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A PROCEDURE FOR DETERMINATION OF THE SURFACE WAVE GROUP VELOCITY *

by

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Abstract

A method is described for determining the group velocity dispersion from the surface wave train. The arrival times of the wave crests, troughs and zero crossings are read from seismograms. The periods and the corresponding travel times are determined with the aid of parabolas fitted by the method of least squares to sections of the set of readings. If the smoothing that arises from this operation is not strong enough, the original readings or/and the computed periods and travel times can be filtered. All calculations are made by computer.

Introduction

During the last two decades activity in the field of seismic surface wave research has been intensive. Perhaps the most influential factors have been the construction of long-period seismographs and the fast development of high-speed digital computers. Owing to progress in instrument design the number of seismograms suitable for surface wave studies is growing explosively. Today computers are almost universally available to research workers. The availability of computers makes it possible to use more advanced methods for data analysis and theoretical

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calculations. It is possible to calculate theoretical dispersion curves for complicated layer models and to use filtering operations and Fourier transform methods for seismic data and so on.

Digitally recording seismographs represent the newest development in instrumental seismology. The data yielded by them is best suited to direct automatic handling. Most of the seismograms available to the investigator today have been recorded by conventional methods, however. When he is using surface waves for structural studies, for instance, he has to collect his material from suitably located stations.

In the beginning of surface wave research most studies were concerned with group velocity. When suitable methods were developed, phase velocity methods became common. If we knew the phase velocity dispersion exactly, there would be no need to study the group velocity at all. It is well known that the phase velocity and group velocity are related to each other by the following equation:

$$U = C + \varkappa \frac{dC}{d\varkappa}, \tag{1}$$

where U is the group velocity, C the phase velocity and $\varkappa$ the wave number. But because we do not get experimental dispersion curves exactly, the determination of both C and U essentially complete each other.

The graphical method developed by Ewing and Press [1] is in common use for the determination of the group velocity dispersion. In this method the travel times (t) of some chosen phases along the surface wave train are measured and plotted on a graph versus the order number (n) of the chosen phases. Usually the travel times of the wave crests and troughs are read. The (n, t) curve built by these points is then approximated with linear segments. The period is determined by the slope of these lines and the corresponding travel times are read from the midpoints of the segments. The procedure is described in Fig. 1. Essentially this is a method of smoothing by eye.

It is desirable to do the smoothing by computer. This is not only for convenience but more to avoid subjective influences during the smoothing procedure. One such method was developed by Pirhonen [3]. He calculated the group velocity by fitting a polynomial to the whole set of (n, t) points. His method was applied successfully in a study on the crust and mantle in Finland (Noponen *et al.*, [2]), where the group velocities between two stations were calculated with its help.

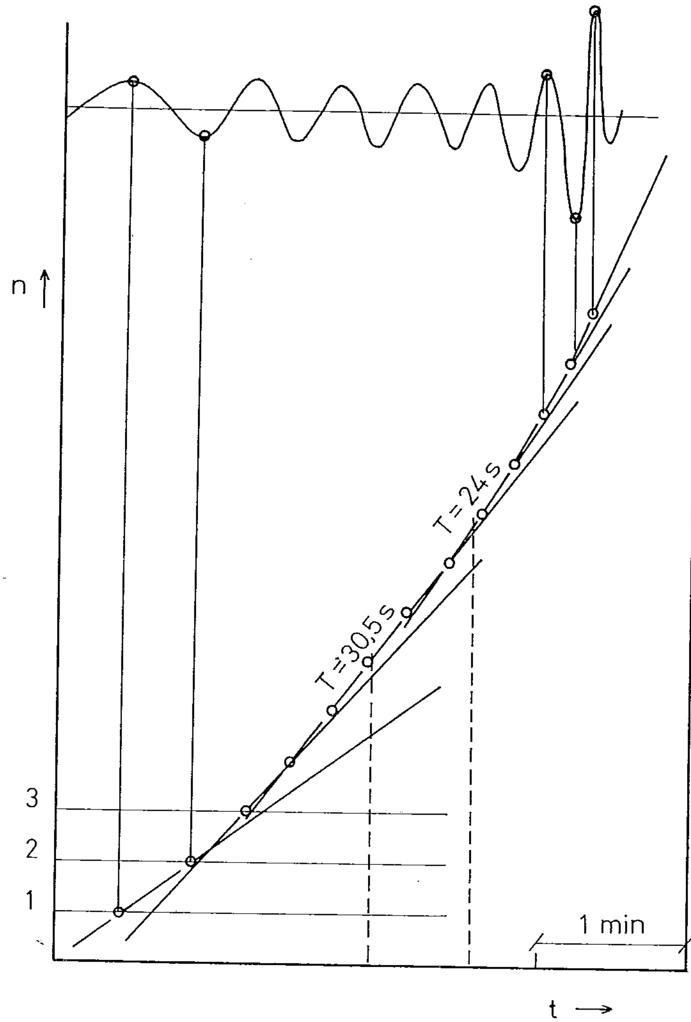

Fig. 1. Determination of the periods with the aid of linear segments.

The procedure

The period (T) is determined by the slope of the (n, t) curve. The general course of this curve (Fig. 1) suggests that it is worth trying to fit it sectionally by parabola

$$t = a + bn + cn^2, \tag{2}$$

where a, b and c are coefficients to be determined by the least square method. The period corresponding to the observation point n is then

$$T = m \frac{dt}{dn} = m (b + 2cn) , \tag{3}$$

where m is the number of points per cycle. When the crests and troughs are read $m = 2$ and if the zero crossing times are also read, it is 4.

In the practical procedure the parabola corresponding to the observation point n is determined by combining its travel time with the measurements at its two or more neighboring points to the left and right. So we have five or more consecutive observations. The parabola at n that fits best in the least squares sense is then determined. Considering $2k + 1$ consecutive points and setting $n = 0$ at the midpoint, the data to be fitted belong to the points

$$n = -k , \ldots , -1 , 0 , 1 , \ldots , k . \tag{4}$$

For T we are to determine the coefficient b and for t the coefficient a of the parabola. The function to be minimized becomes

$$\sum_{j=-k}^{k} (a + jb + j^2 c - t_j)^2 . \tag{5}$$

The condition of minimum gives the following expressions to a and b:

$$a = \frac{\sum_{j=1}^{k} j^4 \sum_{j=-k}^{k} t_j - \sum_{j=1}^{k} j^2 \sum_{j=-k}^{k} j^2 t_j}{(2k + 1) \sum_{j=1}^{k} j^4 - 2 \left(\sum_{j=1}^{k} j^2 \right)^2} \tag{6}$$

$$b = \frac{\sum_{j=-k}^{k} j t_j}{2 \sum_{j=1}^{k} j^2} \tag{7}$$

The desired travel time and period are then

$$t = a \tag{8}$$

$$T = mb \tag{9}$$

If the smoothing is not strong enough, the data can be filtered before or/and after the least square procedure described. A general flowchart of the procedure is presented in Fig. 2.

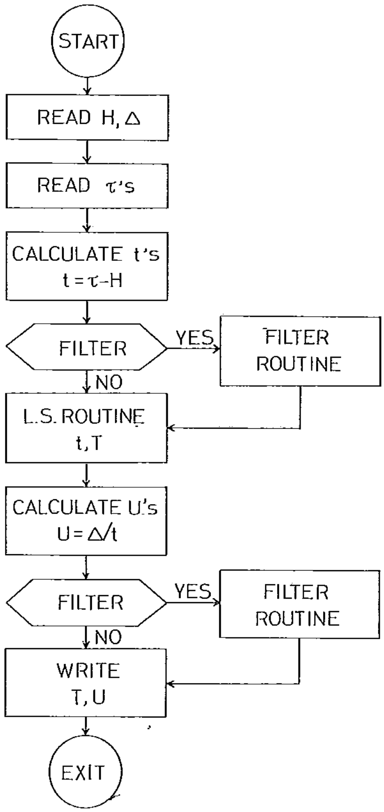

Fig. 2. Flowchart of a procedure for analysing the group velocity dispersion.

H = origin time
Δ = epicentral distance
τ = arrival time
t = travel time
U = group velocity

Results

A computer program is written in FORTRAN following in principle the flowchart of Fig. 2. The method is adapted for use by determining the experimental group velocity dispersion from several surface wave trains. The number of points $(2k + 1)$ used to fit the parabola ranged from five to eleven.

The differentiation is very sensitive to the random variations of the data. So, because the period is found by a differentiation operation, some smoothing of the experimental readings is needed. The least square procedure described may be regarded as a filtering operation. The period and the travel time are determined from the original series of travel times by convolving it with the weighting functions obtained from equations (6) and (7). We can increase the smoothing by taking more neighboring points in each calculation of the parabola. But the use of too many points leads to distortion of the dispersion curve. If the computed dispersion curve seems too irregular, we can filter the original readings before calculating parabolas or/and filter the sets of periods and travel times after the least square operation.

Two types of numerical low-pass filters have been used in this work. Both of them have an even weighting function. Binomial filters have the normalized coefficients of even-order binomials as their weights. By normalization the sum of the weights of a filter is made to be one. Delta filters have weights determined by the equation:

$$w_i = \frac{s + 1 - |i|}{(s + 1)^2}\,, \quad i = -s\,, \ldots, -1\,, 0\,, 1\,, \ldots, s\,. \tag{10}$$

The filter has $2s + 1$ weights.

The computer program is written so that the number of points in the output series is the same as it was in the input. This needs special arrangements for the handling of the points at both ends of the original series of data. In the least square fitting the first two and last two points are calculated from four data points (in this case the equations for the period and travel time are different from those presented before), the next points using a value of two for k and then by increasing values of k until the chosen value is reached. The filters operate similarly; the first and the last point are taken without filtering.

As illustrative examples of the method the dispersion analyses of two Rayleigh wave trains are examined. Both of them were recorded at Nurmijärvi WWSSN station (60°30′32.4″ N, 24°39′5.1″ E) with Press-Ewing long-period seismograph. Information about the earthquakes is presented in Table 1.

In Fig. 3 are presented some details of the effect of the sensitivity of the differentiation to the data noise. This example is an unusually difficult case. The surface wave train analyzed is from an earthquake in the Kurile Islands (Table 1). The use of the five-point parabolic fitting without any other smoothing produces unreliable results. The dispersion curve throws irregularly. The next step presented in Fig. 3 is the use of

Table 1. Earthquake data (according to USCGS).

Date	February 16, 1965	October 24, 1965
Origin time	12 24 08.8	18 15 04.9
Epicenter	39.5 N, 141.8 E	49.7 N, 156.1 E
Depth	33 km	30 km
Location	Honshu, Japan	Kurile Island
Epicentral distance	7553 km	7047 km s

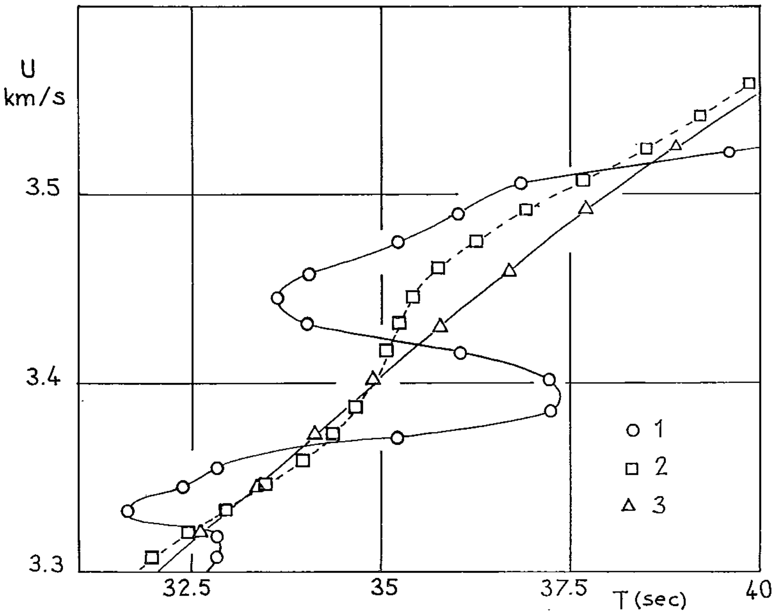

Fig. 3. Details of the dispersion analysis of the Rayleigh waves from the Kurilean earthquake of October 24, 1965.

Input:

1 and 2. Four points per cycle. 3. Two points per cycle.

Operations:

1. Parabolas fitted to five points. No other smoothing. 2 and 3. Parabolas fitted to eleven points. Five-point delta filter before and after the fitting of parabolas.

the five-point delta filter before and after the fitting of eleven-point parabolas. This operation also leads to smoothly oscillating dispersion curve when four points per cycle are used for input. In this case strong smoothing is needed at a period of about 35 seconds. At about 25 seconds

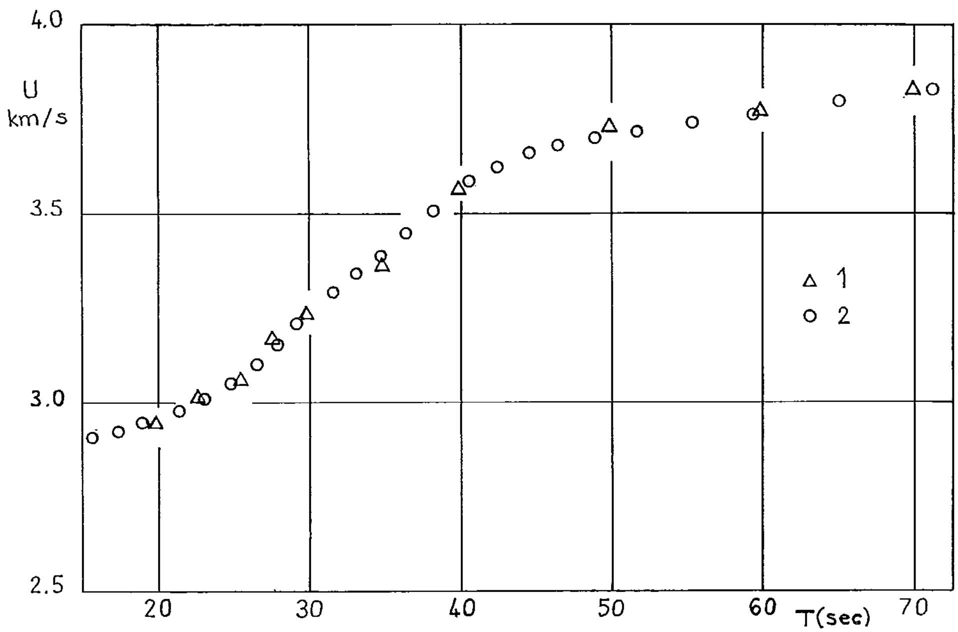

Fig. 4. Comparison of dispersion curves derived with the aid of
(1) linear segments and
(2) the procedure described.
Earthquake of February 16, 1965, at Honshu, Japan.

such difficulties in analysis seems to be common. With two data points per cycle and the same filtering as before (this means doupling of the filter effect), a regular dispersion curve is obtained. This example seems to suggest that it is only necessary to read the arrival times of the wave crests and troughs. But at least for longer periods, say longer than 40 seconds, the use of four points per cycle is valuable to confirm that the procedure does not distort the form of the dispersion curve. It is evident that smoothing by eye constitutes a strong filtering.

A comparison of dispersion curves determined by the procedure described and by the linear segment method is presented in Fig. 4. The earthquake is that of Honshu (Table 1). The figure shows that there is good agreement between the results given by the two methods. The least square process with eleven-point fitting yields more points and a smoother course for the dispersion curve. For period longer than 50 seconds there is no additional smoothing. For shorter periods five-point pelta filtering is applied before and after the fitting of parabolas. For periods shorter than 30 seconds the number of points in the input is reduced to two per cycle. Not all the points computed have been plotted in Fig. 4.

Conclusions

Today we have many earthquake recordings suitable for surface wave studies. Most of this material has been recorded by the conventional photographic method.

One powerful method is to digitize the seismograms, then filter them and handle the data by other methods according to need. But the digitizing is laborious. It is desirable to devise easier ways.

In this paper a convenient method is presented for determining the group velocity of seismic surface waves. The arrival time readings of the crests, troughs and zero crossings are processed by computer. The method of least squares and filtering are used to determine the period and the corresponding travel time.

Comparison of the dispersion obtained by this and other methods indicates that the results are satisfactory.

Naturally, the fitting of a parabola used in this work is not the only possibility. It would be worth while, for example, to try exponential functions. Procedures like the one described here minimize the operator's mechanical routine labor and at the same time reduce his subjective influence on the results.

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