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ZERO-CORRECTIONS FOR TELLUROMETERS OF THE FINNISH GEODETIC INSTITUTE

by

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Abstract

The Finnish Geodetic Institute carried out measurements to determine the zero errors of its three tellurometer units, two of type MRA 3 and one of type MRA 101, on the second half of the Nummela standard baseline, the length of which is

$$432\,027.18 \pm 0.04 \text{ mm}$$

according to light interference comparator measurements.

In order to determine the cyclic part of the zero-correction the calibration distance was changed by 2.5 m steps from 432 m to 412 m.

The correction is expressed with a periodic function

$$k = h + a \cdot \sin (u - \alpha)$$

and the coefficients h, a and α computed by the least square adjustment in three different ways: 1. individual values for each pair, 2. individual values h for each pair but amplitude a and phase shift α common to the pairs with the same master unit, and 3. individual values h for each pair but amplitude a and phase shift α common to all pairs.

The zero-corrections obtained from the second adjustment have been used in field work. The constant h is $0 - +3$ cm for units of type MRA 3 and $-11 - -15$ cm for the unit of type MRA 101, the constant a $2 - 3$ cm and α $90° - 110°$.

Two examples, Seglinge and Vihti, at sides of 2—3 km and 6 km are presented. The cyclic zero-correction improved the consistency of results.

Introduction

Zero-correction for a tellurometer has been determined in particular test measurements. Examples are the work done with MRA 1 and 2 in Denmark ([9]), Sweden ([2]), Norway ([1]) and Canada ([7]).

Zeropoint corrections clearly deviating from zero were obtained for every instrument. In addition some cyclic error was found depending on the dial reading. The test measurements with type MRA 3, made in Ohio, also displayed similar errors ([6]).

When the Finnish Geodetic Institute began its tellurometer measurements in 1965, the sides of the Vihti extension net were measured in order to obtain a total calibration for the measuring instruments ([8]). A preliminary experiment was made in this connection to investigate the possible cyclic zero error in these instruments. It was found that a certain cyclic zero error depending on the dial reading exists in these instruments.

A more detailed investigation into cyclic zero-correction was made in 1969. These observations and their results are reported in this paper.

Observation ground and instruments

The observations were carried out on 11—17 June, 1969 and on 6—10 October, 1969, before and after the period of field work. The tellurometer observations were made by the author, assisted by Mr. Matti Ollikainen. The weather observations were carried out by Mr. Jorma Savolainen in June and by Mr. Juhani Utela in October.

The measurements were made on the second half of the Nummela standard baseline. Fig. 1 shows this half of the baseline. The area is covered with thin forest consisting of about 80% 15 m high pines, 20% 15 m high birches, some 2—3 m high pines. The soil is covered with heath and blueberry brush and moss. The soil is of gravel and dry. The length of the baseline has been determined with a Väisälä comparator seven times ([3] p. 64). The mean of the two latest determinations, in 1966 and in 1968, give the length

$$432\,027.18 \pm 0.04 \text{ mm}$$

for the second half of the baseline. This is the horizontal distance between underground bolts reduced to the elevation of the 0-bolt.

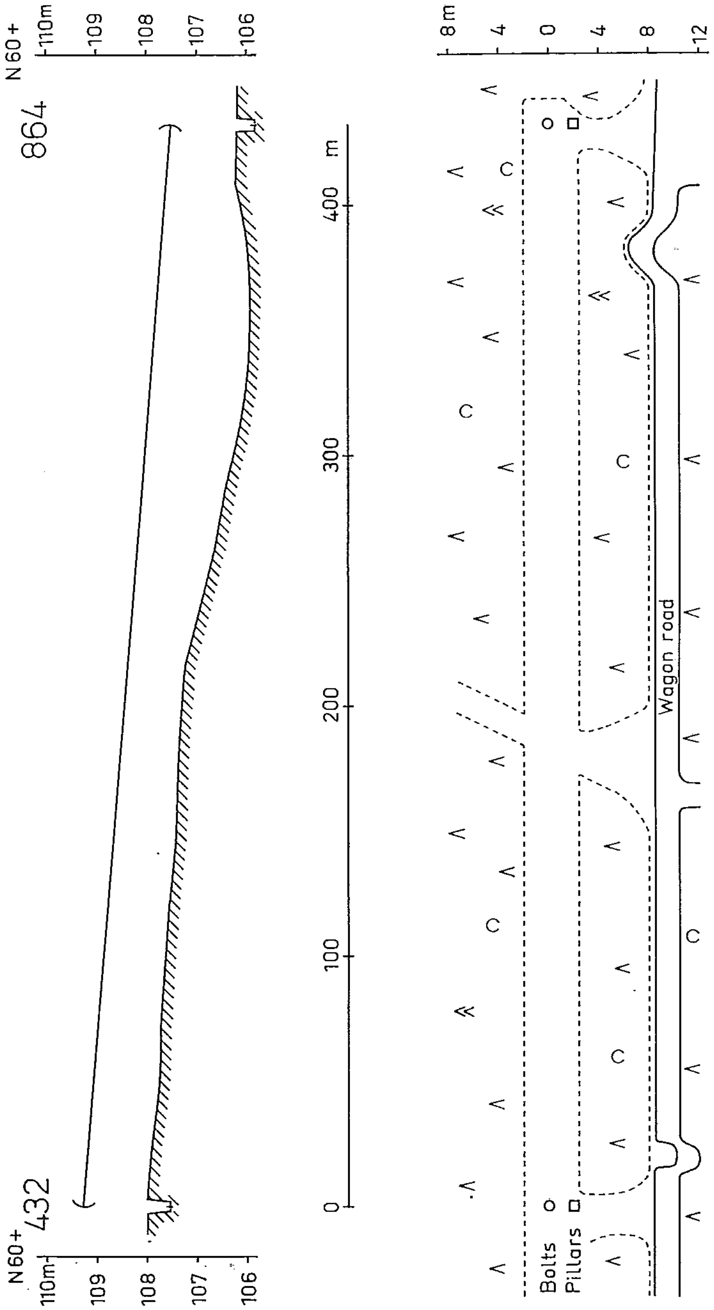

Fig. 1. Vertical projection and horizontal projection of the second half of the Nummela standard baseline. Elevations are in N60-system.

The tellurometers owned by the Finnish Geodetic Institute

MRA 3 No. 574
MRA 3 No. 726
MRA 101 No. 330

have interchangeable master and remote units, which makes it possible for each instrument unit to be used as the master and remote alternatively. In addition they all have dial read-out. The manufacturer gives the nominal frequencies A and A— equal to 7.492 377 Mc/sec for these instruments. The difference frequency A minus A— is 2×7.492 377 = 14.984 754 Mc/sec. So one revolution of the dial corresponds to a change of 10 m in the distance to be measured.

In the observations in June the temperature and water vapour pressure were observed with an Assmann aspiration-psychrometer Fuess No. 31628, thermometers with divisions of 0°.2 C. The air pressure was observed with a barometer Thommen No. 55839.

In October the temperature and water vapour pressure observations were carried out with an electrically ventilated psychrometer Theodor Friedrichs, Hamburg, No. 331. This has gauges of platin resistant wires cast in a glassy mass. A reading device has been constructed by Dr. Seppo Huovila, based on a Wheatstone bridge, with a galvanometer acting as null indicator and used with a precision potentiometer. The gauges and reading device were calibrated to a mercury thermometer before and after the field period.

The air pressure was observed with the barometer Thommen No. 69013.

Observations

The weather during the observation period in June was warm compared with usual conditions in Finland. The weather in October was more similar to normal observation conditions. Maximum, minimum and mean values for the temperatures, water vapour pressures, air pressures and refraction indexes during the observations are given in Table 1. Variations in the meteorological elements caused more variation in the refraction correction in June *i.e.* from +4.5 to +17.3 mm than in October, when it varied from —2.2 to +7.1 mm.

Table 1.

1969 Date	Temperature °C			Water vapour pressure mb			Air Pressure mb			N = (n − 1) · 10⁶		
	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max	Min	Mean
VI 11	21.9	17.4	19.9	4.98	4.22	4.61	999.4	998.6	998.9	295.49	286.72	290.938
12	20.8	16.2	18.9	5.67	5.13	5.36	1001.7	999.2	1000.5	300.26	294.82	296.896
16	24.0	19.9	21.9	6.79	4.86	5.61	1002.8	1001.6	1002.1	300.25	291.20	295.258
17	24.1	18.3	22.0	8.48	4.15	6.55	1003.6	1002.2	1003.0	314.54	285.10	300.822
X 6	5.6	4.8	5.2	6.93	5.47	6.10	1015.2	1014.8	1015.0	316.46	308.68	312.095
7	8.8	8.3	8.6	9.47	8.93	9.17	1010.3	1009.3	1009.8	322.42	319.52	320.840
8	11.6	10.1	10.5	12.53	11.60	11.93	1001.6	999.3	1000.8	329.34	327.56	328.474
9	12.5	9.8	11.0	12.93	10.67	11.58	997.9	992.0	994.8	330.12	319.78	324.609
10	11.6	9.4	10.6	8.80	7.84	8.41	1001.0	998.3	999.9	314.53	308.75	312.033

All distances were measured with each pair of instrument units. The tellurometers were placed on tripods set up at the terminal points. Each tellurometer measurement consists of forward and reverse readings with a ten carrier frequency value. The mean of these is treated as one observation.

Throughout the tellurometer measurements the temperature and water vapour pressure were observed at two minute intervals and the air pressure was observed at five minute intervals. This means that 4—7 temperature and water vapour pressure observations and 2—3 air pressure observations were made during one tellurometer observation. Using the mean of these observations and the formula

$$(n - 1) \cdot 10^6 = \frac{103.49}{T}(p - e) + \frac{86.26}{T}\left(1 + \frac{5748}{T}\right)e$$

for computing the refraction index n the tellurometer observation was corrected for refraction. Thus corrected, the tellurometer value was reduced to correspond to the nominal length of the baseline mentioned in the previous chapter. The nominal distance minus the reduced tellurometer result is a zero-correction at a dial reading.

In order to determine the cyclic part of the zero-correction the distance was changed by 2.5 m steps from 432 m to 412 m. These distances were measured with a steel tape and their accuracy can be considered as ±0.5 mm. The change in distance of 20 m mentioned corresponds to

Table 2.

Distances m	Dial readings cm	Pairs of instruments					
		726/574	574/726	574/330	330/574	726/330	330/726
432 422 412	200	−25.7 ± 6.4 (6)	+ 1.2 ±14.5 (6)	−129.1 ± 5.2 (7)	−138.6 ± 7.8 (7)	−162.3 ± 6.1 (6)	−159.7 ± 10.8 (6)
424.5 414.5	450	−15.5 ±21.9 (4)	+ 6.2 ±19.7 (4)	−112.2 ±15.1 (5)	−122.0 ±18.5 (5)	−150.0 ±18.6 (4)	−147.2 ±25.5 (4)
427 417	700	+32.8 ±16.0 (4)	+56.0 ±18.4 (4)	− 90.8 ± 11.6 (4)	−100.0 ± 14.7 (4)	−107.8 ± 21.3 (4)	−118.2 ± 24.3 (4)
429.5 419.5	950	− 7.2 ±14.4 (4)	+35.8 ±15.7 (4)	−112.4 ± 8.2 (5)	−106.2 ± 12.6 (5)	−133.5 ± 9.1 (4)	−167.5 ± 11.1 (4)

two revolutions of the dial and the corrections therefore form two cycles. Every distance was measured at least once in June and once in October. The arithmetic mean of zero-corrections at each dial reading and its standard error are given in Table 2.

Observed zero-corrections in mm with their standard errors and numbers of observations, given in brackets, for each pair of instruments at different dial readings.

Computations

The observation results show a clear cyclic variation in the correction depending on the dial reading. The correction is expressed as a periodic function

$$k = h + a \cdot \sin(u - \alpha),$$

where k = zero-correction to be determined

h = constant part of the zero-correction

a = amplitude of cyclic variation

$$u = \frac{360^\circ (A - 200)}{1000}\,,\quad \text{where } A \text{ is a fine reading in cm}$$

α = phase shift

The coefficients h, a and α obtained from the least square adjustment for each six combinations of the instrument units are given in Table 3.

Table 3.

Instruments	h mm	a mm	α degree
726/574	− 3.1 ± 7.4	28.3 ± 9.9	+ 98.4 ± 22.2
574/726	+ 25.1 ± 3.8	30.5 ± 5.2	+119.0 ± 10.3
574/330	−111.1 ± 1.2	18.8 ± 1.6	+ 89.7 ± 5.1
330/574	−116.7 ± 2.6	21.5 ± 3.6	+111.6 ± 9.8
726/330	−138.0 ± 3.4	27.9 ± 4.5	+107.2 ± 10.1
330/726	−147.3 ± 9.2	21.6 ±12.6	+ 62.0 ± 35.3

The coefficients of the periodic function $k = h + a \cdot \sin (u - \alpha)$ for each pair of instrument units.

From the results, and especially from the graph (Fig. 2), a similarity in the cyclic errors of every master units can be seen clearly.

On the basis of this it is assumed that the corrections consist of a constant, average correction h of each individual pair of units and of a cyclic correction of the master unit, the correction of which is independent of the remote unit. A second adjustment is therefore made in order to determine

726/574	h_1	a_1	α_1
726/330	h_5		

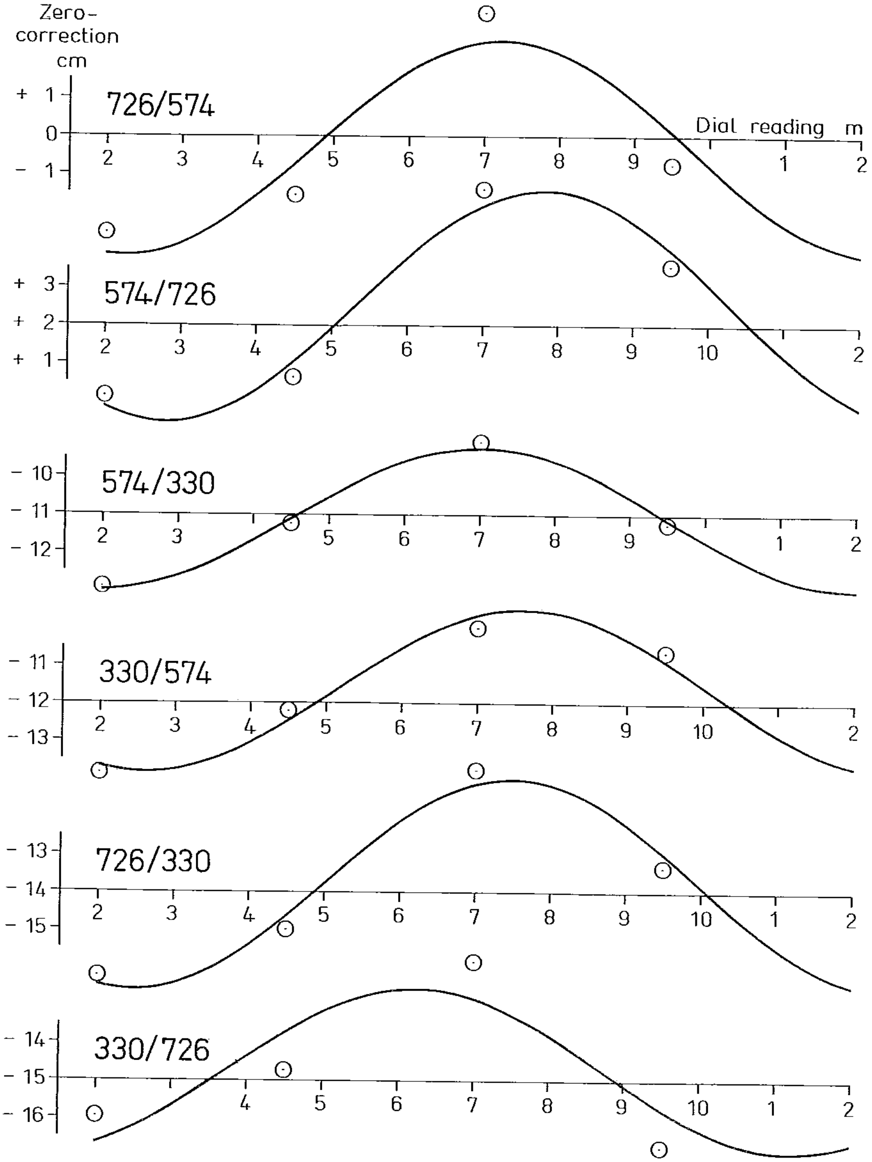

Fig. 2. The curves show the zero-corrections (Table 3) obtained from the firs adjustment, for which the coefficients h, a and α in the expression $k = h + a \cdot \sin(u - \alpha)$ were assumed to be individual for each six pairs of units.

574/726	h_2	a_2	α_2
574/330	h_3		
330/574	h_4	a_3	α_3
330/726	h_6		

The results of the adjustment are given in Table 4.

Instruments	h mm		a mm		α degree	
726/574	− 2.0	±3.4	28.0	±4.2	+102.8	± 9.3
726/330	−138.8	±3.4				
574/726	+ 24.7	±4.7	23.5	±4.4	+106.1	±11.4
574/330	−110.6	±4.4				
330/574	−116.8	±6.1	19.6	±6.1	+ 89.6	±19.1
330/726	−147.3	±6.5				

Table 4. The results of the second adjustment, for which a constant part of the zero-correction was assumed for each six combinations of instrument units but only three different cyclic parts according to three different master units.

The curves of the results in Table 4 are given in Fig. 3.

Because of the similarity of all results a third adjustment was made by assuming again that each instrument combination has its own individual constant part of the zero-correction, but that the cyclic part is common to all combinations. This adjustment gave the results in Table 5.

Instruments	h mm		a mm		α degree	
726/574	− 4.0	±6.2	22.1	±3.4	+100.8	±9.4
574/726	+ 24.5	±6.2				
574/330	−110.6	±5.8				
330/574	−116.6	±5.8				
726/330	−138.7	±6.2				
330/726	−147.1	±6.2				

Table 5. Results of the adjustment assuming the common cyclic part of the correction for all combinations but six individual constant parts of the correction.

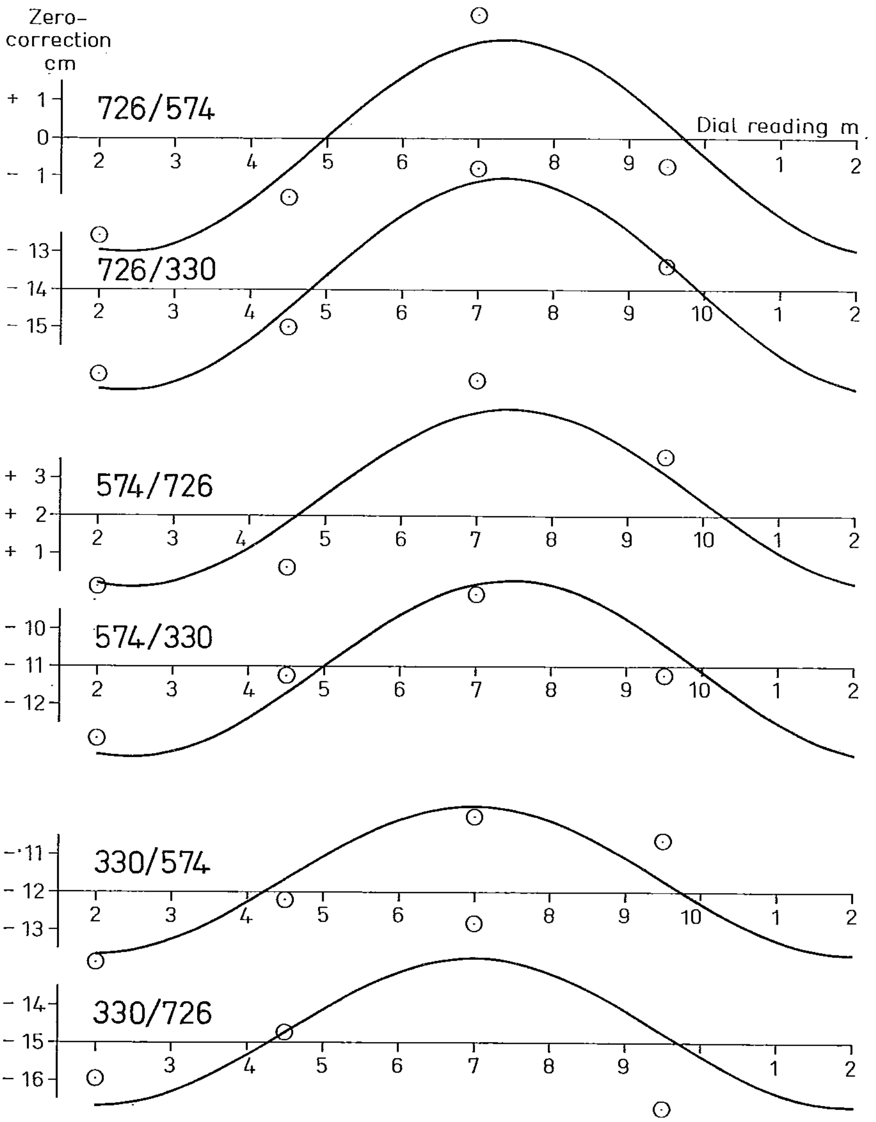

Fig. 3. The curves show the zero-corrections (Table 4) obtained from the second adjustment, for which a constant part of the zero-correction was assumed for each six pairs of units but only three different cyclic pairs according to three different master units.

The observed similarity of all the above results confirms the assumption on a physical basis that the cyclic zero error has its origin in phase measuring technics, common to all instruments of type MRA 3. Instruments of type MRA 101 have a phase lock circuit, which automatically controls the comparison frequencies by locking the remote quartz crystals to a highly stable reference oscillator ([10]). The origin of the cyclic error seems to lie in two other probable components *i.e.* a capacitor and a resistor.

Due to the scarcity of the observation material and the relatively large standard errors of the results no sufficiently detailed conclusions can be made as to differences between the individual instrument units. On the basis of the results obtained here the use of common coefficients would be permissible in the cyclic part of the correction for all instrument units (Table 5). If, however, there might be some differences which are covered by accidental errors but which have certain systematic effects it is justifiable to use the individual coefficients for each master unit (Table 4). Thus all tellurometer observations made with the instruments mentioned here are corrected using the following zero-correction formulas:

$$k = -\ \ 2.0 + 28.0 \cdot \sin(0.36 \cdot A - 174.8) \quad \text{for} \quad 726/574$$

$$k = -138.8 + 28.0 \cdot \sin(0.36 \cdot A - 174.8) \quad \text{for} \quad 726/330$$

$$k = +\ 24.7 + 23.5 \cdot \sin(0.36 \cdot A - 178.1) \quad \text{for} \quad 574/726$$

$$k = -110.6 + 23.5 \cdot \sin(0.36 \cdot A - 178.1) \quad \text{for} \quad 574/330$$

$$k = -116.8 + 19.6 \cdot \sin(0.36 \cdot A - 161.6) \quad \text{for} \quad 330/574$$

$$k = -147.3 + 19.6 \cdot \sin(0.36 \cdot A - 161.6) \quad \text{for} \quad 330/726$$

where A is the fine reading or the dial reading with A-frequency.

Influence of the corrections

Instruments of type MRA 3 normally have several centimeters and instruments of type MRA 101 10—20 cm as the constant part of the zero-correction. In other words, whenever MRA 101 acts in combination, either as master or remote, the zero-correction is large, *i.e.* more than 10 cm. Its influence is significant both on short and long distances.

The cyclic part of the zero-correction, which does not exceed ±28 mm with the instruments tested here, has a clear effect on shorter distances. On longer distances the relatively large error in refraction correction prevents it from being seen.

Two examples of short distances of some kilometers are here described. The Finnish Geodetic Institute has measured distances of 2—3 km at a photogrammetric test area on Seglinge Island and a distance of 6 km in the Vihti extension net.

Seglinge

In May 1969 the Finnish Geodetic Institute measured a triangle net of five points on Seglinge Island in the Aland Archipelago for photogrammetic field calibration (Fig. 4). Adjustment of the angle observations gives an accuracy corresponding to a coordinate accuracy of m = ±0.002 m.

Two sides, AC and BD, were measured with two tellurometer units Nos. 726 and 330, four times with each. The results reduced to horizontal straight lines at the same height level and corrected with a constant, average zero-correction without taking the cyclic correction into consideration are

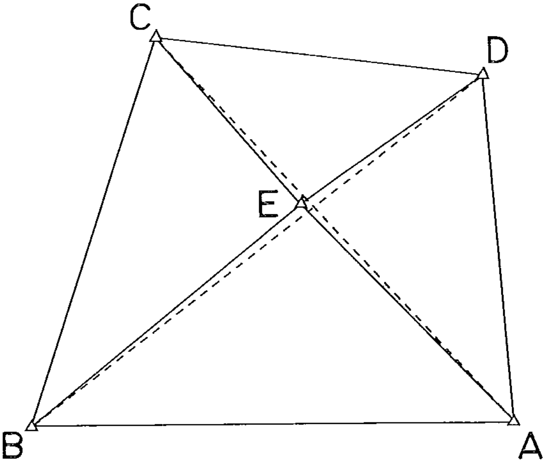

Fig. 4. The triangle net on Seglinge Island. AC and BD measured by tellurometer.

$$AC_{\text{tell}} = 2767.413 \pm 0.014 \text{ m}$$
$$BD_{\text{tell}} = 3020.008 \pm 0.017 \text{ m}\,.$$

When the length of AC is computed with the aid of the adjusted angles from the length of BD above, the length value

$$AC_{\text{tr}} = 2767.472$$

is obtained. So the deviation is

$$AC_{\text{tr}} - AC_{\text{tell}} = + 59 \text{ mm}$$

Using the cyclic corrections determined above the following reduced lengths are obtained

$$AC_{\text{tell}} = 2767.461 \pm 0.008 \text{ m}$$
$$BD_{\text{tell}} = 3020.027 \pm 0.012 \text{ m}\,.$$

When the side AC is derived from the side BD_{tell}, the length value

$$AC_{\text{tr}} = 2767.491$$

is obtained. Now the deviation is

$$AC_{\text{tr}} - AC_{\text{tell}} = + 30 \text{ mm}\,.$$

The consideration of cyclic correction has improved the consistency of results of different instrument units significantly, which can be seen from the drop in the standard errors by one third.

There is no length measurement in this Seglinge net that could give a correct absolute scale. Therefore it is not possible to criticize the total zero-correction using the absolute values of the differences +59 mm and +30 mm. The change in the difference from +59 to +30 gives remarkable evidence of the effect of the cyclic zero-correction on shorter distances.

Vihti

The Vihti extension net includes the invar-wire baseline of the length reduced to the geoid ([4], [5] p. 44), taking i to account the correction of quartz meter length +1.03 μm ([3] p. 60)

$$6\,049.8197 \text{ m}\,.$$

This baseline was measured with tellurometers eight times in 1967 and four times in 1968. The results are compared to the ellipsoidal length mentioned above, which is correct to better than 1 mm.

The results of these comparisons are seen in Table 6. When no zero-corrections are used the tellurometer results deviate greatly from the correct distance. Zero-corrections from the correction formulas on page 11 greatly reduce these deviations. There is no great influence of the cyclic part of the zero-correction compared to the effect of the constant part by chance as the number of last meters of the baseline is about 9.5. At this part of the dial the true zero-corrections are close to average zero-corrections. However, the cyclic part of the zero-correction could be equal or even greater than the average zero-correction. This is the case with MRA 3 units.

Year	Units M/R	No zero-correction		Constant zero-corr.			Cyclic zero-corr.		
		Distance	Diff.	Corr.	Distance	Diff.	Corr.	Distance	Diff.
1967	726/574	6 049.791	+ 29	− 2	6 049.789	+31	− 3	6 049.788	+32
	726/574	.855	− 35	− 2	.853	−33	− 2	.853	−33
	574/726	6 049.750	+ 70	+ 25	6 049.775	+45	+ 26	6 049.776	+44
	574/726	.798	+ 22	+ 25	.823	− 3	+ 27	.825	− 5
	574/330	6 049.889	− 69	−111	6 049.778	+42	−110	6 049.779	+41
	574/330	.917	− 97	−111	.806	+14	−110	.807	+13
	330/574	6 049.964	−144	−117	6 049.847	−27	−123	6 049.841	−21
	330/574	.970	−150	−117	.853	−33	−123	.847	−27
1968	330/726	6 049.965	−145	−147	6 049.818	+ 2	−153	6 049.812	+ 8
	330/726	50.010	−190	−147	.863	−43	−154	.857	−37
	726/330	6 049.963	−143	−139	6 049.824	− 4	−140	6 049.823	− 3
	726/330	.990	−170	−139	.851	−31	−141	.849	−29
Means of differences			−85.2			−3.3			−1.4
Standard errors of means			±25.3			±8.9			±8.4

Table 6. Differences in mm invar-wire result minus tellurometer result without zero-correction, corrected with a constant average zero-correction and cyclic zero-correction.

It can be seen from Table 6 that there is no significant difference between the invar-wire result and the tellurometer result corrected with cyclic zero-corrections.

Conclusions

In 1969 the Finnish Geodetic Institute carried out measurements in order to determine zero-corrections for its two MRA 3 and one MRA 101 type tellurometer units. The Nummela standard baseline, where the calibrations were made, is sufficiently accurate to be considered a true length. The second half of the line has a topography and vegetation on the line and in the close neighbourhood that make the place suitable for such observations. Reflecting surfaces are minimum and the atmosphere, where the radio beam is going, is very homogeneous. The refraction corrections can be determined accurately enough, even though changes take place during measurement.

Each combination of instrument units was used similarly and to the same extent. Based on the physical properties of the tellurometer the adjustments were carried out assuming that the cyclic part of the zero-correction, the amplitude and the phase of each pair of instrument units are independent of the remote unit. The correction formulas were determined so that the constant part of the zero-correction is individual for each pair of units, but the amplitude and phase are common for all pairs which have the same master unit. These correction formulas are used for reduction of the tellurometer observations of the Finnish Geodetic Institute.

From the comparison material described above it can be concluded that the constant part of the zero-correction has a significant effect on the tellurometer result.

Changes in the constant part are possible and should therefore be determined through repeated calibrations.

The cyclic part of zero-correction becomes obvious when sides of different lengths, which generally correspond to different dial readings, are compared to each other. This is of a certain importance in normal geodetic nets and their adjustments.

The observations of the zero-corrections on short distances described above will be continued with an investigation of the influence of constant and cyclic zero-correction on the sides 30—40 km long, which are general in regular field work.

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