scale) Reynolds' stresses $\overline{u'^2}$, $\overline{v'^2}$, $\overline{u'v'}$ play an important role in the flux. The results obtained for the flux divergence indicate that in subtropical latitudes generation exceeds dissipation, while the reverse is true in middle latitudes. Considerable longitudinal differences exist. The eastern parts of the continents, for example, appear to be source (*i.e.* flux divergence) regions and most of the oceans sink (*i.e.* flux convergence) regions for kinetic energy.

The conversion of energy from the mean flow to transient fluctuations (defined as the product of momentum flux and the gradient of mean velocity) is generally negative but relatively small in magnitude. Over the Northern Hemisphere the large-scale transient fluctuations feed kinetic energy to the time-mean motion at an average rate of about 0.3 Wm^{-2}.

1. *Introduction*

The atmospheric flow is often thought to consist of an axisymmetric component and superimposed »eddies» with different zonal wave numbers. Both the axisymmetric component and eddy component may be further thought to consist of a stationary (time-average) part and a transient part. Several diagnostic studies on the energetics of the atmosphere (*e.g.* NEWELL *et al.* [12], OORT and PEIXOTO [14]) have been carried out using this formal scheme, and at present we have a fairly good picture of how the energy balance of the axisymmetric flow and eddies is maintained over the Northern Hemisphere.

The studies of this type do not, however, give any information about the importance, at any particular location, of the fluctuating component of the flow (large-scale turbulence) in the maintenance of the time-mean flow. To study this question we can consider the atmospheric flow during any time period as the sum of the time-mean flow and transient fluctuations and then examine the energies of these two components. Several investigations have been carried out along this line (*e.g.* MONIN [11]; KAO and HURLEY [7]; TSAY and KAO [19], [20]; VAN DEN DOOL [21]; SAVIJÄRVI [17]) for some geographical location, certain isobaric levels or for some relatively short time period. The purpose of the present study is to provide for the Northern Hemisphere geographical distributions for all the terms in the relevant kinetic energy equations which can be evaluated from the available long-term upper-wind statistical data.

The transient fluctuations are in this study defined as deviations from a 5-year time-average. They include, therefore, the effect of all oscillations with periods from a few years down to about one day. These different oscillations owe their existence to different physical mechanisms (*e.g.* annual variation, index cycle, passage of moving baroclinic waves etc.). However, the main part of the Reynolds' stresses considered here is, at least in middle and high latitudes, due to »synoptic» disturbances.

2. *Equations of kinetic energy balance*

We consider an arbitrary variable s as the sum of two components:

$$s = \bar{s} + s' ,$$

where $\bar{s}$ is the mean value over some period of time (in our case usually 5 years) and s' is the transient part of s, *i.e.* the deviation of the instantaneous value of s from $\bar{s}$.

The time-mean kinetic energy of the horizontal motion V (= $ui + vj$; u and v are the zonal and meridional components) can be written as

$$\bar{k} = \frac{1}{2}\overline{V^2} = k_M + k_T, \tag{1}$$

where

$$k_M = \frac{1}{2}\bar{V}^2 \quad \text{(kinetic energy of the mean flow)} \tag{2}$$

and

$$k_T = \frac{1}{2}\overline{V'^2} \quad \text{(kinetic energy of the transient fluctuations)} \tag{3}$$

The equations for the balance of $\bar{k}$, k_M and k_T are obtained from the equations of motion and can be written in the following form:

$$\frac{\partial \bar{k}}{\partial t} = -\nabla\cdot H - \frac{\partial}{\partial p}\overline{k\omega} - \overline{V\cdot\nabla\phi} + \overline{V\cdot F}, \tag{4}$$

$$\frac{\partial k_M}{\partial t} = -\nabla\cdot(H_0 + H_1) - \frac{\partial}{\partial p}\overline{(k_M + \bar{V}\cdot V')\omega} - \bar{V}\cdot\nabla\bar{\phi} + \bar{V}\cdot\bar{F} - C(k_M, k_T) \tag{5}$$

$$\frac{\partial k_T}{\partial t} = -\nabla\cdot(H_2 + H_3) - \frac{\partial}{\partial p}\left(\frac{1}{2}\overline{V'^2\omega}\right) - \overline{V'\cdot\nabla\phi'} + \overline{V'\cdot F'} + C(k_M, k_T) \tag{6}$$

where

$$H = H_0 + H_1 + H_2 + H_3, \tag{7}$$

$$H_0 = k_M\bar{V}, \tag{8}$$

$$H_1 = \overline{(\bar{V}\cdot V')V'}, \tag{9}$$

$$H_2 = k_T\bar{V}, \tag{10}$$

$$H_3 = \frac{1}{2}\overline{V'^2V'} \tag{11}$$

and

$$C(k_M, k_T) = C_H(k_M, k_T) + C_V(k_M, k_T) \tag{12}$$

$$C_H(k_M, k_T) = -\frac{\overline{u'u'}}{a\cos\phi}\frac{\partial\bar{u}}{\partial\lambda} - \overline{u'v'}\cos\varphi\frac{\partial}{a\partial\varphi}\left(\frac{\bar{u}}{\cos\varphi}\right) - \frac{\overline{u'v'}}{a\cos\varphi}\frac{\partial\bar{v}}{\partial\lambda} - \overline{v'v'}\frac{\partial\bar{v}}{a\partial\varphi} + \overline{u'u'}\,\bar{v}\,\frac{\tan\varphi}{a}, \tag{13}$$

$$C_V(k_M, k_T) = -\overline{u'\omega'}\,\frac{\partial \bar{u}}{\partial p} - \overline{v'\omega'}\,\frac{\partial \bar{v}}{\partial p}\ . \tag{14}$$

The terms on the left-hand side of Eqs. (4)–(6) represent local change in $\bar{k}$, k_M and k_T; for annual conditions, these terms are negligibly small. The first terms on the right-hand side of these equations represent the horizontal divergence, and the second terms the vertical divergence of the kinetic energy flux. The discussion here is limited to the averages over the whole air column (in fact, to the layer 100 – 1000 mb); in this case the vertical flux divergence terms disappear. The third terms represent the transformation of total potential energy into kinetic energy by the work done by the pressure forces (ϕ denotes the geopotential) and the fourth terms represent the effect of the work done by the turbulent frictional force $\boldsymbol{F}$; this work is often called turbulent energy dissipation. The term $C(k_M,k_T)$, which appears with different signs in Eqs. (5) and (6), represents the transformation of k_M into k_T.

From the data available for the present study, only the distribution of $\bar{k}$, k_M, k_T, $\nabla\cdot \boldsymbol{H}_0$, $\nabla\cdot \boldsymbol{H}_1$, $\nabla\cdot \boldsymbol{H}_2$ and $C_H(k_M,k_T)$ can be directly determined and, therefore, only a very rough picture be obtained about the fulfillment, in different parts of the Northern Hemisphere, of the kinetic energy balance as described by Eqs. (4)–(6).

No estimates have been made of the generation of kinetic energy by the work of pressure forces. In order to get some idea of the energy dissipation $\overline{\boldsymbol{V}\cdot \boldsymbol{F}}$ (and particularly of how it is partitioned between the time-mean flow and transient fluctuations), we assime that the major part of the dissipation takes place in the planetary boundary layer (PBL) and, following LETTAU [10], we write

$$D^{PBL} = -\overline{\int_{PBL} \boldsymbol{V}\cdot \boldsymbol{F}\,\frac{dp}{g}} = \overline{\tau_0 V_g\ \cos\alpha_0} \tag{15}$$

where τ_0 is the value of the surface stress and α_0 the cross-isobaric angle. If τ_0 is now expressed in terms of the geostrophic drag coefficient C_{Dg} ($\tau_0 = \rho C_{Dg} V_g^2$; ρ is density) we get the expression

$$D^{PBL} = \rho\overline{C_{Dg}\ V_g^3 \cos\alpha_0} \tag{16}$$

which has been used e.g. by KUNG [8] and BAUMGARTNER *et al.* [13] by applying values of C_{Dg} obtained from surface roughness estimates and assuming neutral stability. However, C_{Dg} and α_0 depend considerably upon the static stability (e.g. GARRATT [2]) and the proper evaluation of the geographical distribution of D^{PBL} on the basis of Eq. (16) still requires to be made.

In the present study we assume, following the classical Ekman-Taylor theory for the PBL, that τ_0 is proportional to V_g. Using the wind at the 850 mb level for the geostrophic wind we obtain from Eq. (15)

$$D^{PBL} = \overline{cK\ (850)}\ , \tag{17}$$

where the parameter c can be expected to depend upon the stability and the roughness of the underlying surface. Having no better knowledge of this dependence we take it to be constant and thus write

$$D^{PBL} = c\bar{k}\quad (850) \tag{18}$$

$$D_M^{PBL} = c\, k_M\ (850) \tag{19}$$

$$D_T^{PBL} = c\, k_T\ (850) \tag{20}$$

For c a value of 4.8×10^{-2} Wm^{-2}/m^2s^{-2} was used; this value leads to a value of 2.3 Wm^{-2} for the mean annual dissipation in the Northern Hemisphere (cf. OORT [13]).

The dissipation of kinetic energy in the free atmosphere is much less known than that in the boundary layer, and quantitative estimates for it (e.g. HOLOPAINEN [3], [5]; KUNG [9]) have a large margin of error. For this reason this quantity, which depends on the way in which the flow is partitioned into resolved and subgrid scale components, has been neglected in the present paper.

3. *Data*

The basic data used in the present study are the same two sets of independent 5-year upper-wind statistics used in HOLOPAINEN [6]. The first data covers the years 1950–54, the second the years 1958–62. Both include the information on $\bar{u}$, $\bar{v}$, $\overline{u'u'}$, $\overline{u'v'}$, $\overline{v'v'}$ for the standard pressure levels; the calculations in the present study were made for the atmospheric layer below the 100 mb level.

4. *Results*

4.1 Amount of kinetic energy

Fig. 1 shows the horizontal distributions of $\bar{k}$, k_M and k_T, averaged between 100 and 1000 mb, for the annual-mean conditions. The distribution of $\bar{k}$ shows two major maxima in the middle latitudes over the oceans with the areas of large

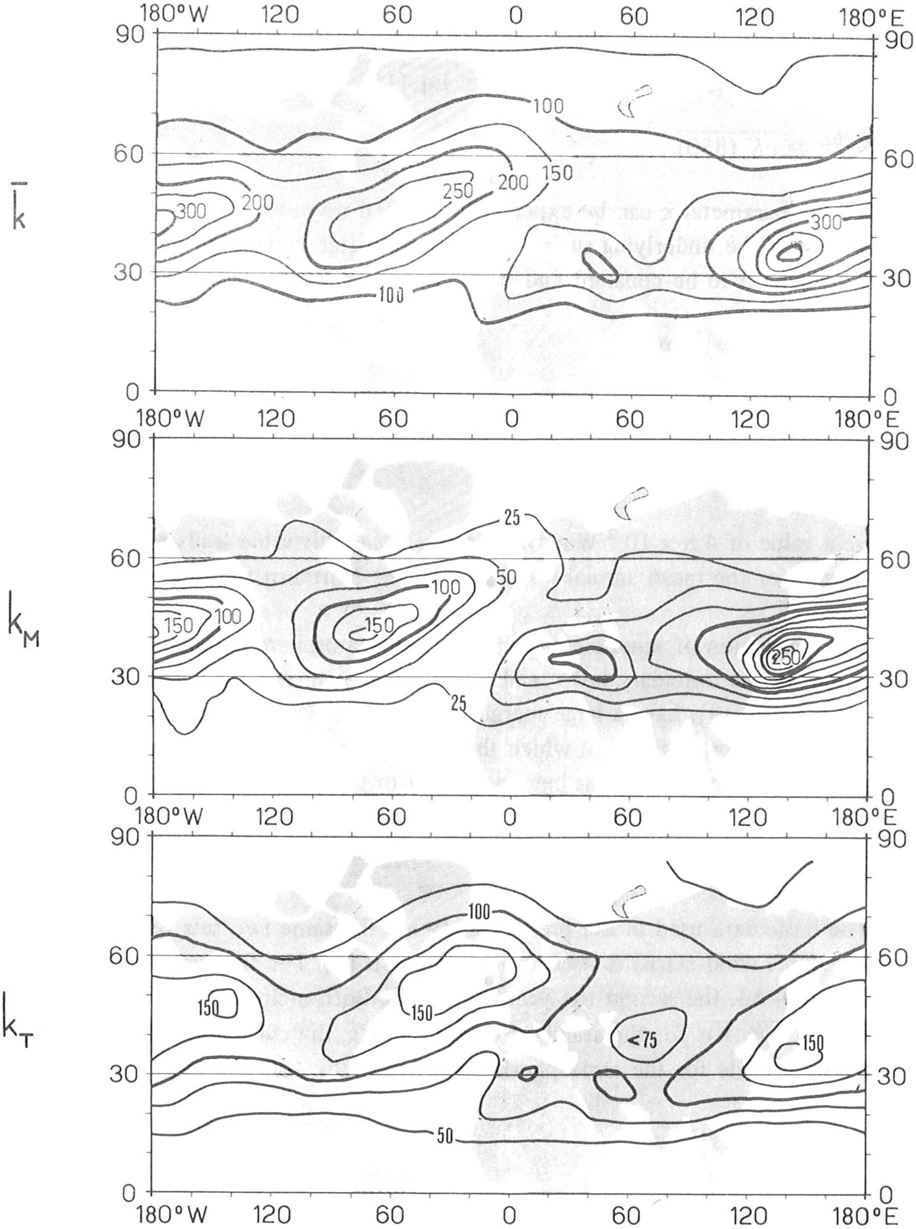

Fig. 1a. Horizontal distribution of total kinetic energy $\bar{k}$ (upper part), kinetic energy of the time-mean flow k_M (middle part) and kinetic energy of the transient fluctuations k_T (lower part), averaged between 100 and 1000 mb, for the annual-mean conditions during the period 1950–54. Unit: J/kg.

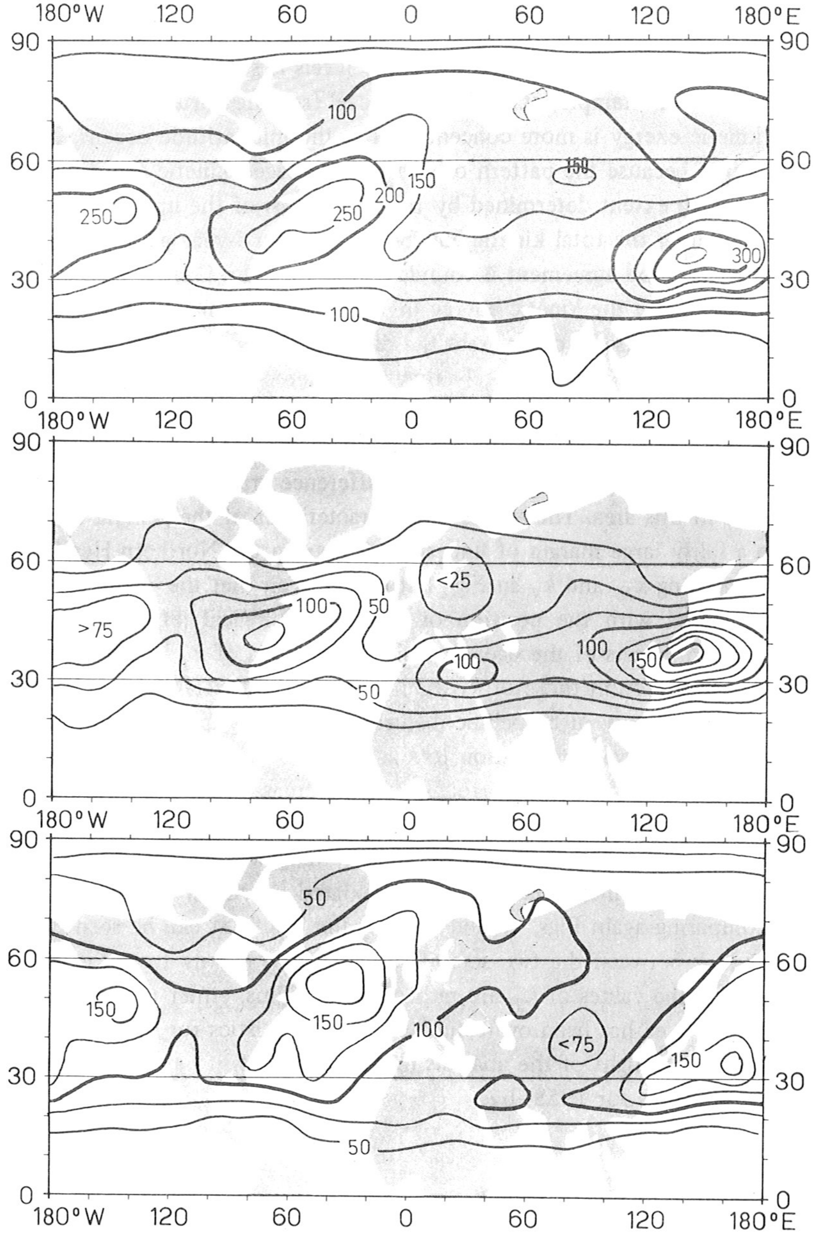

Fig. 1b. As Fig. 1a but for the period 1958–62.

kinetic energy also extending to the eastern coasts of the continents. On the average, however, the continents have much less kinetic energy than the oceans.

The pattern of $\bar{k}$ at different pressure levels is somewhat different from that in Fig. 1. For example, at the 850 mb level (see the pattern of D^{PBL} in Fig. 4) the kinetic energy is more concentrated in the mid-latitude oceans. This difference is natural, because the pattern of vertically-averaged kinetic energy, shown in Fig. 1, is to a large extent determined by the jet streams of the upper troposphere.

Comparing the total kinetic energy for the two 5-year periods (Figs. 1a and 1b), a relatively good agreement is found, except over the Central Pacific in the middle latitudes, where the kinetic energy in 1950–54 was considerably larger than in 1958–62. Because in other areas the differences between Figs. 1a and 1b are rather small, it is not very likely that the differences over the Pacific represent a real difference in the kinetic energy between the two 5-years periods. Rather, since these areas of the Pacific are a »hole» in the network of aerological stations, the differences represent a systematic difference arising due to methods of wind analysis in this area. The »observed» characteristics of the general circulation thus have a fairly large margin of uncertainty, even in the Northern Hemisphere.

Comparing k_M and k_T in Fig. 1 it can be seen that the main part of k_M is associated with the position of the clomatological jet stream over the western coasts of the oceans while the maxima of k_T are located somewhat northeastward from those of k_M. Qualitatively, this latter feature is natural in the light of the theory of baroclinic disturbances, which are likely to make a major (but not the only) contribution to k_T: these disturbances are generated in the area of maximum baroclinity (*i.e.* roughly in the areas of k_M maxima) but reach their maximum kinetic energy a long way downstream. In any case, an outstanding feature of Fig. 1 is that oceans are areas of large k_T, whereas continents are areas of small k_T – a major feature to be simulated by climate models.

Comparing again Figs. 1a and 1b over the Pacific it can be seen that the differences in $\bar{k}$ between the two sets of statistics arise mainly from the differences in k_M, while the values of k_T are rather similar. Thus, either the time-mean wind in the »hole» area has been overestimated in the statistics for 1950–54 or, perhaps more likely in light of the analysis methods applied, it has been underestimated in the statistics for 1958–62.

4.2 Balance of kinetic energy

Figs. 2 and 3 show the horizontal distributions of those terms in Eqs. (4)–(6) that can be computed directly from the available upper-wind statistics. Fig. 4 shows the distribution of terms D^{PBL}, D_M^{PBL} and D_T^{PBL} (see Eqs. (18)–(20)). In Figs. 2 and 3 the results are presented for both of the 5-year periods considered. A great many small-scale differences between the two statistics can be seen. In the following, however, the emphasis is on the broad-scale features, which are common in the results derived from both sets of statistics.

4.2.1 Balance of the mean (total) kinetic energy

The maintenance of the kinetic energy balance in different latitudinal zones has been discussed by several authors (*e.g.* PISHAROTY [16], PALMÉN and NEWTON [15], HOLOPAINEN [4]). However, the longitudinal variation of the way in which the long-term mean balance of kinetic energy is maintained has not been investigated; local studies have been made only for North America (*e.g.* KUNG [9]) and western Europe (HOLOPAINEN [3], [5]), for which areas a homogenous network of analogical stations exists.

According to Eq. (4), the local balance of total kinetic energy is affected by the divergence of kinetic energy flux, work done by pressure forces (which can be interpreted as generation of kinetic energy or as the transformation of available potential energy into kinetic energy) and the work done by (small-scale) turbulent frictional forces.

A mean horizontal divergence of kinetic energy from a certain area means that the area is a source of kinetic energy (*i.e.* generation larger than dissipation) and vice versa. From data available, only the three first terms in the expression

$$\nabla \cdot \boldsymbol{H} = \nabla \cdot \boldsymbol{H}_0 + \nabla \cdot \boldsymbol{H}_1 + \nabla \cdot \boldsymbol{H}_2 + \nabla \cdot \boldsymbol{H}_3 \tag{21}$$

(see Eqs. (7) – (11)) could be evaluated, and vertically-averaged values of these are shown in Fig. 2. All the terms are of the same order of magnitude and show surprisingly similar geographical distributions. Since the fourth term, which represents a third-order quantity in terms of fluctuations, is not likely to be the dominating term in Eq. (21) (VAN DEN DOOL [21]), we can conclude that the distribution of $\nabla \cdot \boldsymbol{H}$ must have the same main features as the first three terms. Accordingly, subtropical latitudes must be areas of flux divergence, and midlatitudes areas of flux convergence. These features have been known since PISHAROTY's [16] work. Not known so far is the large longitudinal variation of $\nabla \cdot \boldsymbol{H}$, apparent in Fig. 2. Thus, for example, the continents (in particular their eastern regions) appear to be areas of divergence of kinetic energy and oceans areas of convergence.

These features are in agreement with the results obtained from certain studies of the local mean balance of kinetic energy. Thus, according to KUNG [9] , for

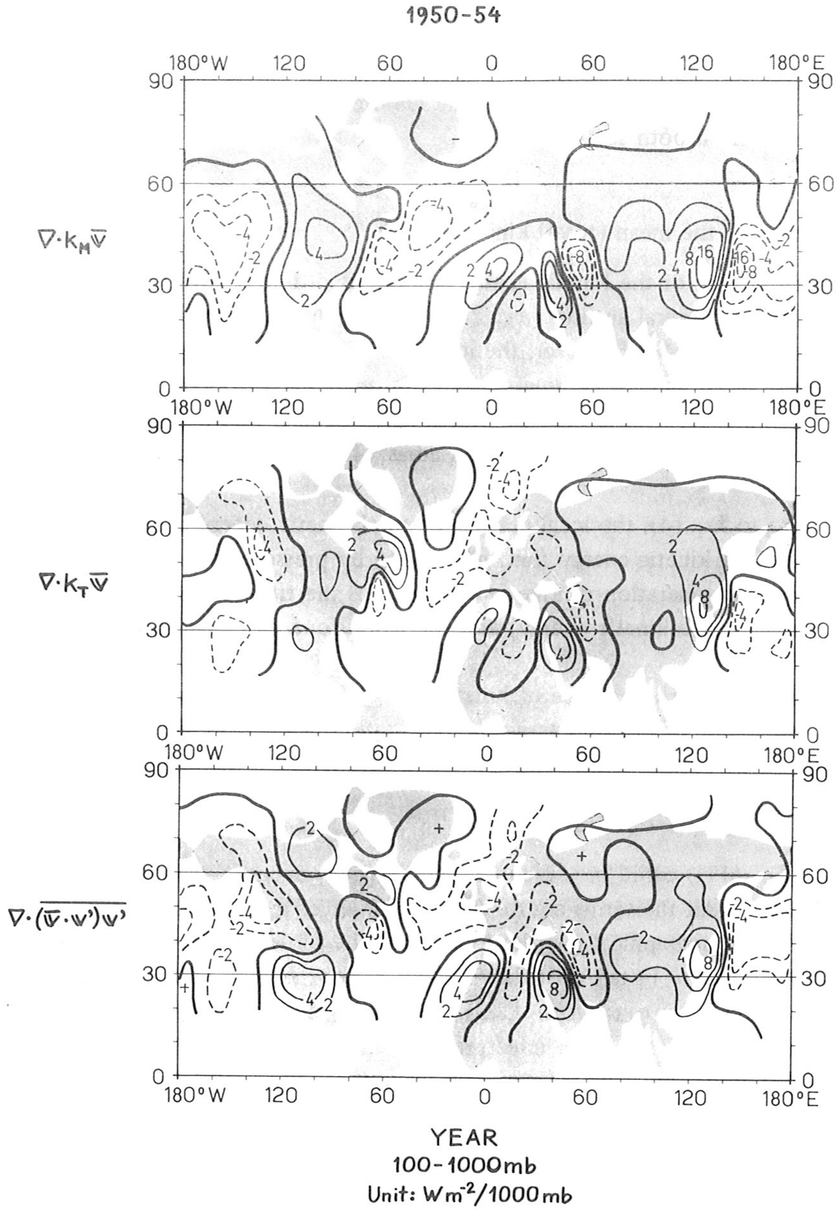

Fig. 2a. Horizontal distribution of $\nabla \cdot H_0$ (upper part), $\nabla \cdot H_1$ (lower part) and $\nabla \cdot H_2$ (middle part) averaged between 100 and 1000 mb, for the period 1950–54. Unit: W m^{-2} (1000 mb)$^{-1}$. For a definition of the symbols, see Eqs. (8), (9) and (10).

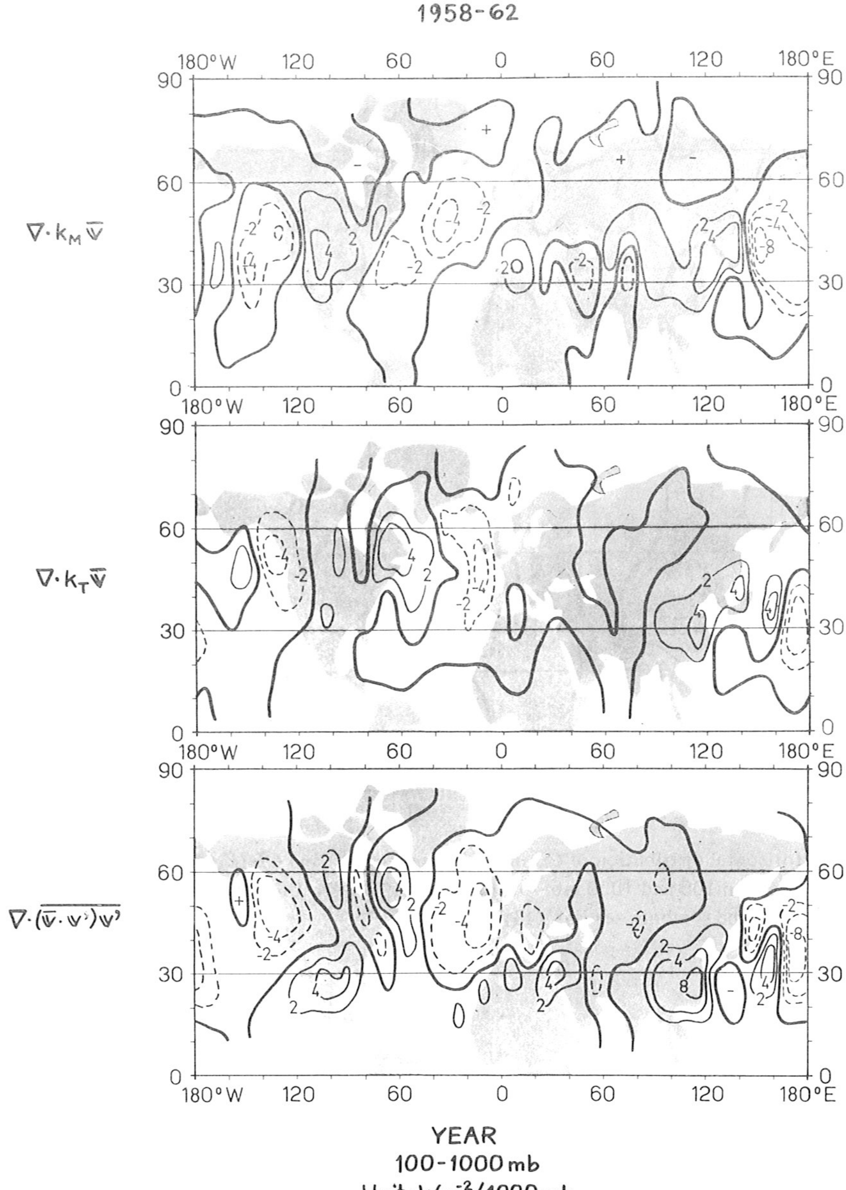

Fig. 2b. As Fig. 2a but for the period 1958–62.

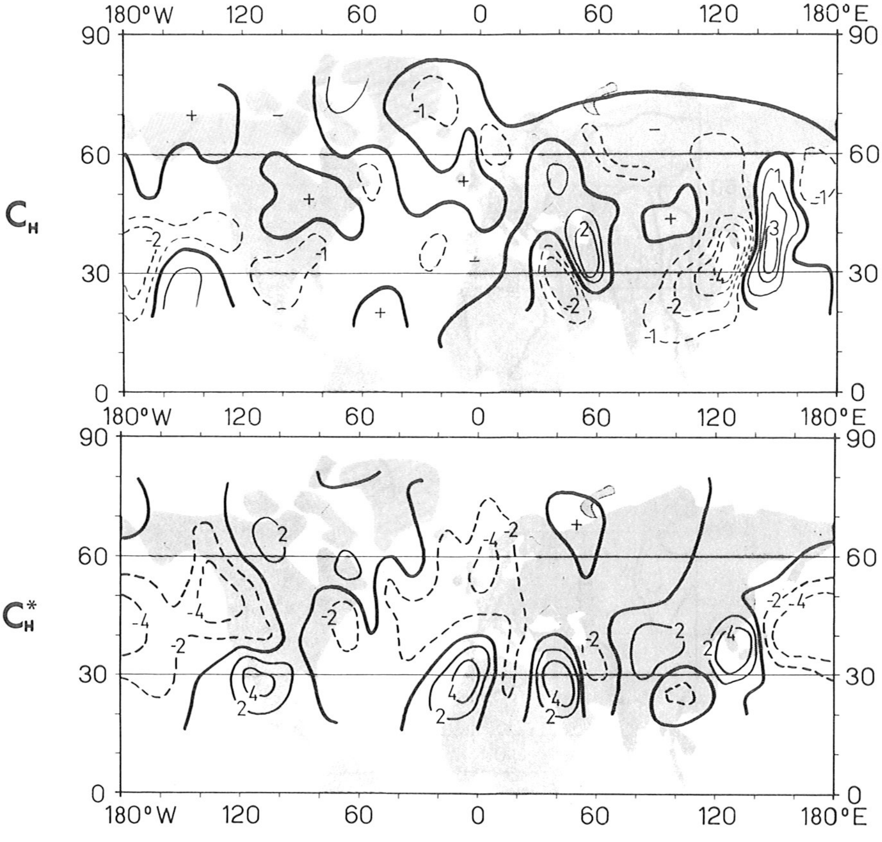

Fig. 3a. Horizontal distribution of C_H (k_M, k_T) (upper part) and C_H^* (k_M, k_T) (lower part), averaged between 100 and 1000 mb, for the period 1950–54. Unit: W m^{-2} (1000 mb)$^{-1}$. For a definition of the symbols, see Eqs. (13) and (27).

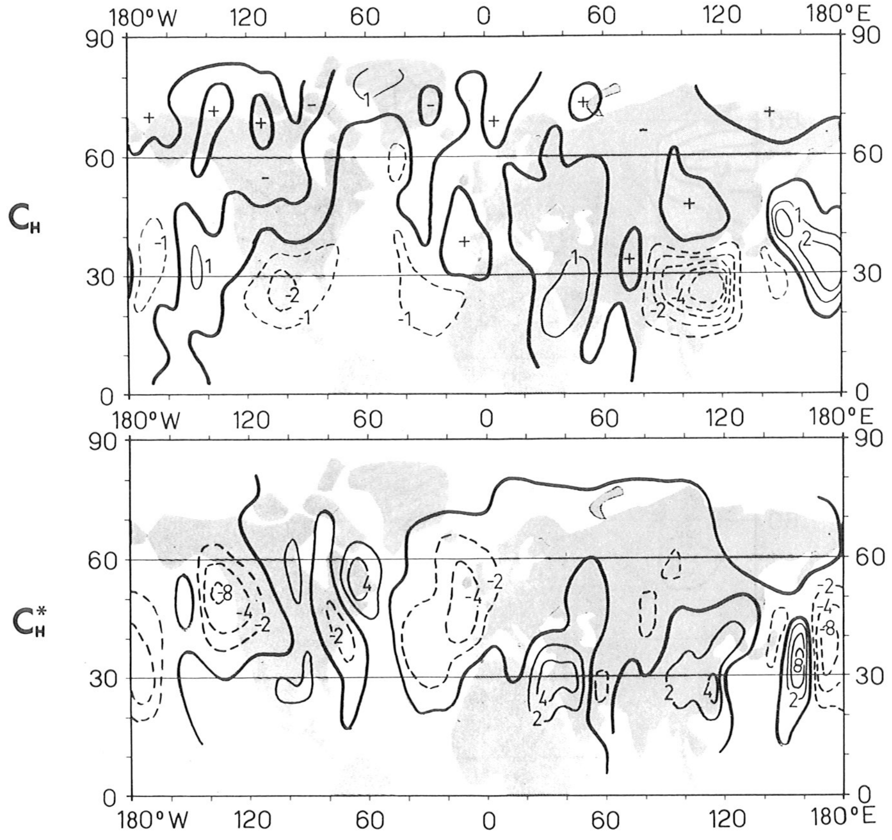

Fig. 3b. As Fig. 3a but for the period 1958–62.

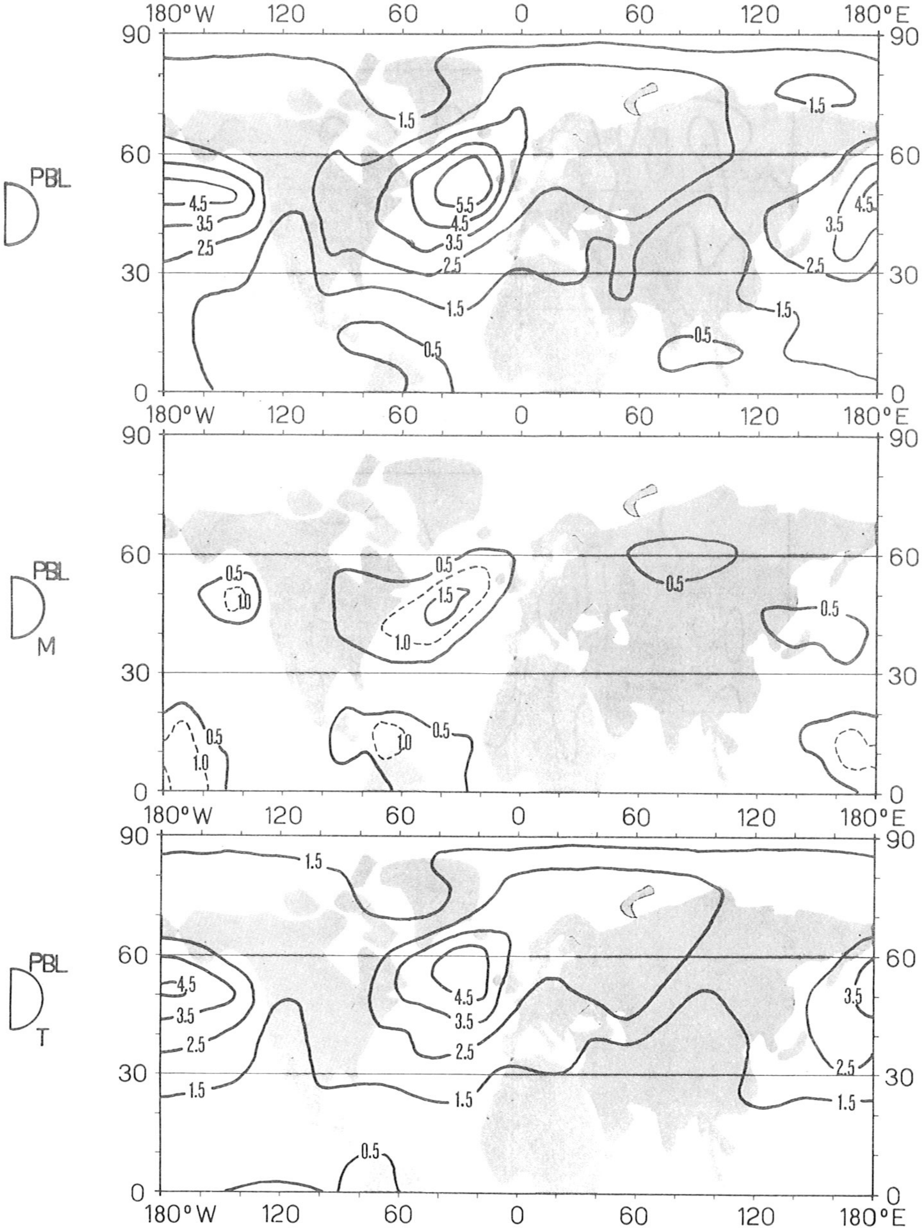

Fig. 4. Horizontal distribution of the quantities $D^{PBL} = c\bar{k}$ (850) (upper part), $D_M^{PBL} = c\,k_M$ (850) (middle part) and $D_T^{PBL} = c\,k_T$ (850) (lower part), computed from the data for the period 1958–62 with $c = 4.8 \times 10^{-2}$ W m^{-2}/m^2 s^{-2}.

example, the North-American continent is an area where generation of kinetic energy (by $-\boldsymbol{V}\cdot\nabla\phi$ process) is larger than the dissipation (by $\boldsymbol{V}\cdot\boldsymbol{F}$ process), and from where there must accordingly be a net mean outflux (flux divergence) of kinetic energy. On the other hand, over the area of the British Isles kinetic energy is, according to HOLOPAINEN [5], lost by the work done by pressure forces, and there is a net influx (flux convergence) of kinetic energy into this region.

The estimated total dissipation in the planetary boundary layer D^{PBL} from Eq. (8) is shown in the upper part of Fig. 4. It has clear maxima over the oceans in the middle latitudes with the largest value, 6 W m^{-2}, occurring over the Atlantic. Over the North American continent the value obtained from Fig. 4, about 2.5 W m^{-2}, agrees relatively well with what has been evaluated by KUNG [9] (as a residual from the kinetic energy equation) for energy dissipation below 850 mb. Similarly, over the British Isles the value of D^{PBL} in Fig. 4, 3.5 W m^{-2}, agrees with the (so far unpublished) values which the author has calculated by the same technique as KUNG for the annual-mean dissipation below 850 mb in this area. On the other hand, these values are considerably larger than those obtained by KUNG [8] and BAUMGARTNER *et al.* [1] using Eq. (16) and applying values of C_{Dg} valid for neutral stability, and also larger than some recent estimates of the intensity of the atmospheric energy cycle (e.g. NEWELL *et al.* [12]) seem to indicate.

The extent to which the pattern of D^{PBL} in Fig. 4 reflects the real geographical variation of the boundary layer dissipation is certainly open for dispute. One may argue that the parameter c in Eqs. (18)–(20) should be smaller over oceans than over continents and therefore the differences in D between oceans and continents should be smaller than those shown in Fig. 4. On the other hand, if D^{PBL} were proportional to the third power of the geostrophic wind (see Eq. (16)), the contrast between oceans and continents would (for constants C_{Dg} and α_0) be even larger than that shown in Fig. 4. It should be stressed here that the main purpose of Fig. 4 is to present not so much the horizontal distribution of D^{PBL} as the partitioning of D^{PBL} into D_M^{PBL} and D_T^{PBL}.

In summary we may say that there is large longitudinal variation in the way in which the time-mean balance of total kinetic energy in maintained. In particular, areas of energy convergence and divergence alternate in the zonal direction in the middle latitudes.

4.2.2 Balance of kinetic energy of the annual mean flow and the associated (large-scale) transient fluctuations

Our calculations on the kinetic energy balance of the annual mean motion (see Eq. (5)) provide estimates of the divergence terms $\nabla\cdot\boldsymbol{H}_0$ and $\nabla\cdot\boldsymbol{H}_1$, the transformation term C_H (k_M, k_T), and a very rough estimate of the dissipation D_M^{PBL} (see Eq. (19)).

The distributions of these terms, shown in Figs. 2, 3 and 4, show that the flux divergences $\nabla \cdot \boldsymbol{H}_0$ and $\nabla \cdot \boldsymbol{H}_1$ both have a relatively large magnitude, and that in most places they act in the same direction; the conversion term and the dissipation term are relatively small. This means that the flux divergence $\nabla \cdot (\boldsymbol{H}_0 + \boldsymbol{H}_1)$ in Eq. (5) has to be balanced mainly by the work done by mean pressure forces. The local balance of k_M is therefore to a first approximation in most places given by

$$0 \approx -\nabla \cdot (\boldsymbol{H}_0 + \boldsymbol{H}_1) - \overline{V} \cdot \overline{\nabla\phi} \tag{22}$$

where the tilde indicates an average with respect to pressure. The distribution of $\nabla \cdot \boldsymbol{H}_0$ and $\nabla \cdot \boldsymbol{H}_1$ indicates that the areas of energy production by mean pressure forces ($-\overline{V} \cdot \overline{\nabla\phi}$) alternate in the longitudinal direction very much in the same way as those for the total kinetic energy. Because $\nabla \cdot \boldsymbol{H}_1$ is determined by Reynolds' stresses, we can also conclude that these stresses are very important for the maintenance of the local balance of k_M.

It should in emphasized that Eq. (22) refers to the vertical average from the surface up to 100 mb. The horizontal flux divergence terms have largest magnitude in the upper troposphere and are small in the lower troposphere. It is thus likely that in the boundary layer the energy equation of the mean motion essentially reduces to an approximate balance between the energy dissipation and the work done by mean pressure forces. However, a quantitative investigation of the vertical variation in the budget of the kinetic energy would require an evaluation of the vertical flux divergence terms, which was not feasible in the present study.

Considering now, in the light of Eq. (6), the balance of k_T, it is first to be pointed out that only $\nabla \cdot \boldsymbol{H}_2$ (Fig. 2) and C_H (k_M, k_T) (Fig. 3) have been calculated and a rough estimate obtained for D_T^{PBL} (Fig. 4). Examining only the orders of magnitude of these terms, it can be seen that the divergence term $\nabla \cdot \boldsymbol{H}_2$ is of the same magnitude as the dissipation term; their patterns, however, are quite different. On the other hand, the conversion term is relatively small and unimportant in the local balance of k_T, except in some small subtropical areas. The flux divergence $\nabla \cdot \boldsymbol{H}_3$, representing a third-order term in fluctuation quantities, could not be computed from the data available. There is some evidence (VAN DEN DOOL [21]) that this term is of the same order of magnitude as $\nabla \cdot \boldsymbol{H}_2$. If so, a first approximation to the balance of the kinetic energy of the large-scale turbulence in the middle and high latitudes would be

$$0 \approx -\nabla \cdot (\boldsymbol{H}_2 + \boldsymbol{H}_3) - \overline{V' \cdot \nabla\phi'} + \overline{V' \cdot F'} \tag{23}$$

It is interesting to note that the small conversion term $C_H(k_M, k_T)$ (see Fig. 3) is in most places negative. This »negative viscosity» phenomenon (STARR [18]) is well known from studies in which the atmospheric flow is considered as a sum of the axisymmetric component and superimposed waves; the longitudinal distribution of $C_H(k_M, k_T)$ has not, however, been investigated. The zonal averages of this term are shown in Fig. 5 for the two 5-year periods (1950–54 and 1958–62). The hemispheric (15°N – 90°N) average of the computed term is equal to 0.3 W m^{-2} for both periods, which is the same as the estimate given by OORT [13] for the »mixed space-time domain».

4.2.3 The ambiguity in the considerations of energy conversion

There is some ambiguity in the interpretation of which terms in the energy equations for an open system represent an energy conversion from one form to another. For example, instead of Eqs. (5) – (6) we can write

$$\frac{\partial k_M}{\partial t} = -\nabla \cdot \boldsymbol{H}_0 - \frac{\partial}{\partial p} k_M \bar{\omega} - \bar{\boldsymbol{V}} \cdot \overline{\nabla \phi} + \bar{\boldsymbol{V}} \cdot \bar{\boldsymbol{F}} - C^\star(k_M, k_T) \tag{24}$$

$$\frac{\partial k_T}{\partial t} = -\nabla \cdot (\boldsymbol{H}_1 + \boldsymbol{H}_2 + \boldsymbol{H}_3) - \frac{\partial}{\partial p}\left(k_T \bar{\omega} + \overline{(\bar{\boldsymbol{V}} \cdot \boldsymbol{V}')\,\omega} + \frac{1}{2}\overline{V'^2 \omega}\right) - \\ - \overline{\boldsymbol{V}' \cdot \nabla \phi'} - \overline{\boldsymbol{V}' \cdot \boldsymbol{F}'} + C^\star(k_M, k_T)\,. \tag{25}$$

Here we have

$$C^\star(k_M, k_T) = C_H^\star(k_M, k_T) + C_V^\star(k_M, k_T), \tag{26}$$

$$C_H^\star(k_M, k_T) = -\boldsymbol{V} \cdot \boldsymbol{A}_H\,, \tag{27}$$

and

$$C_V(k_M, k_T^\star) = -\boldsymbol{V} \cdot \boldsymbol{A}_V, \tag{28}$$

where $\boldsymbol{A}_H$ and $\boldsymbol{A}_V$ are the »frictional» forces which arise from the (large-scale) horizontal and vertical Reynolds' stresses, respectively:

$$\boldsymbol{A}_H = A_\lambda \boldsymbol{i} + A_\varphi \boldsymbol{j}, \tag{29}$$

$$\begin{cases} A_\lambda = -\dfrac{1}{a\cos\varphi}\dfrac{\partial}{\partial\lambda}\overline{u'u'} - \dfrac{1}{a\cos^2\varphi}\dfrac{\partial}{\partial\varphi}(\overline{u'v'}\cos^2\varphi), & (30) \\ A_\varphi = -\dfrac{1}{a\cos\varphi}\dfrac{\partial}{\partial\lambda}\overline{u'v'} - \dfrac{1}{a\cos\varphi}\dfrac{\partial}{\partial\varphi}(\overline{v'v'}\cos\varphi) - \dfrac{\overline{u'u'}}{a}\tan\varphi & (31) \end{cases}$$

$$\boldsymbol{A}_V = -\frac{\partial}{\partial p}\overline{\boldsymbol{V}'\omega'} \tag{32}$$

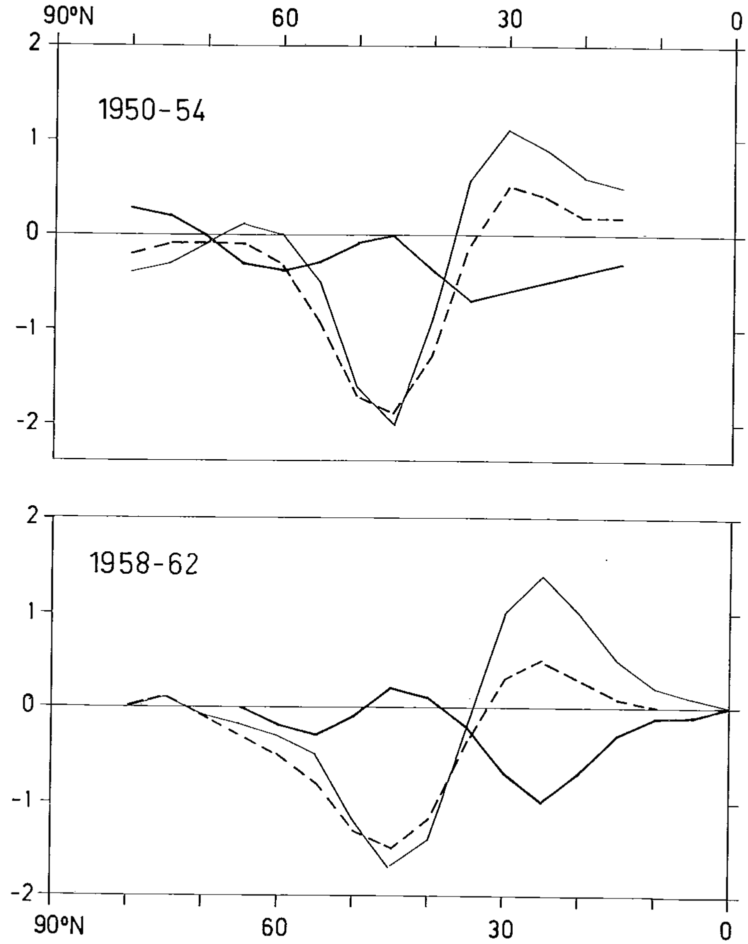

Fig. 5. Latitudinal distribution of the terms in Eq. (33):

$C_H(k_M, k_T)$: heavy continuous line (see Eq. (13))

$C_H^{\star}(k_M, k_T)$: broken line (see Eq. 27))

$\nabla \cdot H_1$: thin continuous line (see Eq. (10))

averaged with respect to longitude and pressure. Unit: W m^{-2} (1000 mb)$^{-1}$.

The relationship that connects equations (5)–(6) and (24)–(25), is

$$C_H(k_M, k_T) = C_H^{\star}(k_M, k_T) - \nabla \cdot H_1 \tag{33}$$

We can interpret either $C_H(k_M, k_T)$ or $C_H^{\star}(k_M, k_T)$ to represent the conversion of mean flow kinetic energy to transient flow kinetic energy. In the former scheme the flux H_1 is thus considered as part of the flux of k_M, in the latter scheme as part of the flux of k_T.

The distributions of the vertically-averaged values of the different terms in Eq. (33) are shown in Fig. 2 ($\nabla \cdot H_1 = \nabla \cdot \overline{(\bar{V} \cdot V') V'}$) and Fig. 3; the mean meridional distribution of these terms is shown in Fig. 5. There is general agreement between the results obtained from the two sets of statistics. It can be seen that $C^{\star}_H(k_M, k_T)$ is a term of relatively large magnitude compared with $C_H(k_M, k_T)$. Related to this is the fact that $C^{\star}_H(k_M, k_T)$ and $-\nabla \cdot H_1$ tend to counterbalance each other. One reason for this counterbalancing and the simultaneous smallness of $C_H(k_M, k_T)$ in Eq. (33) is the fact that the order of magnitude of $\nabla \cdot H_1$ and $C^{\star}_H(k_M, k_T)$ is essentially determined by $\overline{u'^2} (\approx \overline{v'^2})$, whereas that of $C_H(k_M, k_T)$ is, in the light of Eq. (13) and the quasinondivergent nature of $\bar{V}$, determined by $(\overline{u'^2} - \overline{v'^2})$ and $\overline{u'v'}$, which are both small compared with $\overline{u'^2}$ and $\overline{v'^2}$.

A relevant question now is, which one of the terms $C(k_M, k_T)$ and $C^{\star}(k_M, k_T)$ should be interpreted as representing an energy conversion from k_M to k_T, and consequently, which one of the two sets of equations, (5)–(6) and (24)–(25), should be used in studies of the energy balance of the mean flow and the transient fluctuations. Eqs. (5)–(6) are analogous to those used in studies of small-scale turbulence, where the conversion term is always positive and can be interpreted as an irreversible conversion of mean flow energy to turbulent energy. Our results also show that for the equation of kinetic energy of the large-scale turbulence, Eq. (5) might be preferable because, unlike Eq. (25), it does not have two large, almost-counterbalancing terms, and because in a first approximation picture $C_H(k_M, k_T)$ may even be neglected, except in some small regions where large gradients in the mean field occur. On the other hand, the scheme represented by Eqs. (24)–(25) would perhaps be physically more understandable: according to the expression for $C^{\star}_H(k_M, k_T)$, k_T is converted into k_M in those places where the mean flow is accelerated by the turbulent frictional force (as, for example, over the oceans in the middle latitudes) and vice versa. However, there seems to be no clear physical reason for preferring any of these schemes, both of which give the same numerical value when integrated over a closed system.

5. *Concluding remarks*

This paper presents the climatological distribution of those budget terms for kinetic energy which can be evaluated with some confidence from the available data. In addition to giving new information on atmospheric energetics, the results should be useful for verification of the 3-dimensional climate models.

The generation term in the kinetic energy balance cannot be estimated directly from observations. The climatological distribution of this term (and its different partitionings) will be obtained only after the numerical analysis (data assimilation) schemes have produced estimates of vertical velocity and associated quantities. The term, for which the climatology is likely to remain longest unresolved, is the dissipation of kinetic energy in the free atmosphere.

Acknowledgements: The author is indebted to Dr. ABRAHAM H. OORT from the Geophysical Fluid Dynamics Laboratory, Princeton, for putting at his disposal the analyzed wind statistics data for the period May 1958 – April 1963.

REFERENCES

1. BAUMGARTNER, A., MAYER, H. and W. METZ, 1977: Weltweite Verteilung des Rauhigketsparameters z_0 mit Anwendung auf die Energiedissipation an der Erdoberfläche. *Meteor. Rundschau,* 30, 43–48.
2. GARRATT, J.R., 1977: Rewiev of drag coefficients over oceans and continents. *Mon. Wea. Rev.,* 105, 915–929.
3. HOLOPAINEN, E.O., 1964: Investigation of friction and diabatic processes in the atmosphere. Soc. Sci. Fennica, Comment. Phys.-Math. XXIX 9.
4. –»– , 1969: On the maintenance of the atmosphere's kinetic energy over the northern hemisphere in winter. *Pure and Applied Geophysics,* 77, 104–121.
5. –»– , 1973: An attempt to determine the effects of turbulent friction in the upper troposphere from the balance requirements of the large-scale flow: a frustrating experiment. *Geophysica,* 12, 151–176.
6. –»– , 1978: On the dynamic forcing of the long-term mean flow by the large-scale Reynolds' stresses in the atmosphere. Submitted for publication in J. Atm. Sci..
7. KAO, S.K. and W.P. HURLEY, 1962: Variation of the kinetic energy of large-scale eddy currents in relation to the jet stream. *J. Geophys. Res.,* 67, 4233–4242.
8. KUNG, E., 1963: Climatology of aerodynamic roughness parameter and energy dissipation in the planetary boundary layer over the Northern Hemisphere. In: *Annual Report, Univ. of Wisconsin, Dept. of Meteor.* 37–96.
9. –»– , 1967: Diurnal and long-term variation or the kinetic energy generation and dissipation for a five-year period. *Mon. Wea. Rev.,* 95, 593–606.
10. LETTAU, H., 1959: Wind profile, surface stress and geostrophic drag coefficients in the atmospheric surface layer. *Adv. Geophys.,* 6, 241–257.
11. MONIN, A.S., 1956: On the macro-turbulent exchange in the earth's atmosphere (in Russian). *Izvestiya Academy of Sciences USSR, No. 4,* 452–463.
12. NEWELL, R.E., KIDSON, J.W., VINCENT, D.G. and G.J. BOER, 1974: *The general circulation of the tropical atmosphere and interactions with extratropical latitudes, Vol. II.* The MIT Press.
13. OORT, A., 1964: On estimation of the atmospheric energy cycle. *Mon. Wea, Rev.,* 92, 483–493.
14. –»– and PEIXOTO, J.P., 1974: The annual cycle of the energetics of the atmosphere on a planetary scale. *J. Geophys. Res.,* 79, 2705–2719.
15. PALMEN, E. and C.W. NEWTON, 1969: *Atmospheric Circulation Systems.* Int. Geoph. Ser., Vol. 13, Academic Press.

16. PISHAROTY, P., 1954: *The kinetic energy of the atmosphere.* Final Rep. Gen. Circ. Proj., No. AF 19 (122)-48. Univ. Calif. at Los Angeles, Dept. of Meteor., XIV, 1–61.
17. SAVIJÄRVI, H., 1976: The interaction of the monthly mean flow and large-scale transient eddies in two different circulation types. Part I: The forcing effect of large-scale turbulence and kinetic energy balance. *Geophysica,* **14**, 23–46.
18. STARR, V., 1968: *Physics of negative viscosity phenomena.* McGraw-Hill Book Company.
19. TSAY, C.Y. and S.K. KAO, 1973: An investigation of the spectral structure of atmospheric waves near a jet stream. *Tellus,* **25**, 111–131.
20. –»–, –»–, 1974: An analysis of wave structure near the center of maximum turbulent kinetic energy. *Ibid.* **26**, 294–312.
21. VAN DEN DOOL, H., 1975: *Three studies on spectral structures of the horizontal atmospheric motion in the time domain.* PhD Thesis from the Department of Meteorology, University of Utrecht.