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CALCULATION OF RESPONSE SPECTRA FROM SHORT-PERIOD WWSSN RECORDS USING THE FOURIER TRANSFORM

by

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Abstract

The low-sensitivity short period seismogram of the Novaya Zemlya explosion on October 18, 1975, recorded at Nurmijärvi station was used to calculate response spectra. The trace spectrum was calculated by applying the fast Fourier transform to the digitized data. The spectrum of the oscillator movements was obtained by dividing and multiplying the trace spectrum by the complex transfer functions of both the seismograph and the oscillator, respectively. The time histories of the vibration of the oscillators with different natural frequencies were obtained from the spectra by inverse Fourier transform. The maximum values of these time histories were sought and presented as relative response spectra. The calculated true ground motions are also given. The levels of the response spectra based on bedrock measurements are low, although the event was felt in some high buildings in Finland; the maximal spectral values of P waves in a N–S direction with 8 percent damping in the frequency band 0.4 – 4.0 Hz were: d = 0.01 mm at 0.88 Hz, ν = 0.07 mm/s at 1.29 Hz and a = 1.1 mm/s^2 at 3.76 Hz.

1. *Introduction*

The response spectrum is a common measure of the effect of an earthquake on various kinds of constructions; it is also the basic key used in designing earthquake-resistant structures. Although the ground quakes only to a limited extent in Finland, some seismic events are felt from time to time, and since structures which are more and more sensitive to quaking are constructed these days, interest in the effects of quaking is, of course, growing. Conventionally the response spectrum is calculated by convolving accelerograms with the response time function of linear systems with a single degree of freedom (HUDSON, [4]). The ordinary convolution technique is, however, tedious and takes a lot of computer time. Other methods have, therefore, been developed and used for this purpose. The damped Fourier spectrum presented by UDWADIA and TRIFUNAC [6] gives almost the same information and can be calculated more rapidly. An alternative procedure to convolution is the solution of a second order differential equation of a linear system with a single degree of freedom. BEAUDET and WOLFSON [1] have developed a method which gives a numerical solution for this differential equation using recursive filtering of the accelerograms.

In many aseismic countries no accelerograms are commonly available, and ordinary seismograms must therefore be used for the analysis. When the effect of the seismograph is removed by dividing the trace spectrum by the seismograph complex response, a straight-forward way appears to be to calculate the response spectrum. The ground spectrum is multiplied by the complex response of the oscillating system. The resulting spectrum is converted back to the time domain by the Fourier transform and finally the maximum amplitude must be sought. In 1975 a low-sensitivity output of the Nurmijärvi (NUR) short-period horizontal Benioff seismometer was recorded for test purposes. The recorded material included some nuclear explosions at Novaya Zemlya. Owing to the low magnification of this seismograph, the high-power explosions, which were also felt, were recorded without overlapping or any microseismic noise. In the present study the horizontal N-S component of P and S waves from the event of October 18, 1975, was analyzed in order to test the method described above.

2. *The response spectrum*

Using spectral methods, the response spectrum is determined by first calculating the complex Fourier spectrum $X(\omega_e)$ from the time function $x(t)$. The spectrum of the ground motion is then

$$G(\omega_e) = X(\omega_e)/S(\omega_e) \tag{1}$$

where $S(\omega_e)$ is the complex response or transfer function of the seismograph. The spectrum $R(\omega_e)$ of the motion of a linear system with a single degree of freedom (SDF) is now obtained by multiplying the ground motion spectrum $G(\omega_e)$ by the transfer function $H(\omega_e)$ of the SDF system

$$R(\omega_e) = H(\omega_e) \cdot G(\omega_e) \tag{2}$$

where

$$H(\omega_e) = \frac{\omega_e^2 (i\omega_e)^n}{\omega_0^2 - \omega_e^2 + i2h\omega_e^2\omega_0^2} \tag{3}$$

In equation (3) n has the values 0, 1, 2, for displacement, velocity, and acceleration, respectively. ω_0 and h are the natural frequency and the damping of the oscillating system, and ω_e is the frequency of the ground motion. Applying the inverse Fourier transform to $R(\omega_e)$, the relative motion of SDF is obtained:

$$r(h, \omega_0; t) \leftrightarrow R(h, \omega_0; \omega) \tag{4}$$

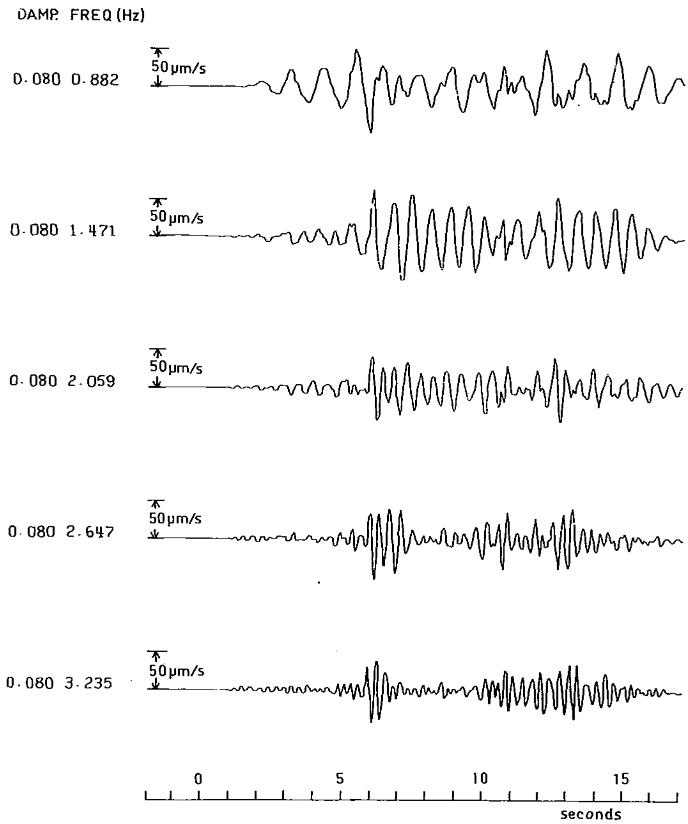

Fig. 1. Examples of the velocity variations of the oscillators with five different natural frequencies and a damping of 0.08 calculated from the P wave portion of a seismogram.

In this formula the parameters h and ω_0 are also indicated for purposes of clarity. The maximum amplitudes, sought from such time series $r(t)$ and plotted as a function of ω_0, yield the response spectrum. As an example, Fig. 1 gives the relative velocities $r(t)$ of linear harmonic oscillators with five different natural frequencies and with damping $h = 0.08$. In practice only one of the spectra needs to be calculated because the other two pseudo displacement and acceleration spectra can be obtained simply by dividing and multiplying by ω_e or using a four-way log grid (Figs. 4 and 5). Generally the displacement, velocity and acceleration of the ground motion are also interesting. They can easily be calculated as a by-product using the inverse Fourier transform from the ground motion spectra.

3. *The response of the seismograph*

Following the work of CHAKRABARTY and CHOUDHURY [3] on the response of the Benioff-type seismograph, and converting their formulas into the complex form, the response of the seismograph can be written as follows:

$$S(\omega_e) = -i\omega_e^3 C' / [(\omega_1^2 - \omega_e^2 + i2\epsilon_1 \omega_e)(\omega_g^2 - \omega_e^2 + i2\epsilon_g\omega_e)(E_0 + iE_1)] \qquad (5)$$

where

$$E_0 = (R + s)(r + s)$$

$$E_1 = L_0\omega_e(r + s)$$

$$\epsilon_1 = \epsilon/(1 + C^2)$$

$$\omega_1^2 = \omega^2 + 2\epsilon\,\omega_e \frac{C}{1 + C^2}$$

$$C = L_0\,\omega_e/(R + s)$$

The symbols used here are general and are the same as those in the work of CHAKRABARTY *et al.* [3]. In addition, C' is the magnification level constant and $i = \sqrt{-1}$. In formula (5) the inductance of the transducer coil was taken into consideration but the coupling and inductance of the galvanometer as well as the air damping of the seismometer were neglected.

4. *Data processing*

The nuclear explosion on October 18, 1975, in the Novaya Zemlya region was recorded on the bedrock at Nurmijärvi seismic station by a low-sensitivity N–S Benioff seismograph. According to the PDE cards of USGS, the location of the event was 70.8°N and 53.7°E, origin time = 08 59 56.3, m_b = 6.7 and M_s = 5.1; hence the epicentral distance to Nurmijärvi was 1733 km and the azimuth 36°. The instrumental constants of the seismograph were as follows: T = 1.0 s, h = 0.47, T_g = 0.75 s, h_g = 0.86, $\sigma^2 \leqslant 0.0006$ and magnif. = 2100 at 1 s period. The speed of the photosensitive paper used on the recorder was 60 mm/s and, according to SAVILL *et al.* [5], the inductance of the seismometer, L_0 = 6.8 H. The original record was enlarged optically about ten-fold and 17 seconds from the beginning of the P and S wave records were digitized into 182 sampling pairs x_i, t_i by a semi-automatic analog-to-digital converter model P.C.D. type ZAE. 2A with unequal time intervals. The equal spaced data with the sampling of 15 Hz were then produced by the standard subprogram of the program library of the Institute of Seismology. This program fits a second order polynomial to three adjacent x_i, t_i pairs and reads the new values for x with the desired equal time intervals. At the same time some digitizing errors are removed, too. In spite of this procedure, some incorrect values still exist in time series. The influence of these errors appears especially on the high frequency part of the spectrum; hence this part of the spectrum above 5 Hz has been cut off. Because the main lobe of the Benioff seismograph's sensitivity lies between 0.7 and 5 Hz, nothing essential from the recorded information was omitted in this cutting operation. If the quality of the data had been better, as is the case for instance with digitally recorded data, this cutting would not, of course, have been necessary. In order to minimize the errors caused when only one part of the continuous record is taken for analysis, both ends of the time series were tapered by a cosine bell function.

The digitization also causes other errors, of course. The incorrect zero line, for instance, produces errors at low frequencies and since the sensitivity of the seismograph here is low, these errors are greatly magnified when the true ground spectrum is calculated. To avoid this the response $S(\omega_e)$ is replaced by a modified response $S'(\omega_e)$, which is identical to $S(\omega_e)$ at the frequency band 0.3–17 Hz, but which has an amplitude kept constant at the 3 % level of the peak magnification at other frequencies. The phases of $S(\omega_e)$ and $S'(\omega_e)$ are identical over the entire frequency range. These modifications have no effect at the frequency band concerned. Another troublesome problem arises when the continuous Fourier integrals are replaced by discrete Fourier transform. With the aid of the discrete Fourier transform the periodic functions (the window length T as a period) are presented as harmonic components. The ground is actually supposed to repeat its movements again and again within this same period. The oscillator is not yet at rest when the same excitation starts again. In order to give the oscillator time to rest, zeroes must be added to one or other end of the time series. The number of zeroes needed to rest the oscillator depends on the damping and period of the oscillator. From the damping term of the oscillator $\exp(-h\omega_0 t)$ one can easily derive the equation

$$t = \ln(100/p)\,(h\omega_0)^{-1} \tag{6}$$

When the zeroes corresponding to time t have been added, p percent of the earlier movements is still present when the new excitation starts. In this work two time values $t = 3T$ (fractional damping of 0.008) and $t = T$ (fractional dampings of 0.08 and 0.2) are used. These values guarantee that p is always smaller than or equal to 10. In addition a noteworthy problem arises when the ground spectrum is multiplied by the response of the oscillating system. The resonance peak of the oscillator response with small damping values is very narrow. When the line spectrum $G(\omega_e)$ of the ground motion is now multiplied by the oscillator response $R(\omega_e)$ it is possible that the spectral lines of $G(\omega_e)$ will not coincide with the large values of the oscillator response near its resonance peak. To avoid this, the natural frequencies of the oscillator were chosen from the population of the frequency lines of the ground motion spectrum. This difficulty and the problem of the periodicity of the excitation force make it impossible to calculate the response spectrum with very low damping values using the method described here. These difficulties are not serious, however, when the damping values of real buildings are used in the calculations.

All calculations were made using the Burroughs B6500 computer of the Computing Center of the University of Helsinki. The Fourier spectra were calculated with the aid of the well-known Fast Fourier algorithm.

5. *Results and discussion*

Although the true ground movements are not necessary for the calculating procedure presented here, they are none the less interesting results and are also useful for checking possible digitization errors. These motions will be given here first because they are usually considered as input quantities. Fig. 2 shows the tapered P wave seismogram as well as the displacement, velocity and acceleration of the ground motion. The maximal horizontal values of the bedrock vibration in the N–S direction are as small as 5.5 μm, 24 μm/s and 510 μm/s^2 for displacement, velocity and acceleration, respectively. The same information for the S wave part of the seismogram is presented in Fig. 3. The corresponding maximum values in this case are 8.9 μm, 31 μm/s and 425 μm/s^2. These S wave values are slightly larger except for acceleration, which is larger for P waves evidently because of the higher frequency content. Considering that the waves arrived with an azimuth of

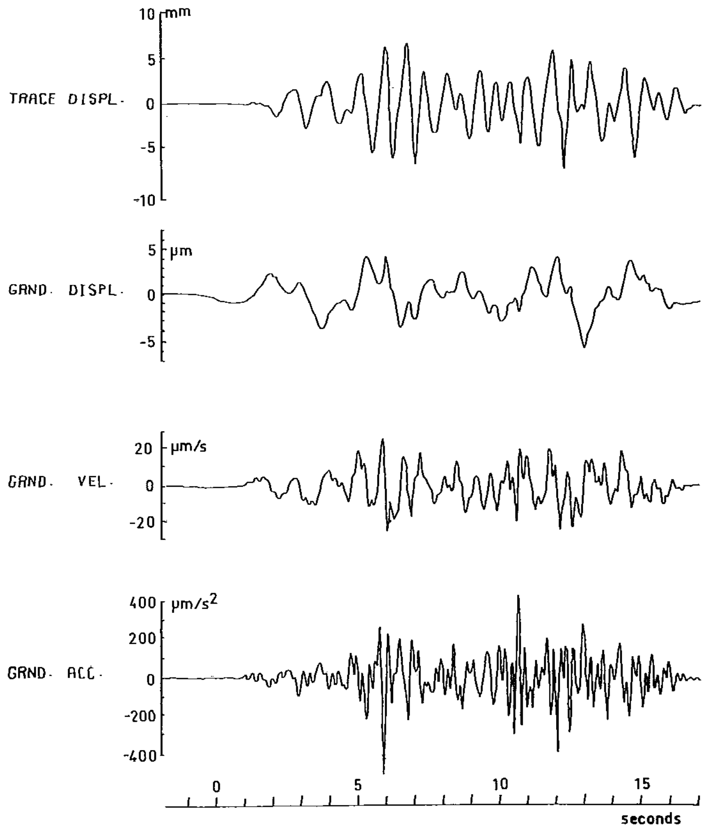

Fig. 2. From top to bottom: the P wave trace of the seismogram (after the digitization and tapering process) and the calculated ground displacement velocity and acceleration values.

about 36 degrees from the N–S direction, all the total horizontal quantities of the S wave have been bigger. The response spectra of P and S waves in the N–S direction are presented in Figs. 4 and 5. The velocity spectra were calculated with 3 different damping values and drawn on the four-way log grids from which also the pseudo-displacement and pseudo-acceleration spectra are to be read. As expected, the spectral values are very small. For the P wave, the maximum displacement, velocity and acceleration of an 8 % damped oscillator are only 0.012 mm at 0.41 Hz, 0.07 mm/s at 1.29 Hz and 1.1 mm/s^2 at 3.76 Hz, respectively. The spectra for S waves are at almost the same level but now the low frequency part of the spectrum is more powerful, which is, of course, typical of S waves, the

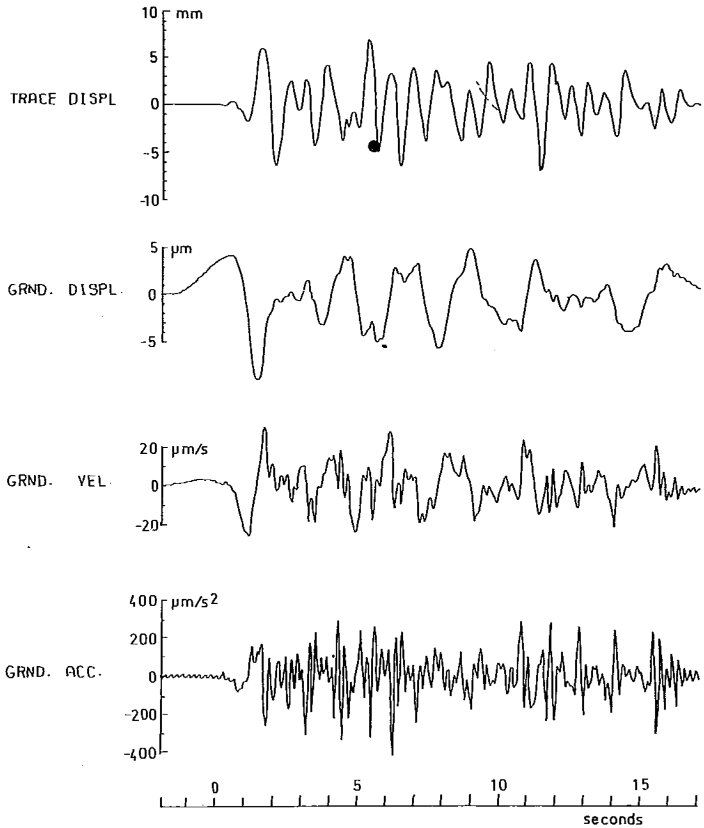

Fig. 3. From top to bottom: the S wave trace of the seismogram (after the digitization and tapering process) and the calculated ground displacement velocity and acceleration values.

corresponding values being 0.023 mm at 0.41 Hz, 0.07 mm/s at 0.76 Hz and 0.76 mm/s^2 at 2.82 Hz. As before, these spectra are based on the N–S component of the ground motion. If the azimuth of the arrival waves is taken into consideration and the P wave is supposed to be purely longitudinal and the S wave transversal, the values of the S wave spectrum must be multiplied roughly by 1.7 and those of the P wave by 1.2 in order to get the total horizontal spectra. This results that the total horizontal response spectra generated by the S wave lie at higher levels at all frequencies studied here. It is difficult to compare these results with others from the Scandinavian region because no spectrum calculated from the data of a nuclear event felt has ever been found. BÅTH *et al.* [2] give some

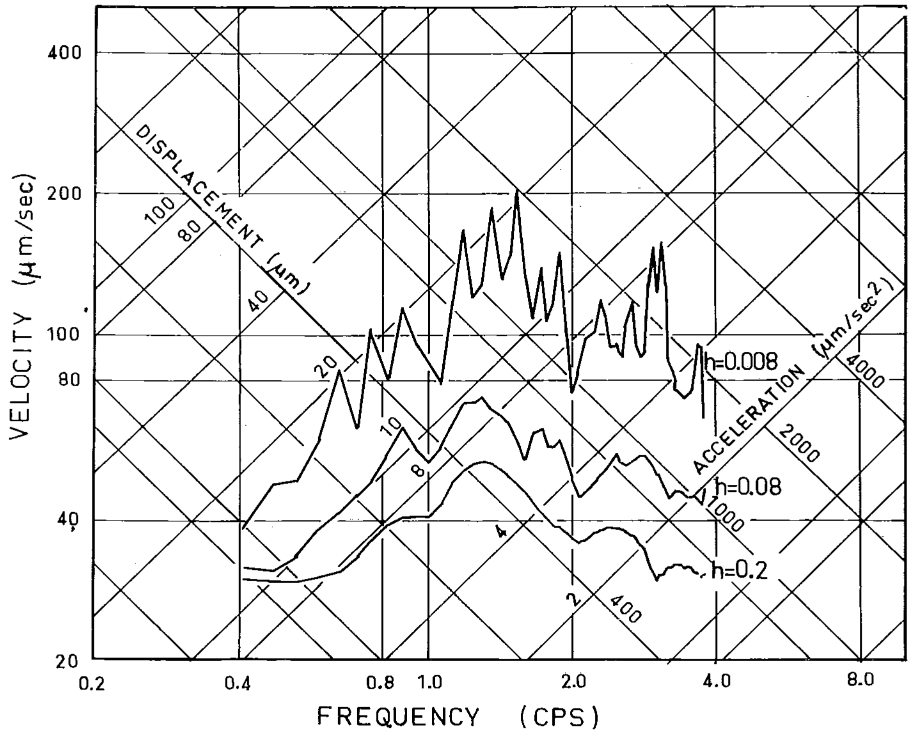

Fig. 4. The relative response spectra calculated from the P wave portion of a Novaya Zemlya nuclear explosion recorded in the N–S direction on bedrock at the NUR station (Finland) October 18, 1975.

results determined by a different method, and which are based on a Crenet-Coulomb record of the P wave for the Novaya Zemlya event not felt [m_b (CGS) = 5.9, m (UPP, KIR) = 6.3] from Oct. 21, 1967. Their maximum values (about 0.012 mm, 0.045 mm/s and 0.6 mm/s^2) of a 10 % damped oscillator at a distance of 2400 km can only be compared with difficulty with the results of this present study, because they are based on the vertical component of motion, and because the magnitude of this event vary according to different authors.

The spectral method and its application described in this paper provide a useful way of calculating response spectra where, instead of accelerograms, only records of ordinary seismographs are available. The main difficulties are involved in the digitization of the seismograms and in the processing of discrete data. These are not serious when the spectra are calculated with the parameter values of real constructions.

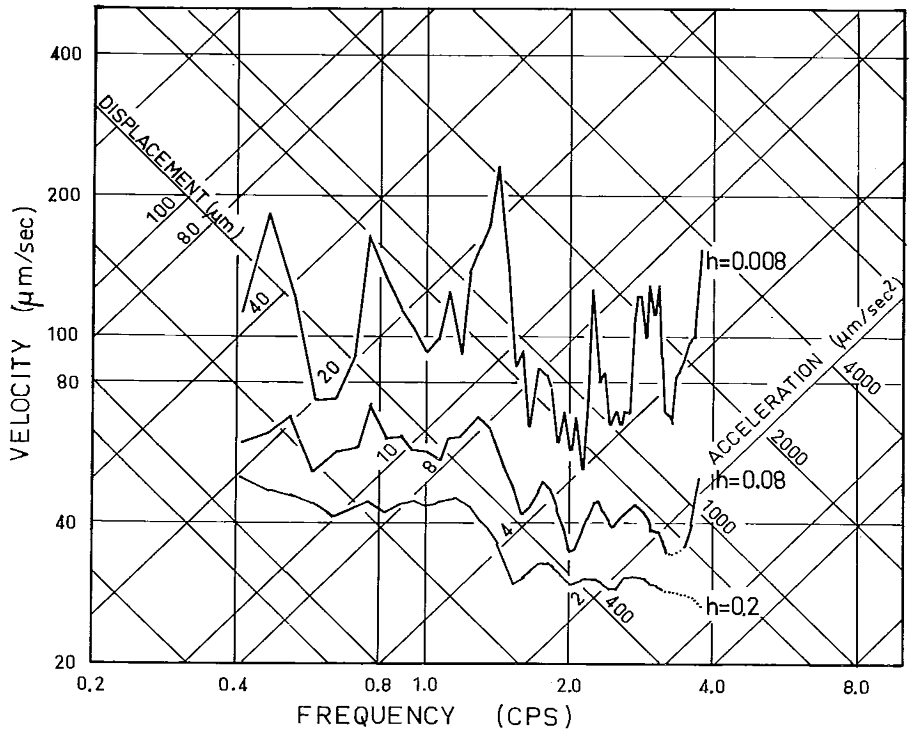

Fig. 5. The relative response spectra calculated from the S wave portion of a Novaya Zemlya nuclear explosion recorded in the N-S direction on bedrock at the NUR station (Finland) October 18, 1975.

The spectral values calculated for the event of Oct. 18, 1975, are so insignificant that they cannot by themselves explain the fact that this event was felt at epicentral distances that are the same as the epicentral distance to the recording site. The explanation can obviously be found in the soil structures under the buildings where the shaking was felt.

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