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CONTROL OF ELECTROMAGNETIC SEISMOGRAPH SENSITIVITY

by

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Abstract

The various formulae for current attenuation in the galvanometer circuit of electromagnetic seismographs are summarized for transducers with one (signal) coil and for transducers with an auxiliary coil for damping control. The distortion of the magnification curves versus magnification level was tested for three typical cases: (i) sensitivity adjustment over a wide range without changing the basic seismometer and galvanometer constants (*i.e.* periods and damping of both systems), (ii) sensitivity adjustment on a small scale around a given standard magnification level, (iii) transformation of the basic constants for large steps of magnification and then regulation of magnification on a small scale as in (ii).

The first case applies to long-period seismographs in WWSSN where, for the whole interval of maximum magnification 375 – 6,000, the value of the coupling coefficient is changed from negligibly small up to about 0.6. The maximum distortion of the magnification curve is 60 per cent. A current attenuation of 20 per cent of the particular magnification level raises the distortion of the magnification curve by less than 1 per cent, with the exception of the maximum coupling in which case 10 per cent discrepancies occur. In the second case the broad-band SKD seismograph with standard characteristic has only one maximum magnification level, 1,000, with a coupling coefficient of 0.25. A current attenuation of the same magnitude as above necessitates the distortion of the magnification curve by up to 8 per cent. The SKM-3 short-period seismographs used with 4 different types of standard amplitude characteristics are given as an example of the third case. Their basic constants are changed for a particular magnification level so to keep the standard course of the magnification curve invariable. At a maximum magnification of 100,000 and a current attenuation of ±20 per cent, the curve distortion is up to 10 per cent for characteristics II and IV, only up to 5 per cent for characteristic III and up to 2 per cent for characteristic I.

The changes in the magnification curve should be taken into account if the seismograph is calibrated at a different sensitivity than is used in operation. A magnification adjustment independent of period is possible for a small coupling coefficient and for small coupling coefficient differences only.

1. *Introduction*

The time accuracy in routine analyses of seismograms is nowadays satisfactorily high since seismic stations have been equipped with X-tal clocks. In order to solve dynamical problems of seismic wave propagation, ground amplitude movement must be determined at the same level of accuracy. For this reason attention is being drawn more and more to the calibration of individual seismographs and further to the adjustment of the amplitude response of seismograph sets to specified standard requirements. In order to obtain identical magnification curves for electromagnetic seismographs, all basic constants (i.e. seismometer and galvanometer periods, damping constants of both systems and the coupling coefficient) and the scaling factor need to be adjusted. The sensitivity is controlled mainly by an attenuator circuit located between the seismometer signal coil and the recording galvanometer; this makes it possible to adjust the current amplitude flowing through the galvanometer. The physical parameters of individual seismographs often show some permissible deviations from the corresponding nominal values of the instrument type. For this reason the fine continuous adjustment of the magnification level should, to same extent, be applied to reach the value of permitted tolerances in the absolute magnification. On the other hand, for large changes in magnification, another attenuator with suitable constant attenuation steps (*e.g.* with factor 2, *i.e.* 6 dB) is preferred.

Different attenuator settings keep the damping constants of seismometer and galvanometer unchanged. This makes it possible for the amplitude and phase response to be independent of the maximum magnification if the influence of the used range of coupling coefficient is negligible. Otherwise both magnification curves and phase delay will alter their course with the attenuator setting. Full transformation of the basic constants is then inevitable in order to keep the amplitude and phase response constant for a given coupling coefficient range.

The deviation of a resistance circuit with a single T-type attenuator has been published in several papers [1, 2, 5, 7, 8]. Most methods use a parameter defined by the ratio of the current flowing through the galvanometer to the current flowing in another part of the attenuator circuit. This approach is reasonable, especially for relative changes in magnification. When the prescribed absolute magnification is adjusted the coupling coefficient should be taken into account. The influence of the attenuator setting on seismograph response stability is then easier to work out.

The present paper considers the possible applications of attenuators and the magnification adjustment of some electromagnetic seismographs with non-negligible coupling coefficients.

2. *Attenuator circuits*

The sensitivity of the electromagnetic seismograph is adjusted by means of a simple T-type attenuator inserted between the seismometer transducer coil and the recording galvanometer. The basic function of the attenuator is to control the current flowing through the galvanometer without altering the damping of either seismometer or galvanometer. In seismographs, which do not have adjustable open circuit damping, the external resistances of the signal and galvanometer coils should be independent of the attenuator setting. If the seismometer transducer has a special coil for additional electromagnetic damping, only the external resistance of the galvanometer need be kept constant. Changes in the damping constant due to the recording circuit of the seismometer are then balanced by this auxiliary damping circuit. Some of the procedures used for calculating attenuator resistances are further summarized.

2.1 Single attenuator

2.1.1 Single-coil seismometer transducers

The resistance circuit diagram of the seismograph is given in Fig. 1 [5]. Resistances R_1 and R_2 are the internal resistances of the seismometer signal coil and galvanometer, respectively. They include the corresponding parts of the line resistance to the attenuator. This is composed of three resistances X_1, X_2 and X_3. Signal attenuation is measured mainly by the ratio of current I_2 flowing through the galvanometer to current I_1 flowing through the seismometer signal coil and/or to current I_3 flowing through shunt X_3.

For the attenuation factor $\mu = I_2/I_1$, introduced by Hagiwara, the resistances of the attenuator can be written as follows:

$$X_1 = X_3(Z_1/Z_2 - \mu)/\mu - R_1 \quad \text{(1a)}$$

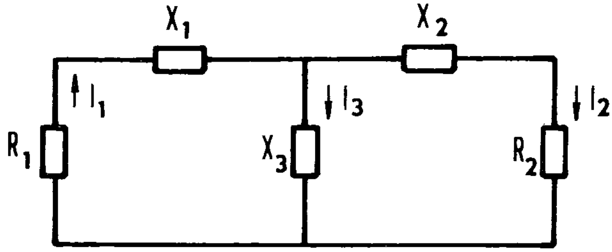

Fig. 1. Diagram of single T-type attenuation circuit of electromagnetic seismograph.

$$X_2 = X_3(1 - \mu)/\mu - R_2 \tag{1b}$$

$$X_3 = \mu Z_1 Z_2/(Z_1 - \mu^2 Z_2) \tag{1c}$$

where Z_1 equals resistance R_1 plus external seismometer resistance and Z_2 equals resistance R_2 plus external galvanometer resistance: $Z_1 = R_1 + X_1 + (X_2 + R_2)X_3/(X_2 + R_2 + X_3)$, $Z_2 = R_2 + X_2 + (X_1 + R_1)X_3/(X_1 + R_1 + X_3)$.

Resistances Z_1 and Z_2 are determined by the required damping constant of the seismometer $D_s = a_{sg}/Z_1 + D_{s0}$ and of the galvanometer $D_g = a_g/Z_2 + D_{g0}$, where a_{sg} and a_g are the total critical resistances of seismometer and galvanometer, respectively. D_{s0} and D_{g0} are the open-circuit damping constants of the corresponding systems. As usual, critical resistance here denotes the total resistance of a circuit that yields an electromagnetic damping constant equal to unity. Critical resistance is sometimes defined as that total resistance which, together with open circuit damping, gives a damping constant equal to unity. The damping constants D_{s0} and D_{g0} correspond to mechanical and air damping of the vibrating system without any additional electromagnetic damping in the auxiliary damping coils, if such coils are used.

The attenuation factor $\mu = X_3/(X_3 + X_2 + R_2)$, where $0 \leqslant \mu \leqslant 1$, depends on the shunt current I_3. The lower limit corresponds to zero sensitivity ($X_3 = 0$), and the upper limit to the maximum attainable sensitivity of the seismograph ($X_3 = \infty$).

For a different current ratio, $\mu_0 = I_{20}/I_{10}$, where $I_{20} = I_2/i$, the attenuation factor is

$$\mu = \mu_0 i \tag{1d}$$

The current I_{10} is equal to I_1 for all cases of the same transducer movement since resistance Z_1 is constant. When the amplitude response is independent of the coupling, the value i, the current ratio, defines the relative change of seismograph magnification. Under these conditions this is also true for the other attenuation parameters given in this paragraph.

In [7] parameter $k = I_1/I_2$, *i.e.* the reciprocal value of μ, is introduced. Here $\infty > k \geqslant 1$, and using the same symbols as in the preceding case, the resistances can be rewritten as follows:

$$X_1 = kZ_1(kZ_1 - Z_2)/(k^2Z_1 - Z_2) - R_1 \tag{2a}$$

$$X_2 = k(k-1)Z_1Z_2/(k^2Z_1 - Z_2) - R_2 \tag{2b}$$

$$X_3 = kZ_1Z_2/(k^2Z_1 - Z_2) \tag{2c}$$

For the relative change of currents $i = I_2/I_{20}$ we get

$$k = k_0/i \tag{2d}$$

where $k_0 = I_{10}/I_{20}$.

The definition of shunt losses can also be used to calculate the parameter $A = I_3/I_2$ [1]. Using the adopted notation for the parameters the formulae for the resistances are

$$X_1 = Z_1 - X_3A/(1 + A) - R_1 \tag{3a}$$

$$X_2 = AX_3 - R_2 \tag{3b}$$

$$X_3 = Z_1Z_2/[(1 + A)Z_1 - Z_2] \tag{3c}$$

The shunt losses lie in the range $\infty \leqslant A \leqslant 0$, where the upper limit holds true for zero sensitivity and the lower limit for the maximum sensitivity of the seismograph. If the shunt losses are defined by $A_0 = I_{30}/I_{20}$ then for $i = I_2/I_{20}$ we get

$$A = (A_0 + 1 - i)/i \tag{3d}$$

since $I_2 + I_3 = I_{20} + I_{30} = I_1 = I_{10}$ if Eqs. (3a–c) are satisfied.

In [8] the fractional part of the maximum sensitivity m was introduced, the maximum being indicated by unity. The $m = I_2/I_{2\max}$, *i.e.* the ratio between current I_2 allowed to pass through the galvanometer and the maximum possible current $I_{2\max}$. After modifying the notation, the formulae derived by Neumann have the form

$$X_1 = \{m^2R_1 + Z_1[(Z_1 - R_1)/Z_2 - m]\}/(Z_1/Z_2 - m^2) \tag{4a}$$

$$X_2 = \{m^2R_2 + Z_1[(Z_2 - R_2)/Z_2 - m]\}/(Z_1/Z_2 - m^2) \tag{4b}$$

$$X_3 = (Z_1 - R_1 - X_1)/(1 - m) \tag{4c}$$

From a comparison with the definition of the attenuation factor μ it follows that $m = \mu$ and the theoretical value for m, therefore, again lies between 0 and 1.

The transformation between different attenuations m and m_0 is then

$$m = m_0 i \tag{4d}$$

The last example of an attenuator calculation is not based on a current ratio but directly on the seismograph coupling coefficient σ^2 [2]. Using the parameter $\alpha = (a_{sg} a_g / D_s D_g \sigma^2)^{\frac{1}{2}}$ the resistances are defined by

$$X_1 = X_3(\alpha/Z_2 - 1) - R_1 \tag{5a}$$

$$X_2 = X_3(\alpha/Z_1 - 1) - R_2 \tag{5b}$$

$$X_3 = \alpha Z_1 Z_2/(\alpha^2 - Z_1 Z_2) \tag{5c}$$

For the ratio $i = I_2/I_{20}$ it holds that $\sigma = i\sigma_0$ and therefore

$$\alpha = \alpha_0/i \tag{5d}$$

For the sake of clarity all formulae are summarized in Table 1 for a single coil transducer.

The definite attenuator parameters are equivalent and their application is mainly determined by the input data at our disposal. For relative changes in magnification a knowledge of the circuit parameters is sufficient because the attenuation factor can be determined uniquely. All formulae for individual resistances given by different parameters can be converted into one form, taking into account the relationship between attenuation parameters. If we keep the attenuation factor μ as the basic parameter, we get

$$k = 1/\mu, \qquad A = 1/\mu - 1, \qquad m = \mu, \qquad \alpha = Z_1/\mu \tag{6a–d}$$

The attenuation factor μ will be considered in the following treatment. To attain a given attenuation the real resistances $X_1 \geqslant 0$, $X_2 \geqslant 0$, $X_3 > 0$ should be obtained from (1a–c). For changing the magnification level the value i is preferred, giving $\mu = \mu_0 i$ following which the same formulae will be used.

The general formulae can be simplified in some special cases. If the condition $Z_1 = Z_2$ is fulfilled we have for the maximum attainable sensitivity $\mu = 1$

$$X_1 = X_2 = 0, \qquad X_3 = \infty \tag{7a–c}$$

The maximum coupling coefficient in this case is $\sigma^2_{max} = (D_s - D_{s0})(D_g - D_{g0})/(D_s D_g) \leqslant 1$. If the condition $R_1 = R_2$ is also satisfied, the positive resistances X_1,

Table 1. List of parameters and attenuator resistances.

Transducer with	Definition of parameter	Theoretical range	X_1	X_2	X_3	Parameter for current ratio $i = I_2/I_{20}$
one coil	$\mu = I_2/I_1$	$0 < \mu \leqslant 1$	$\frac{X_3}{\mu}\left(\frac{Z_1}{Z_2} - \mu\right) - R_1$	$\frac{X_3}{\mu}(1-\mu) - R_2$	$\frac{\mu Z_1 Z_2}{Z_1 - \mu^2 Z_2}$	$\mu = \mu_0 i$
one coil	$k = I_1/I_2$	$\infty > k \geqslant 1$	$\frac{kZ_1(kZ_1-Z_2)}{k^2Z_1-Z_2} - R_1$	$\frac{k(k-1)Z_1Z_2}{k^2Z_1-Z_2} - R_2$	$\frac{kZ_1Z_2}{k^2Z_1-Z_2}$	$k = k_0/i$
one coil	$A = I_3/I_2$	$\infty > A \geqslant 0$	$Z_1 - X_3A/(1+A) - R_1$	$AX_3 - R_2$	$\frac{Z_1Z_2}{(1+A)Z_1-Z_2}$	$A = \frac{A_0+1-i}{i}$
one coil	$m = I_2/I_{2m}$	$0 < m \leqslant 1$	$\frac{m^2R_1 + Z_1\left(\frac{Z_1-R_1}{Z_2} - m\right)}{\frac{Z_1}{Z_2} - m^2}$	$\frac{m^2R_2 + Z_1\left(\frac{Z_2-R_2}{Z_2} - m\right)}{\frac{Z_1}{Z_2} - m^2}$	$\frac{Z_1 - R_1 - X_1}{1-m}$	$m = m_0 i$
one coil	$\alpha = \sqrt{\frac{a_{sg}a_g}{D_sD_g\sigma^2}}$	$\infty > \alpha \geqslant \sqrt{\frac{a_{sg}a_g}{D_sD_g}}$	$X_3\left(\frac{\alpha}{Z_2} - 1\right) - R_1$	$X_3\left(\frac{\alpha}{Z_1} - 1\right) - R_2$	$\frac{\alpha Z_1Z_2}{\alpha^2 - Z_1Z_2}$	$\alpha = \alpha_0/i$
two coils	$\mu = I_2/I_1$	$0 < \mu \leqslant 1$	$\frac{Z_1(Z_2 - R_2 - X_2)}{\mu Z_2} - R_1$	C	$\frac{Z_1(Z_2-R_2-X_2)}{Z_1-\mu Z_2}$	$\mu = \frac{Z_1}{Z_{10}}\mu_0 i$

X_2, X_3 account for the whole range of μ

$$X_1 = X_2 = R_1(1 - \mu)/(1 + \mu), \qquad X_3 = 2R_1\mu/(1 - \mu^2) \tag{8a–c}$$

Here the parameter μ is identical with i if the relation with maximum current I_{20} at $\mu = 1$ is used. The magnification adjustment from zero to maximum sensitivity limited by the attainable σ^2_{max} has no limits for resistors.

If the sum of the internal resistances of the seismometer and galvanometer coils, $R'_1 + R'_2$, is greater than the required total resistance of circuit Z_1, the value $R_1 = Z_1/2$ may be used in (8a–c). The attenuator can be built for attenuation factors μ at which $Z_1/2 + X_1 \geqslant R'_1$ and $Z_1/2 + X_2 \geqslant R'_2$. Similarly for $R'_1 + R'_2 < Z_1$ the calculation is provided for $R_1 = R_2 = Z_1/2$. If $R'_1 \leqslant Z_1/2$ and $R'_2 \leqslant Z_1/2$ then the missing resistance should be added to reach $R'_1 = R'_2 = Z_1/2$ and the whole range of attenuation is then applicable. If only one of the resistances R'_1, R'_2 is greater than $Z_1/2$ it is possible to build the attenuator for attenuation factors which simultaneously yield $Z_1/2 + X_1 \geqslant R'_1$, and $Z_1/2 + X_2 \geqslant R'_2$, as in the preceding case.

If $R_2 = rR_1$, where $r \neq 1$, the attenuator resistances should be

$$X_1 = R_1(r - \mu)/(1 + \mu) \tag{9a}$$

$$X_2 = R_2(1 - \mu r)/r(1 + \mu) \tag{9b}$$

$$X_3 = R_1(1 + r)\mu/(1 - \mu^2) \tag{9c}$$

Here μ is again equal to i and denotes the fractional part of the maximum magnification at $\mu = 1$. To get positive resistances X_1 and X_2 the relations $\mu \leqslant r$ and $\mu \leqslant 1/r$, respectively, must hold true. To fulfil both conditions, for $r > 1$ the attenuation factor $\mu \leqslant 1/r = \mu^\star$ and for $r < 1$ $\mu \leqslant r = \mu^\star$. From the theoretical attenuation range $0 \leqslant \mu \leqslant 1$ it is therefore not possible to use the interval $\mu^\star \leqslant \mu \leqslant 1$.

If $R'_1 + R'_2 < Z_1$, some resistance must be added in series with the seismometer and galvanometer coils to obtain $R_1 + R_2 = Z_1$. This resistance may be included with either the seismometer or the galvanometer when calculating the attenuator resistances from (9a–c). The resistance ratio $R_2/R_1 = r$ thus depends on the co-ordination of additional resistance. For $R'_1 + R'_2 > Z_1$ the resistance ratio holds good for initial resistances R_1, R_2, $R_1 + R_2 = Z_1$. The applicable attenuation range holds good only for values at which $R_1 + X_1 \geqslant R'_1$ and $R_2 + X_2 \geqslant R'_2$.

The values of X_1/R_2 in Fig. 2 are dependent on μ for some ratio r. If we put $r' = 1/r$ then $X_1/R_2(r) = X_2/R_1(r')$ and the graphs account for X_2 resistances for reciprocal values of r.

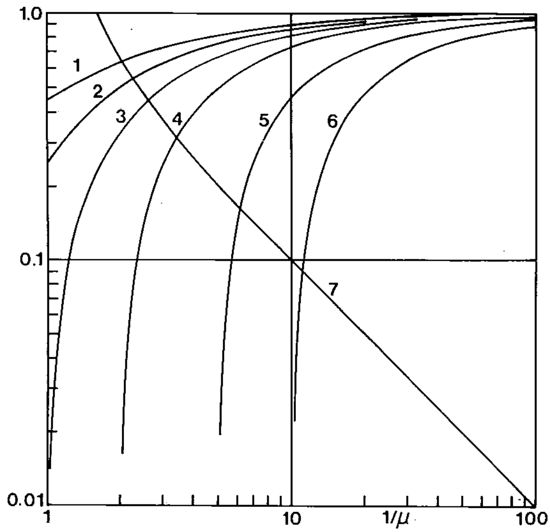

Fig. 2. Attenuator resistances ratio X_1/R_2 for r = 0.1, 0.2, 1, 2, 5, 10 (1–6) and/or X_2/R_1 for $1/r$ and $X_3/(R_1 + R_2)$ (7) versus $1/\mu$.

In general case $Z_2 = zZ_1$, $z \neq 1$, in addition to two conditions for resistances X_1 and X_2, the third condition that for shunt resistance X_3 takes place. The attenuation factor $\mu = 1$ cannot be attained at all and neither, of course can $\sigma^2_{max} < 1$. Given dependence on the relation between R_1 and R_2 only certain possible ranges of μ are valid. This problem was solved in detail by Chaudhury [4].

2.1.2 Transducer with auxiliary coil damping control

In special transducers with two independent coils, the damping of the seismometer is determined by the current in the recording circuit and by the auxiliary current in the additional second coil. This coil has internal resistance R_3 and critical damping resistance a_d; the corresponding maximum constant of electromagnetic damping is $D_d = a_d/R_3$. Only the electromagnetic damping constant of the galvanometer should remain unchanged (Z_2 = const.) and Z_1 changes with attenuation setting. The relations $Z_1 = a_{sg}/D_{sg} \geqslant a_{sg}/(D_s - D_{s0})$ must hold because the damping constant of the signal circuit $D_{sg} \leqslant D_s - D_{s0}$.

If we put $X_2 = C \geqslant 0$ where C is a constant and if we take the same definition of the attenuation factor μ as in the preceding case of one coil transducer, we get

$$X_1 = Z_1(Z_2 - R_2 - X_2)/(\mu Z_2) - R_1 \qquad (10a)$$

$$X_2 = C \tag{10b}$$

$$X_3 = Z_1(Z_2 - R_2 - X_2)/(Z_1 - \mu Z_2) \tag{10c}$$

$$X_4 = a_d/(D_s - D_{s0} - D_{sg}) - R_3 \tag{10d}$$

where X_4 is the resistance in series with the internal resistance R_3 of the damping coil, $D_{sg} = a_{sg}/Z_1$. The necessary condition for a real attenuator circuit is that resistances X_1, X_3, X_4 are all positive.

Since by magnification control Z_1 is not constant, the total current I_1 through the signal coil is different in spite of the influence of the same transducer movement. In this case

$$i = (Z_{01}/\mu_0)/(Z_1/\mu), \tag{11a}$$

$$\mu = i\mu_0 Z_1/Z_{01} \tag{11b}$$

and the ratio of attenuator factors μ/μ_0 is not equal to the current ratio i as was the case in (1d). The resistance ratio $Z_1/Z_{01} \neq 1$, where Z_{01} is the total circuit resistance of the signal coil for attenuation factor μ_0, which depends for the same magnification on additional seismograph parameters. For this reason the attenuator resistances are expressed directly by the ratio i as follows:

$$X_1 = (X_{01} + R_1)/i - R_1 \tag{12a}$$

$$X_2 = C \tag{12b}$$

$$X_3 = \mu_0(X_{01} + R_1)Z_2/(Z_{01} - \mu_0 i Z_2) \tag{12c}$$

Here X_{01} corresponds to (10a) for the original value $\mu = \mu_0$. The value of resistance X_4 is calculated from (10d).

Additional independent adjustment of seismometer damping is useful in extending the possible interval of magnification control in comparison with the single-coil transducer. For $Z_1 = Z_2$, and $R_1 \neq R_2$ it is $Z_{01} = R_1 + R_2$, $\mu_0 = 1$, $X_{01} = 0$, $X_{02} = 0$, $X_{03} = \infty$ at maximum possible sensitivity, and for $i < 1$ the attenuator resistances are

$$X_1 = R_1(1 - i)/i \tag{13a}$$

$$X_2 = 0 \tag{13b}$$

$$X_3 = R_1/(1 - i) \tag{13c}$$

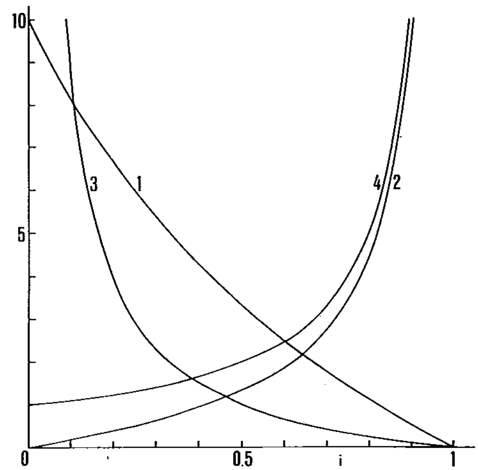

Fig. 3. The values of 10 X_1/R_1 (1), X_3/R_1 (2) of single coil transducer and X_1/R_1 (3), X_3/R_1 (4) of the double coil transducer for cases $R_1 = R_2$.

The resistors X_1 and X_3 are both positive for all values of i and there are no physical limitations if the condition

$$Z_1 = R_1 + X_1 + R_2X_3/(R_2 + X_3) \geqslant R_1 + R_2 \tag{14}$$

is satisfied giving a damping constant of the signal circuit D_{sg} not greater than that at maximum sensitivity. It should be possible to compensate for the decrease in electromagnetic damping from the signal circuit by additional damping of the auxiliary coil, thereby keeping the total damping constant of the seismometer unchanged. Eq. 14 can be rewritten in the form

$$r \leqslant (1 - i + i^2)/i \tag{15}$$

If we have $R_2 \leqslant R_1$ $(r \leqslant 1)$ relation (15) is valid without exception. For $R_2 > R_1$ the current attenuation is forbidden in the range from maximum sensitivity up to some limit, as in the case of the single coil transducer. It is possible to design the correct attenuator only for $i < r^\star$.

Fig. 3 shows the course of resistances X_1 and X_3 for a transducer with an auxiliary damping coil compared with the same resistances of the attenuator of a single-coil transducer. The simple case of $R_1 = R_2$ with the current ratio between maximum magnification ($\mu = 1$) and zero sensitivity ($\mu = 0$) is given. There are particularly great differences between the X_1 and X_2 values (for a single-coil transducer $X_2 = X_1$, for a double-coil transducer $X_2 = 0$). Only the X_3 resistances show not great differences at a higher current ratio.

2.2 Double attenuator

Seismometers with one coil for both signal and electromagnetic damping adjustment have the advantage that when $Z_1 = Z_2$, two independent attenuators can be used in series. This is not possible for a double-coil transducer system with variable resistance Z_1.

If the attenuator factors of individual T-terms are μ_1 and μ_2, the attenuation factor of the whole attenuator circuit is $\mu_t = \mu_1\mu_2$. The values of both attenuator resistances X_{11}, X_{21}, X_{31} and X_{12}, X_{22}, X_{32}, where the first subscript is the number of the resistance in the attenuator and the second is the number of the attenuator (Fig. 4), are calculated again with the aid of Eqs. (1a–c).

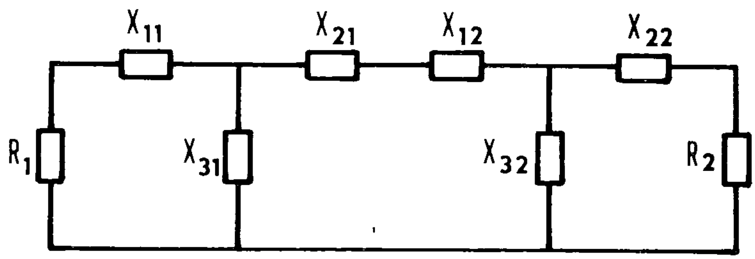

Fig. 4. Diagram of double T-type attenuator circuit of electromagnetic seismograph.

To check the resistances, the single T-term attenuator with resistances X_1, X_2, X_3 calculated for the attenuation factor μ_t from (1a–c) should be equal to the corresponding resistances of the double attenuator given by the following formulae

$$X_1 = X_{11} + X_C/X_{32} \tag{16a}$$

$$X_2 = X_{22} + X_C/X_{31} \tag{16b}$$

$$X_3 = X_C/(X_{12} + X_{21}) \tag{16c}$$

where $X_C = (X_{12} + X_{21})X_{31}X_{32}/(X_{12} + X_{21} + X_{31} + X_{32})$.

2.3 The accuracy of attenuation adjustment

The above formulae give correct values for all circuit resistances, which enables us to obtain the prescribed current attenuation by keeping the electromagnetic

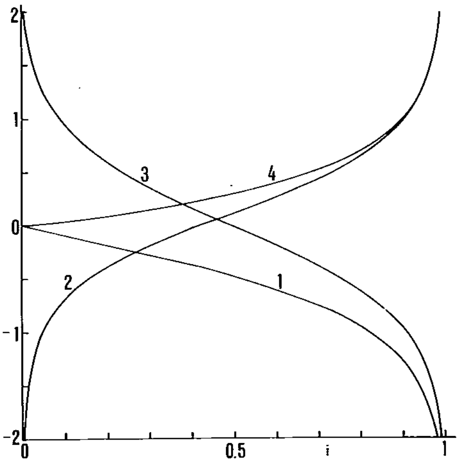

Fig. 5. The logarithmic values of X_1/R_1 (1) and X_3/R_1 (2) of single coil transducer and X_1/R_1 (3), X_3/R_1 (4) of double coil transducer for cases $R_1 = R_2$.

damping of seismometer and galvanometer constant. The accuracy with which these conditions are met depends on the adjustment of the individual resistors.

Using the step attenuator with several position-selector switches there is no problem in getting fixed resistances of sufficient accuracy for standard selected and trimmed electrotechnical resistors. For the single-coil transducer starting with the maximum sensitivity, at which the attenuation factor is μ_0, we have, for 6 dB steps, attenuation factors equal to $\mu_0/2^{n_1}$ ($n_1 = 2, 3,..., N$). The t per cent permissible tolerance in sensitivity requires attenuation factors equal to $\mu_0(1 - 2tn_2/100)$ ($n_2 = 1, 2,..., n$). Using the double attenuator with both attenuation steps different, it is then possible to get arbitrary attenuation factors in the range μ_0 up to $\mu_0/2^N$ with an error smaller than t per cent if n is a minimum integer with a value not less than $25/t$.

With a double-coil transducer it is convenient to start with the maximum attenuation factor μ_0; the decrease in static magnification is then determined in the same way but using current ratio steps: $i = 1/2^n$ and/or $i = 1 - 2tn_2/100$. The corresponding attenuation factors must be calculated from (11b) with the aid of the Z_1 value at individual attenuator settings.

The attenuation factor range determined by continuous attenuation has some practical limits. For $\mu_0 \to 1$, the synchronous adjustment of resistors X_1 and X_3 using the resistance potentiometers with a common shaft is difficult to perform with sufficient accuracy. For this reason some residual attenuation is applied at minimum attenuation. Then for $\mu_0 < 1$, *e.g.* in the special case shown in Figs. 3 and 5, the resistors X_1 and X_2 may be approximated by a linear potentiometer and X_3 by a logarithmic potentiometer. The errors of such approximations depend on the maximum attenuation factor used and its range. For this reason the continuous setting can be applied only over a small range, *e.g.* to trim the step attenuator of double attenuator.

3. *Attenuator setting and displacement sensitivity*

Current I_2 flowing through the galvanometer determines the magnification of the seismograph. A change in current causes a corresponding change in magnification if the coupling coefficient is negligibly small. In the reverse case the ratio of magnification at different periods in the passband may differ substantially from the given current ratio. The current ratio then corresponds only to magnification outside the seismograph passband. To prevent this undesirable effect the basic constants of the seismograph T_s, D_s, T_g, D_g should be controlled. This method is correct but troublesome; by matching the magnification level to only a small extent, the conventional current attenuation with some error can be used.

3.1 Magnification and attenuation factors

For the derivation of the attenuation versus magnification relation we use the following form of the general equation for magnification M (displacement sensitivity) [2]

$$M = \frac{2A}{l} \sqrt{\frac{K_s}{K_g}} \sqrt{\frac{4D_s D_g \sigma^2}{T_s T_g}} \frac{1}{\sqrt{T^{-2} + a + bT^2 + cT^4 + dT^6}} \tag{17}$$

where A is the recording distance of the galvanometer, l is the reduced pendulum lenght of the seismometer, K_s is the moment of inertia of the seismometer and K_g is the moment of inertia of the galvanometer. This relation is valid for the ideal linear seismometer and galvanometer with resistors in the electrical circuit only. The formula can be modified for translational seismometers making l equal to one and replacing K_s by the mass of the seismometer.

The term $V_0 = 2A/l\sqrt{K_s/K_g}$ is independent of the basic seismograph constants and the ground movement period T. The second factor $V_1 = [4D_sD_g\sigma^2/(T_sT_g)]^{\frac{1}{2}}$ depends on the basic constants and the coupling coefficient σ^2. The last term $U = (T^{-2} + a + bT^2 + cT^4 + dT^6)^{-\frac{1}{2}}$ depends on the basic constants, the coupling coefficient and the ground period.

Since $a = m^2 - 2p$, $b = p^2 - 2mq + 2s$, $c = q^2 - 2ps$, $d = s^2$; $m = 2(D_sT_s^{-1} + D_gT_g^{-1})$, $p = T_s^{-2} + T_g^{-2} + 4D_sD_gT_s^{-1}T_g^{-1}(1 - \sigma^2)$, $q = 2(D_sT_s^{-1}T_g^{-2} + D_gT_g^{-1}T_s^{-2})$, $s = T_s^{-2}T_g^{-2}$ the amplitude response function parameters a, b and c are determined by the coupling coefficient and in this way by the attenuation setting. Here the original notation of parameters a, b, c, d, m, p, q, s is given. Later in this section the symbols d, m, s are also used for other parameters.

If T_s, D_s, T_g, D_g do not change with magnification adjustment, and the influence of the change in coupling coefficient on U can be neglected, then for a given period M is only proportional to σ. This is true for small coupling coefficients and at periods outside the passband of the seismograph, since for $T \to 0$ it follows from (17) that $M = V_0V_1T$ and for $T \to \infty$ that $M = V_0(4D_sD_g\sigma^2T_s^3T_g^3)^{\frac{1}{2}}T^{-3}$. In the passband the invariability of amplitude response should be checked for individual cases of combined seismograph constants.

Using the circuit parameters the term V_1 can be rewritten in the form

$$V_1 = \sqrt{\frac{4\bar{a}_{sg}\bar{a}_g}{Z_1Z_2}\frac{X_3^2}{(R_1 + X_1 + X_3)(R_2 + X_2X_3)}} \tag{18}$$

Here $\bar{a}_{sg}$ and $\bar{a}_g$ are the critical total resistances of the seismometer signal coil and the galvanometer, respectively, reduced to the free period of a corresponding system equal to 1 second: $\bar{a}_{sg} = a_{sg}/T_s$, $\bar{a}_g = a_g/T_g$.

Using the relation

$$\sigma^2 = \mu^2\frac{Z_2}{Z_1}\frac{D_s - D_d - D_{s0}}{D_s}\frac{D_g - D_{g0}}{D_g} \tag{19}$$

we get

$$V_1 = \frac{2\mu}{Z_1}\sqrt{\bar{a}_{sg}\bar{a}_g} \tag{20}$$

and further the magnification

$$M = V_0 \frac{2\mu}{Z_1} \sqrt{\bar{a}_{sg} \bar{a}_g}\, U \tag{21}$$

The necessary attenuation factor μ for given magnification M at period T is then

$$\mu = \frac{M(T)}{U(T)} \frac{Z_1}{2V_0} \frac{1}{\sqrt{\bar{a}_{sg} \bar{a}_g}} \tag{22a}$$

$$\mu = \frac{M(T)}{U(T)} \sqrt{\frac{K_g}{K_s}} \frac{l}{4A} \sqrt{\frac{T_s T_g}{(D_s - D_d - D_{s0})(D_g - D_{g0})} \frac{Z_1}{Z_2}} \tag{22b}$$

The values of M and U, *e.g.* at maximum amplitude response, are constants for given periods T_s and T_g and damping constants D_s and D_g, respectively. V_0, $\bar{a}_{sg}$, $\bar{a}_g$ are also constants for a given seismometer and galvanometer. Z_1 is determined uniquely by the prescribed electromagnetic constant of the seismometer. If the right-hand side of (22a) and (22b) is greater than one, the required sensitivity cannot be obtained. For $\mu \leqslant 1$ it is possible to adjust the required sensitivity if all other conditions for attenuator resistors are fulfilled.

Let us consider the relations among current ratio i, attenuation factors, the ratio μ/μ_0, the corresponding magnification ratio $m = M/M_0$ and the coupling coefficient ratio $s = \sigma/\sigma_0$. With single-coil transducer where Z_1 and Z_2 are constants and the influence of coupling coefficient on the course of the amplitude curve is negligible

$$i = \mu/\mu_0 = m = s \tag{23a}$$

For a double-coil transducer Z_1 of the attenuator has to be changed and according to (22a)

$$i = \mu/Z_1 / \mu_0/Z_{01} = m = s \tag{23b}$$

where Z_{01} corresponds to Z_1 at $\mu = \mu_0$.

To keep the amplitude response constant for an arbitrary coupling coefficient a transformation of the basic seismograph constants should be made [11]. If the constants T_s, T_g, D_s, D_g apply to the coupling σ, and T_{0s}, T_{0g}, D_{0s}, D_{0g} to σ_0, the magnification ratio is according to (17)

$$m = s\sqrt{D_s D_g / (D_{0s} D_{0g})} \tag{23c}$$

because by the correct transformation $T_s T_g = T_{0s} T_{0g}$.

When the amplitude response shape varies with absolute magnification, keeping the basic constants T_s, T_g, D_s, D_g unchanged, the magnification ratio is

$$m = sU(T,\sigma)/U(T,\sigma_0) \tag{23d}$$

The above relationships can be used for checking the magnification curve distortion using the current ratio measurements because $i = s$.

3.2 Distortion of magnification curves with attenuation

Three different concepts of the magnification adjustment of seismographs with non-negligible coupling will be further discussed:
(1) Sensitivity adjustment over a wide range without changing the basic seismometer and galvanometer constants,
(2) Sensitivity adjustment over a small range around some standard magnification level,
(3) Transformation of basic constants for large steps of magnification and regulation of magnification over a small range as in (2).

The first case is demonstrated by the long-period seismograph of the World-Wide Standard Seismograph Network (WWSSN) [12]. Here the same constants apply to all levels of maximum magnification from 375 up to 6,000. Maximum sensitivity corresponds to a coupling coefficient of 0.6 – 0.8 (Table 2). In the second case the SKD intermediate band seismograph with nominal magnification 1,000 and coupling coefficient $\sigma^2 = 0.25$ was chosen [6]. For the third case one of the short period standard characteristic A IV of the SKM-3 seismograph with different basic constants at two levels of maximum magnification, 100,000 and 50,000, was used for detailed discussion. The other three short-period characteristics were used for rough estimations [2, 3, 10]. The last characteristics act as standards in the

Table 2. Seismograph constants.

Seismograph type	T_s(s)	D_s	T_g(s)	D_g	Maximum magnification	Coupling coefficient
Press-Ewing	15	0.9	98.1	1.0	up to 6,000	up to 0.6–0.8
SKD	25	0.5	1.2	8.0	1,000	0.25
SKM-3 I	1.23	0.495	0.583	0.495	100,000	0.07
II	1.12	0.644	0.305	1.60	100,000	0.31
III	1.88	0.400	0.573	0.675	100,000	0.20
IV	2.18	0.533	0.293	1.48	100,000	0.79
IV	1.75	0.531	0.366	1.83	50,000	0.16

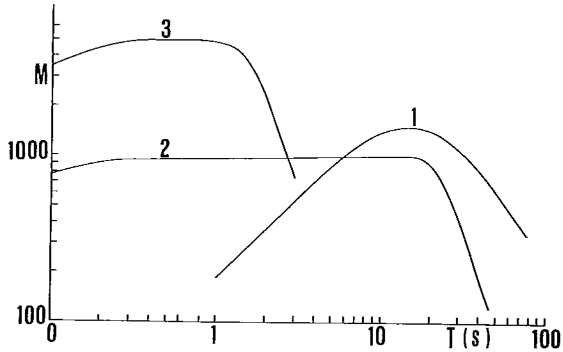

Fig. 6. Magnification curves of standard Press-Ewing long-period seismograph (1), SKD intermediate-period seismograph (2) and SKM-3 short-period seismograph with characteristic A IV (3).

network of seismic stations of the Commission of the Academies of Socialist Countries for Planetary Geophysics (KAPG).

The magnification curves of the seismographs mentioned above are presented in Fig. 6.

3.2.1 Press-Ewing long-period seismograph

To take into account the influence of coupling on the magnification curve we introduce the function h

$$h = \frac{\sigma}{\sqrt{T^{-2} + a + bT^2 + cT^4 + dT^6}} \tag{24}$$

which includes the terms of (17) with all coefficients varying with the coupling coefficient. For the prescribed range of the coupling coefficient $0 < \sigma^2 < \sigma^2_{max}$, we approximate h by means of the polynomial $h = \Sigma a_j \sigma^j$ and similarly the inverse relation $\sigma = \Sigma b_j h_j$, both for constant period T. For a given coupling coefficient these polynomials enable us to obtain the value h for the determination of the absolute magnification, and conversely for the required magnification at period T to calculate the necessary coupling for determining the attenuator resistances.

The function h was normalised at $\sigma^2 = 0{,}8$ to a value equal to unity and then the coefficients a_j and b_j were derived by least-square fitting. The use of 3rd degree polynomials ensures relative errors smaller than 2 per cent for h at $\sigma^2 > 0.05$ and for σ at $\sigma^2 > 0.01$ (Tables 3 and 4). For $\sigma^2 \to 0$ the linear relation $h \sim \sigma$ must

Table 3. Coefficients a_j for approximation $h = \Sigma a_j \sigma^j$.

T (seconds)	a_0	a_1	a_2	a_3	Coefficient of normalization
5	-0.00038	1.15827	-0.00293	-0.04700	0.2489
15	0.00135	1.04431	0.09734	-0.01737	0.1313
20	0.00059	0.92537	0.02526	0.21265	0.1227
25	-0.00281	0.87566	-0.20864	0.53787	0.1223
30	-0.00659	0.87327	-0.44435	0.80533	0.1292
35	-0.00851	0.87931	-0.55843	0.92559	0.1439
40	-0.00821	0.87882	-0.54063	0.90626	0.1666
45	-0.00657	0.87504	-0.44257	0.80109	0.1970
50	-0.00455	0.87454	-0.31796	0.66196	0.2348

Table 4. Coefficients b_j for approximation $\sigma = \Sigma b_j h^j$.

T (seconds)	b_0	b_1	b_2	b_3
5	-0.00012	0.86594	-0.00729	0.03582
15	-0.00122	0.95627	-0.07840	0.01709
20	-0.00285	1.12229	-0.21622	-0.01000
25	-0.00510	1.31083	-0.43896	0.02585
30	-0.00675	1.45378	-0.63834	0.08361
35	-0.00742	1.51593	-0.73111	0.11482
40	-0.00731	1.50485	-0.71360	0.10828
45	-0.00670	1.44859	-0.62887	0.07927
50	-0.00584	1.37346	-0.52077	0.04559

be used to avoid greater errors due to the non-zero coefficients a_0 and b_0. The corresponding coefficients $a_1 = 1.118$ and $b_1 = 0.894$, respectively, which hold true for a non-variable shape of the amplitude response, deviate in the passband of the seismograph from a non-linear course, especially at periods 30–40 seconds. This phenomenon is evident using, according to (23d), the parameter $d = \dot{m}/s = h/h_0 \cdot \sigma_0/\sigma$ for comparison of magnification at different coupling coefficients. The deviation of d from value 1 is a measure of the magnification curve distortion.

In Fig. 7 the parameter d is plotted for several periods against the coupling coefficient σ; the value $\sigma_0 = 0.1$ is chosen for comparison as the standard course of the amplitude response at a negligibly small coupling coefficient. For small periods up to 15 seconds the distortion lies in the range ±5 per cent. Testing the magnification level by the sine-wave method is quite reasonable for the whole coupling coefficient range. For periods of 30–40 seconds the magnification dis-

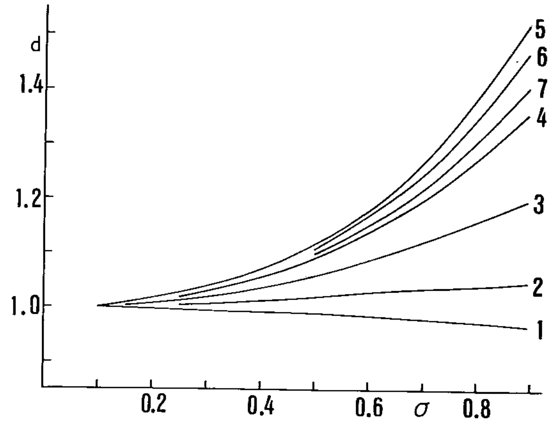

Fig. 7. Distortion of magnification curve of long-period seismograph at periods 5, 15, 20, 25, 35, 45 and 50 seconds (1–7) versus coupling coefficient σ.

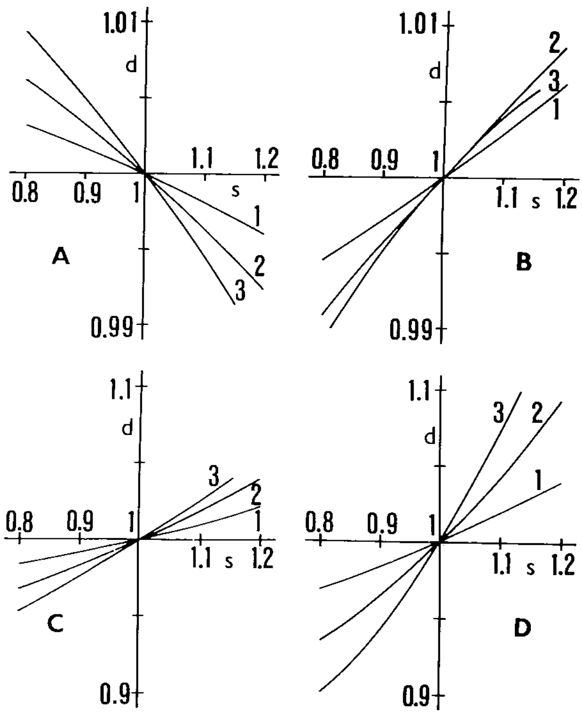

Fig. 8. Distortion of magnification of long-period seismograph at period 5 (A), 15 (B), 20 (C) and 30 (D) seconds on three sensitivity levels $\sigma_0^2 = 0.2$ (1), $\sigma_0^2 = 0.4$ (2), $\sigma_0^2 = 0.6$ (3) with attenuation adjustment s.

tortion is smaller than 10 per cent at $\sigma_0^2 < 0.2$, more than 20 per cent at $\sigma_0^2 = 0.4$ and as high as 60 per cent at the theoretical coupling maximum. The step attenuator adjustment between 24 dB and 0 dB necessitates an equally large change in amplitude response. This must be borne in mind if the calibration is made with a substantially smaller seismograph sensitivity adjustment than that used during operation.

With a fine adjustment of magnification over a small range about some mean coupling value it is more advantageous to use the amplitude response at this mean coefficient as the comparison standard. The amplitude response distortion for periods 5, 15, 25 and 30 seconds in the range $0.8 \leqslant s \leqslant 1.2$ for $\sigma_0^2 = 0.2, 0.4$ and 0.6 is given in Fig. 8. The magnification adjustment in the range −1.9 dB, to +1.6 dB yields a distortion at 5 and 15 seconds of less than 1 per cent and only for the maximum coupling coefficient $\sigma_0^2 = 0.6$ does it reach 10 per cent at longer periods. The curve for $\sigma_0^2 = 0.6$ is limited here by the maximum value of $s = 1.155$ ($\sigma^2 = 0.8$).

3.2.2 Broad-band SKD seismograph

For this type of seismograph the nominal coupling coefficient is only 0.25. The conventional attenuation is tested over a small range to simulate the equivalent magnification due to deviations in the constructional parameters of SKD seismometers and GK-VII galvanometer. As in the preceding case, the range $0.8 \leqslant s \leqslant 1.2$ is considered at a mean coupling of $\sigma_0^2 = 0.25$. The magnification distortion is presented in Fig. 9. The greatest deviations are at periods of 15–20 seconds (up to 8 per cent); for longer and/or shorter periods the deviations decrease. This is

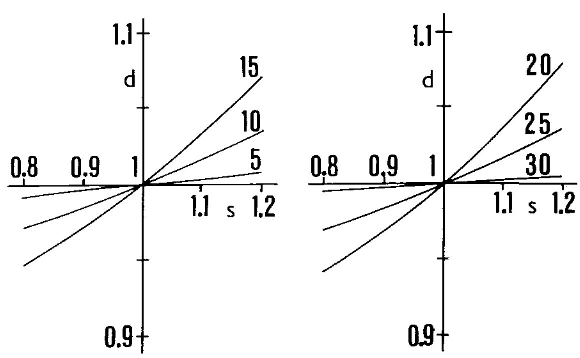

Fig. 9. Distortion of standard magnification of SKD seismograph at periods 5, 10, 15, 20, 25 and 30 seconds with attenuation adjustment s in the range 0.8–1.2.

a quite acceptable range of deviation from the standard course. Using other seismometer and galvanometer types the same characteristic can be attained even when the coupling coefficient clearly differs from the value mentioned (0.25) due to the transformation of the basic constants [9]. For $\sigma^2 < \sigma_0^2$, magnification adjustment using the attenuator yields even smaller deviations from the standard course. On the other hand, for $\sigma^2 > \sigma_0^2$ the effect of the attenuator is less advantageous.

3.2.3 Short-period SKM-3 seismograph

The short-period characteristics A I–IV are the recommended characteristics derived from the point of view of the seismic noise amplitude-frequency distribution. The characteristic A IV is the one most used in practice at KAPG seismic stations. The same maximum magnification requires the highest coupling coefficient among short-period seismographs (see Table 2) and thus the amplitude response deviations for current attenuation should also be the greatest.

The magnification curve distortion calculated for periods of 0.5, 1, 1.5 and 2 seconds and for $0.8 \leqslant s \leqslant 1.2$ is given in Fig. 10. (Only for the 100,000 maximum magnification is the increase in magnification limited to $s = 1.125$, corresponding to the theoretical maximum value of the coupling coefficient $\sigma^2 = 1$.) At a maximum magnification of 50,000 the magnification distortion is smaller than 5 per cent and at 100,000 is about 10 per cent for a 10 per cent attenuation. The

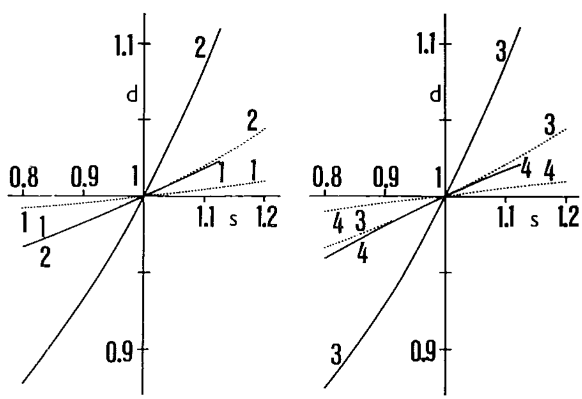

Fig. 10. Distortion of standard magnification curve of short-period SKM-3 seismograph with characteristic IV at periods 0.5 (1), 1 (2), 1.5 (3) and 2 (4) seconds with attenuation adjustment in the range 0.8–1.2 for maximum magnification 100,000 (solid lines) and 50,000 (dotted lines). In the intervals where both curves cannot be differentiated the course for $M = 100{,}000$ with two numbers is given.

Table 5. Deviation d (per cent) from standard curve with maximum magnification 100,000 for s equal to 0.8 and 1.2 at some periods T(s).

Characteristic / Period T(s)	I	II	III	IV
0.5	-0.1 0.1	-5.0 6.5	0 -0.1	-3.3 2.3*)
1.0	-1.4 1.7	-6.9 9.3	-2.7 3.6	-12.3 12.0*)
1.5	-0.2 0	-0.2 -0.2	-2.9 5.7	-12.8 11.1*)
2.0	0.3 -0.3	1.3 -1.7	-0.5 0.5	-4.2 2.2*)

*) For s equal to maximum theoretical attainable value 1.125.

greatest deviations are in the period interval 1–1.5 seconds in the passband of seismograph. To achieve a smaller distortion of the magnification curve it is preferable to readjust both the periods and the damping constants of the seismograph for the required coupling coefficient [10].

The characteristic II with the second highest coupling coefficient at the 100,000 maximum magnification has a distortion range similar to that of type IV with the same maximum magnification. The values of the magnification curve deviations at periods 0.5, 1, 1.5 and 2 seconds for the limit values of the attenuation adjustment s = 0.8 and 1.2 are given for comparison in Table 5. Only the other two characteristics I and III with $\sigma_0^2 = 0.2$ and 0.07 at 100,000 maximum magnification produce a reasonable distortion of the magnification curve. The deviation of the amplitude response from the standard course is acceptable over the whole range of magnification adjustment tested.

4. *Conclusion*

The formulae listed provide a brief summary of the procedures commonly used in seismometry for the calculation of the resistances of an attenuator circuit. The relationships among the different attenuation factors facilitate the use of different methods if the amplitude response and the maximum magnification are known and/or only the relative change of magnification is required. The simple tests of magnification curve distortion in the range of attenuator adjustment help us to determine the limits of the deviations of operational magnification from some standard magnification course. This is particularly important when the seismograph is calibrated at a magnification level substantially different from the one at which it is to operate, and if the effect of the coupling coefficient is non-negligible in at least one of these cases.

Control of the magnification level by an electric circuit only, even over a small range, may sometimes cause comparable distortions in the amplitude response, so that the effect of such adjustments is questionable. A sufficiently high accuracy of attenuation, and the stability of the attenuator with respect to time are the necessary prerequisites for keeping the seismograph response constant. The errors in the attenuator circuit and the influence of environmental conditions on the long-term magnification stability will be discussed in a separate paper.

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