551. 463. 6: 536. 6

THE HEAT CONTENT OF THE GULF OF BOTHNIA

by

S. Uusitalo

Institute of Marine Research, Helsinki

Abstract

The heat content of the Sea of Bothnia and that of the Bay of Bothnia have been calculated from data collected primarily during the Aranda cruises. Graphical integration method was used.

1. *Introduction*

Considering the great significance of oceans and seas as a factor regulating the climate, some investigators have undertaken the task of determining the heat content of certain water bodies. (Johnson, 1940 [4]; Wallerius, 1940 [10]). A few of them have dealt with Finnish or nearby waters. The most extended of them is the treatment of Jurva (1937 [6]). Others (Simojoki, 1946 [9]; Hankimo, 1964 [2]; Hela, 1951 [3]; Jurva, 1926 [5]) have taken only limited sea areas into consideration.

Jurva [6] calculated at first the line integrals of temperature from the surface on the sea downwards for each station along the Finnish coast using the trapezoidal rule. In this manner he obtained the heat content per cross sectional area of water column (or, what is about the same, the heat content of a water column of a cross section of 1 m^2). This quantity he called the Heat Sum. He then formed the monthly means of the heat sums and averaged them over a span of several years. Using these he calculated regional mean values for equal depths. Then he determined heat contents of columns between consecutive curves of equal depth using bathymetric charts and, by summing these he found the total heat contents of sea areas in question. In this way he obtained the mean maximal heat content of Finnish waters north of the latitude 59° N to be $6.59 \cdot 10^{19}$ cal. The area concerned is about 168 000 km^2.

Simojoki [9] has performed calculations concerning the Bogskär area in the northern Baltic. His method was to determine that part of the heat in the water body which transferred into the air through the upper surface. He found the heat penetrating into the water during a warm season to be $426 \cdot 10^3$ kcal m^{-2} and that passing into the air during a cold season to be $467 \cdot 10^3$ kcal m^{-2} thus exceeding the former amount by $41 \cdot 10^3$ kcal m^{-2}. The difference was due to advection from other parts of the sea.

Using meteorological factors, Hankimo [2] has determined the heat flow through the water surface at Finngrundet in the southern Bothnian Sea. Between March 13, 1961 and September 13, 1961 he found it to be 40.8 kcal cm^{-2}.

The aim of the present work is to calculate the heat content of the Bothnian Sea and that of the Bothnian Bay from data taken from different parts of the sea areas in question.

2. *Determination of the heat content*

The heat content of a given system can be defined as the (positive or negative) amount of heat to be carried from the system to make the temperature equal to 0°C. Therefore, the heat content of a water body of volume V can be expressed as

$$Q = \int_V \varrho c t \, dV, \tag{1}$$

where

ϱ = density
c = specific heat
t = temperature

of water.

The heat content of any ice present is taken into account by subtracting it from (1). The influence of an ice and snow cover will be discussed later. (See p. 162).

For our purpose it is sufficient to assume that the density and specific heat of water are constants. The errors caused by these assumptions are small compared to errors due to the uncertainty of the temperature. After these simplifications the heat content (1) reads

$$Q = c\varrho \int_V t \, dV \, . \tag{2}$$

This may be written using a triple integral

$$Q = c\varrho \int dy \int dx \int t \, dz \, , \tag{3}$$

where the Cartesian coordinates x, y, z are directed to the east, north and downwards respectively. Naturally, the integration must be extended over the whole water body in question. The small deviation of the sea surface from a plane may be neglected. The integral in (3) may be expressed as three separate integrals

$$\begin{aligned} I &= c\varrho \int t \, dz, \\ K &= \int I \, dx \, , \\ Q &= \int K \, dy \, . \end{aligned} \tag{4}$$

The new quantities I and K have specified meanings, too. If we determine the heat content of a water column of a small cross section $dx \, dy$ from bottom to surface, using (3), and divide the result by $dx \, dy$, we get just I. Thus, I is the heat content of a water column per unit cross section. Then

$$I/(c\varrho D)$$

is the mean temperature of the column. Here the depth is denoted by D.

To find the meaning of K we set two planes perpendicular to the y-direction at a small distance dy apart. If we divide the heat content of the water between these planes by dy, we obtain K. Thus, K is the heat content per unit thickness of a sheet between two meridional cross sections. Then

$$K/(c\varrho A)$$

is the mean temperature of a meridional cross section. The area of the cross section is denoted by A.

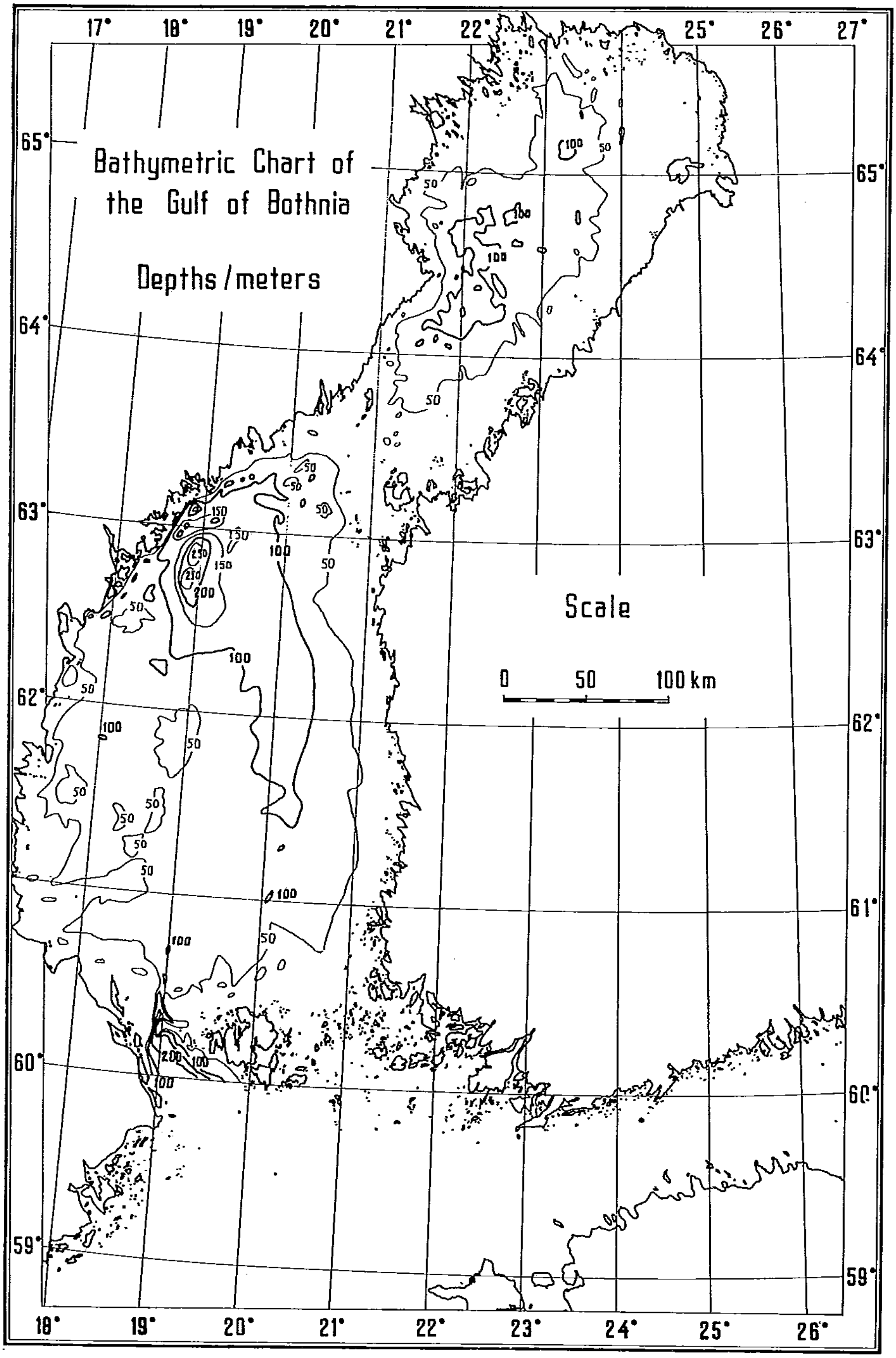

Fig. 1. Bathymetric chart of the Gulf of Bothnia.

In this connection it may be recalled that

$$Q/(c\varrho V) \tag{5}$$

represents the mean temperature of the total water body, the volume of which is V.

3. *Depths*

The Gulf of Bothnia is rather shallow. (See Fig. 1.) On an average it measures 60 meters. It is divided by the Quark into two parts, the southern one being the Bothnian Sea and the northern one the Bothnian Bay.

The depth of the Bothanian Sea is 65 meters on an average. But the bottom topography is very complicated. The general pattern of it may be explained here. The northwestern part is deep with places exceeding 250 meters. From this deep a channel continues to the southeast growing shallower farther south. Another branch, which is not so deep as the former one, goes parallel to the western coast. Between these two deeps lies a ridge with less than 50 meters of water on an average. This ridge begins from the southwestern coast and extends to the area west of the center of the sea.

The mean depth of the Bothnian Bay is about 43 meters. The bottom configuration is simpler than that of the Bothnian Sea. The depth exceeds in places 100 meters. The deepest area is to the west or southwest of the center.

4. *Water movements and temperature distribution*

Several causes set the water in basins into motion. They are the wind, inflow and outflow (rivers, channels *etc.*), density differences, precipitation and evaporation. Because of the irregularity of the wind, the motion of the water masses tends to be rather irregular in the Bothnian Sea as well as in the Bothnian Bay. But the reason, why it is not altogether irregular, is to be found in the Coriolis force. Because of this force the water masses assume a counterclockwise motion superposed by the irregularities. The main features of this circular motion are depicted in Fig. 2. (*Cf.* PALMÉN, [8]).

It can be assumed that the temperature changes along a stream line are more regular than across it. In the case of a steady state motion this

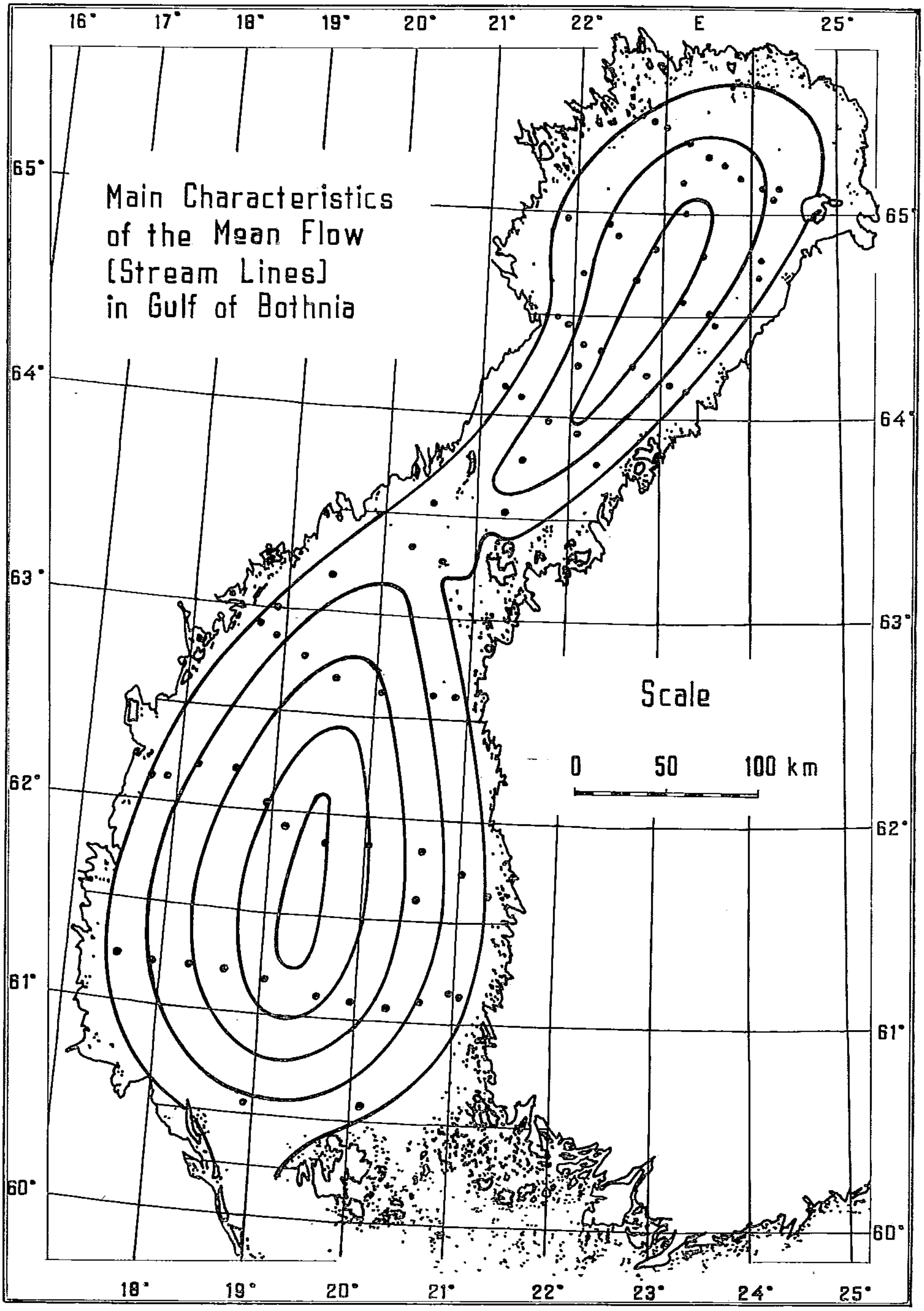

Fig. 2. Simplified main characteristics of the stream lines in the Gulf of Bothnia. The dots show the locations of the hydrographic stations.

assumption seems very plausible because the water particles along a single stream line have about the same life history and they are therefore strongly correlated, whereas the correlation of temperatures on different stream lines at a certain distance apart needs not necessarily be so close.

When temperature interpolations are to be made in sich a water body, the results seem to be better when they are obtained from data on the same stream lines. In the case of the Gulf of Bothnia the irregularities of the water movements cause the streamline pattern to differ from an ideal one. Therefore the interpolation along stream lines may not give as favorable results as expected. Nevertheless, this idea was adopted in the form, that interpolations across the stream lines were made over short distances only, while interpolations in the direction of stream lines could be extended over longer distances.

5. *Observations*

The material for this investigation was collected primarily during «Aranda» cruises which have been made routinely during summer seasons only. On the Gulf of Bothnia there are several hydrographical stations marked by dots in Fig. 2, which normally will be visited on every research cruise. Therefore the observations made on these stations offer in general a pretty good basis for investigations. The samples for chemical composition at these stations were collected mostly with a Knudsen sampler or Nansen bottles and the temperatures were determined at the same time. Also a bathythermogram was taken. Sometimes when sampling could not be performed, it was still possible to use a bathythermograph. Data collected in this way are available for August 1961, June 1962 and September 1962, but not for March 1962, which is one of the occasions for wich we would like to perform our calculations. No regular observations have been made during winter season. Some discrete samplings and measurements have been carried out on board of the icebreakers of the Finnish government. Unfortunately, no such data were available for the winter of 1962, but instead for the spring of 1963. In addition, observations of June 1962 and of June 1963 exist as well. Therefore it was decided to use the given data to construct the best possible values to replace the missing ones.

The temperature values of March 1962 were replaced by values found from the formula

$$t_1 = t_2 t_3 / t_4$$

where

t_1 = temperature in March 1962
t_2 = temperature in June 1962
t_3 = temperature in March 1963
t_4 = temperature in June 1963.

This formula was used separately for each depth at each station. Naturally, some interpolations and extrapolations were necessary to find a suitable set of data for March 1963. Here it was assumed that the freezing point of water is at 0°C. This does not seriously affect the accuracy of calculations. The real freezing point lies about at -0.2°C.

6. *Actual performance of the calculations*

The calculations were performed in a space network the principal directions of which were the directions of the parallels, the meridians and the verticals. The horizontal mesh width was taken to about 50 km and the vertical one as 10 m. Additional points were used in some places to increase the accuracy of the calculations. To accomplish the calculations it was necessary to interpolate the given temperatures for the grid points.

A special interpolation method was adopted. For the sake of simplicity, the stream lines were assumed to be horizontal. It was easy to observe that a certain stream line went through the point P, where the temperature had to be obtained. (See Fig. 3.) In the vicinity of P four data points A, B, C, D, were chosen in such a way that they were located on both sides of the stream line and not too far from it. The intersections of AB and CD with the stream line were denoted by Q and R respectively. Following denotations were used:

$$\frac{AQ}{AB} = \alpha \, , \frac{BQ}{AB} = \beta \, , \frac{CR}{CD} = \gamma \, , \frac{DR}{CD} = \delta \, , \frac{PQ}{QR} = \eta \, , \frac{PR}{QR} = \vartheta \, . \tag{6}$$

The distances PQ, PR and QR were measured along the streamline. By linear interpolation then was obtained

$$\begin{aligned} t_Q &= \beta t_A + \alpha t_B \, , \\ t_R &= \delta t_C + \gamma t_D \, , \\ t_P &= \vartheta t_Q + \eta_R t \, , \end{aligned} \tag{7}$$

where the indices of the temperatures refer to the points in question. From (7) it is found that

$$t_P = \beta\vartheta t_A + \alpha\vartheta t_B + \delta\eta t_C + \gamma\eta t_D \, . \tag{8}$$

For checking purposes it is of value to note that the sum of the coefficients in this formula is $= 1$. At the same time this also indicates that t_P is a weighted mean of the given data. If data values used in different interpolations were located on the same set of vertical lines, then the formula (8) was used with the same set of coefficients.

In some places the interpolation method did not work. Then suitable modifications of it were used. Sometimes an extrapolation instead of interpolation could be used, or, instead of the original stream line another line, differing only slightly from the original one, was adopted. Then it could happen that in the calculations only three data points were used instead of the four regular ones.

In most cases observed temperatures were available for integrations, but it happened several times that one or more of the data stations were shallower than the point where the temperatures had to be determined. Then the lacking values were extrapolated from values in the neighbourhood taking the general behaviour of the temperatures at those depths into account. The distances for the calculations of the coefficients (6) in (8) were measured on a map. Such combinations of data for which the segment QR (See Fig. 3) was a very curved line, were avoided.

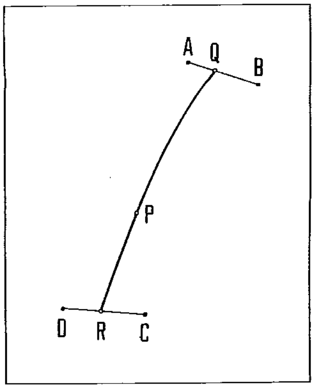

Fig. 3. A chart showing a part of a streamline QR and four stations A, B, C and D used to interpolate the temperature at a point P.

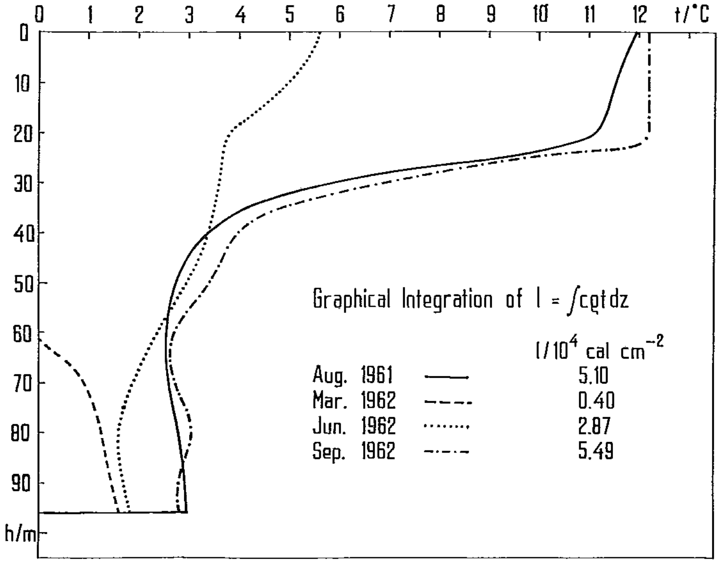

Fig. 4. Temperature — depth curves at the point $\varphi = 61.5°\mathrm{N}$, $\lambda = 10°\mathrm{E}$. The areas bound by the curves and abscissa and ordinate axes are equal to the heat contents per unit cross sectional area of the water column at the point in question at different times.

The integrations over z, x and y were performed graphically. To this end the equations (4) were scaled as follows:

$$\begin{aligned} I &= c\varrho a_t a_z \int (t/a_t) \cdot (dz/a_z)\ , \\ K &= a_I a_x \int (I/a_I) \cdot (dx/a_x)\ , \\ Q &= a_K a_y \int (K/a_K) \cdot (dy/a_y)\ . \end{aligned} \tag{9}$$

The scaling factors were

$$\begin{aligned} a_x &= 10^4\,\mathrm{m\,u^{-1}} & a_t &= 0.5°\mathrm{C\,u^{-1}} \\ a_y &= 2 \cdot 10^4\,\mathrm{m\,u^{-1}} & a_I &= 5 \cdot 10^3\,\mathrm{cal\,cm^{-2}u^{-1}} \\ a_z &= 5\,\mathrm{m\,u^{-1}} & a_K &= 5 \cdot 10^{10}\,\mathrm{cal\,cm^{-1}u^{-1}}. \end{aligned}$$

Here u is the length unit used for the scaled figures. The scaling factors were chosen in such a way that the figures drawn for each particular integration properly filled the area reserved.

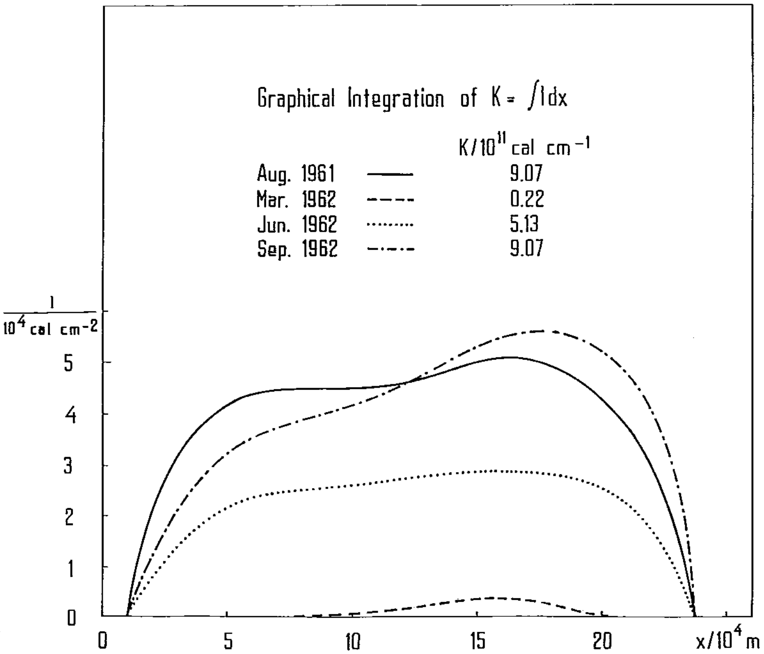

Fig. 5. Heat contents per unit cross sectional area along the latitude $\varphi = 61.5°N$. The areas bound by the curves and the abscissa axis equal to the heat contents per unit thickness of the water lamellae along the latitude $\varphi = 61.5°N$ at different times.

In these calculation it was found advisable to use the depth values not at the points themselves because of the roughness of the bottom, but rather the averaged depths in circles, centered at the grid points, were taken. The radii of the circles were determined separately in each case. Indeed, they did not exceed half the mesh width in any case. In the neighbourhood of the coastline the radii were chosen to be smaller.

An example of the integration procedure is given in Figures 4, 5 and 6. The integrals in (9) are determined consecutively as surface areas bounded by curves and parts of the axes. The measurement was accomplished by means of a planimeter. The calculated heat contents of the water of the Bothnian Sea and that of the Bothnian Bay are given in Tables 1 and 2 for different times. The values of heat content over unit mass and mean temperatures are also incorporated in these tables. In the calculation the specific heat of the water was taken as $c = 0.98$ cal g^{-1} $°C^{-1}$ and the density $\varrho = 1.00$ g cm^{-3}. The volume of the Bothnian

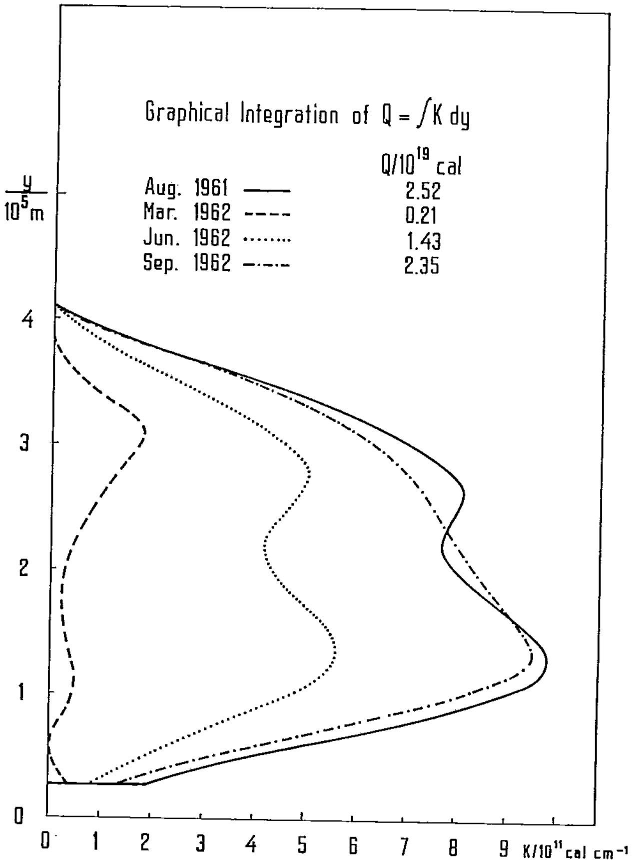

Fig. 6. Heat contents of the Bothnian Sea in different east-westerly cross sections per unit thickness. The areas bounded by the curves and the ordinate axis are equal to the total heat contents of the Bothnian Sea at different times.

Table 1. Heat content, heat content per unit mass and mean temperature of the water of the Bothnian Sea for different times.

	$Q/10^{19}$ cal	(Q/m)/cal g^{-1}	$\bar{t}$/°C
August, 1961	2.52	5.93	6.05
March, 1962	0.21	0.49	0.50
June, 1962	1.43	3.37	3.44
September, 1962	2.35	5.53	5.64

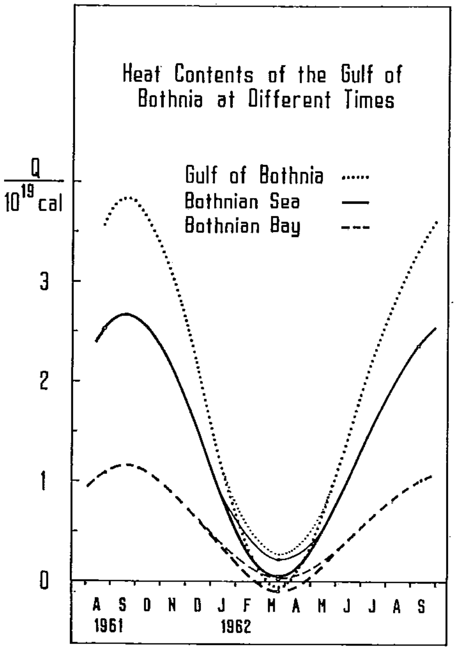

Fig. 7. Graphs showing the heat content variation of the total Gulf of Bothnia as well as of its parts, the Bothnian Sea and the Bothnian Bay. The heavy lines show the variations with all influencing factors, *i.e.* during winter time the heat content of the ice cover is included when it exists. The thin lines give the heat contents of the water only, *i.e.* the influence of the ice cover has been omitted.

Sea was calculated to $V_S = 4.25 \cdot 10^{18}$ cm^3 and that of the Bothnian Bay to $V_B = 1.52 \cdot 10^{18}$ cm^3. The variation of heat content with time is plotted schematically in Fig. 7.

Table 2. Heat content, heat content per unit mass and mean temperature of the Bothnian Bay for different times.

	$Q/10^{19}$ cal	(Q/m)/cal g^{-1}	$\bar{t}$/°C
August, 1961	1.08	7.09	7.23
March, 1962	0.02	0.13	0.13
June, 1962	0.48	3.15	3.21
September, 1962	1.01	6.63	6.76

7. *Influence of Ice and Snow Cover on the Heat Content*

The melting of ice and snow binds heat energy. Therefore ice and snow cover diminish the heat content of the sea. Also, if the temperature of the ice and snow cover is below the freezing point, it gives a negative contribution to the total heat content of the sea. This amount of heat is indeed small compared to the inaccuracy of temperatures in sea water. Therefore we omit its influence. The heat content of ice and snow is

$$Q_m = -q(\varrho_i V_i + \varrho_s V_s), \tag{10}$$

where

q = melting heat of ice and snow
ϱ_i = density of ice
V_i = volume of ice
ϱ_s = density of snow
V_s = volume of snow.

In the numerical calculations we used the values $q = 80$ cal g^{-1} and $\varrho_i = 0.9$ g cm^{-3}. The term for snow in (10) was neglected because the thickness of snow was properly taken into account in the determination of the ice thickness. In this connection it may be recalled that the eventual salt content of ice diminishes the melting heat value considerably. For example at $S = 4^0/_{00}$, $t = -1°\mathrm{C}$, $q = 62$ cal g^{-1}. (Landolt-Börnstein, 1952). But this effect is compensated to a large extent by the increment of specific heat of the ice. For example at $S = 4^0/_{00}$, $t = -2°\mathrm{C}$, $c_i = 4.63$ cal g^{-1} °C^{-1} versus $c_i = 0.48$ cal g^{-1} °C^{-1} at $S = 0^0/_{00}$, $t = -2°\mathrm{C}$. Therefore neglecting the influence of salt content in ice does not induce any considerable error to our calculations.

The maximal ice cover of the year 1962 did occur in the latter half of March. Then the whole Bothnian Bay and 95% of the surface of the Bothnian Sea were covered by ice. This situation was chosen for the heat content determination of the waters as well of the Bothnian Sea as of that of the Bothnian Bay.

The ice volume was evaluated by the following method. The surface area of different types of ice were measured by a planimeter from ice charts. (*Cf.* Publ. Nr 206 of the Institute of Marine Research. [1]) Then these areas were multiplied by the corresponding average ice thicknesses and the products were added. In this way the ice volumes in the Bothnian Sea and in the Bothnian Bay were found to be 22.2 km^3 and 19.2 km^3 respectively. The corresponding amounts of heat needed for melting the ice are $0.16 \cdot 10^{19}$ cal and $0.14 \cdot 10^{19}$ cal, when 80 cal g^{-1} is assumed to be the melting heat of ice and 0.9 g cm^{-3} the density of it. When these contributions (taken as negative) are added to the heat contents of the waters, the total heat content of the Bothnian Sea and that of the Bothnian Bay in March 1962 are obtained to be $0.05 \cdot 10^{19}$ cal and $-0.12 \cdot 10^{19}$ cal respectively. In Fig. 7 there are double heat content curves plotted for times when sea areas in question were partly or totally covered by ice. The thinner curve gives the heat content of the water only, whereas the thicker one takes care of the melting heat of the ice as well. It is seen from this figure that the difference of maximal and minimal heat content in the Bothnian Sea and in the Bothnian Bay are $2.60 \cdot 10^{19}$ cal and $1.27 \cdot 10^{19}$ cal respectively.

8. *Discussion*

Several factors affect the accuracy of the results. The interpolation method used may cause errors of the magnitude 0.5°C. This causes the error in heat content for the Bothnian Sea to be

$$\begin{aligned}\Delta Q_t &= c\varrho V\Delta t\\ &= 1.0\ \text{cal g}^{-1}\ {}^\circ\text{C}^{-1} \cdot 1.0\ \text{g cm}^{-3} \cdot 4.25 \cdot 10^{18}\ \text{cm}^3 \cdot 0.5{}^\circ\text{C}\\ &= 0.21 \cdot 10^{19}\ \text{cal.}\end{aligned}$$

The corresponding value for Bothnian Bay will be $0.076 \cdot 10^{19}$ cal. It is true that errors of different sign cancel each other to a large extent. Therefore the actual error caused by the interpolation method may be much smaller.

Another reason for inaccuracy in heat content is caused by the variation in water level. An estimate for the magnitude in Bothnian Sea can be found assuming that the water level may rise by 0.5 m, say. The water amount needed may naturally originate in the Baltic proper. The mean temperature of the water moved over may be 6°C. The additional heat content will then be

$$\begin{aligned}\Delta Q_D &= c\varrho At\Delta D\\ &= 1.0\ \text{cal g}^{-1}\ {}^\circ\text{C}^{-1} \cdot 1.0\ \text{g cm}^{-3} \cdot 66500\ \text{km}^2 \cdot 6\ {}^\circ\text{C} \cdot 0.5\ \text{m}\\ &= 0.020 \cdot 10^{19}\ \text{cal.}\end{aligned}$$

Similar considerations show the error or uncertainty in the heat content of the Bothnian Bay due to water level variations to be about $0.017 \cdot 10^{19}$ cal.

Other factors play only an insignificant role in the fluctuations of the heat content. Only the inflow from rivers and the precipitation are mentioned here.

Comparison of our values with those of JURVA [6] implies a rather good agreement. It is found for example that our heat content maximum in the Bothnian Sea ($2.65 \cdot 10^{19}$ cal) is a little smaller than his value $3.25 \cdot 10^{19}$ cal and that our minimum value $0.21 \cdot 10^{19}$ cal fits with his value $0.25 \cdot 10^{19}$ cal. In the Bothnian Bay again, our maximum $1.15 \cdot 10^{19}$ cal is a little larger than his corresponding value $1.0 \cdot 10^{19}$ cal. Our minimum value $0.02 \cdot 10^{19}$ cal compares well with his zero value. The differences are understandable though, remembering that Jurva had to rely on observations at coastal stations only. Furthermore our results refer to a specific year whilst Jurva has used averaged values over a series of years.

The difference of maximal and minimal heat contents is in the Bothnian Sea $2.60 \cdot 10^{19}$ cal. (*Cf.* Fig. 7.) The heat flow through the sea surface is thus

$$2.60 \cdot 10^{19} \text{ cal}/66500 \text{ km}^2 = 3.9 \cdot 10^4 \text{ cal cm}^{-2}. \tag{11}$$

This figure compares well with the values $4.67 \cdot 10^4$ cal cm^{-2} and $4.26 \cdot 10^4$ cal cm^{-2} given by SIMOJOKI [9] for the Bogskär area in the northern Baltic. It is evident that his figures must be larger than ours, because Bogskär is situated south of the Bothnian Sea.

The figure (11) is also in a very good agreement with HANKIMO's [2] results at Finngrundet in the southern Bothnian Sea. He found for the total heat flow the value $4.08 \cdot 10^4$ cal cm^{-2}.

REFERENCES

1. Anonymous, 1962: Jäätalvi 1961/1962 Suomen merialueilla. Icewinter 1961/1962 along the Finnish coasts. *Publ.* 206 *of the Institute of Marine Research.* Helsinki.
2. Hankimo, Juhani, 1964: Some computations of the energy exchange between the sea and the atmosphere in the Baltic area. *Contrib. Finnish Meteor. Office* 57.
3. Hela, Ilmo, 1951: On the energy exchange between the sea and the atmosphere in the Baltic area. *Ann. Acad. Scient. Fennica.* Ser. A. **97**, p. 47.
4. Johnson, Nils G. (= Jerlov, Nils G.), 1940: Östersjöns värmeekonomi. *Svenska Hydrografiska-Biologiska Kommissionens Skrifter.* Ny serie: hydrografi XV. Göteborg.
5. Jurva, Risto, 1926: Kohtisuoran lämpövirran voimakkuudesta meressä Pohjois-Itämerellä Bogskärin majakan seuduilla. *II Määräaikainen Tutkijainkokous* 11—14. I. 1926. Helsinki.
6. —»— 1937: Laskielmia meriemme lämpövarastosta. *Suomi Merellä* 12, 162—191.
7. Landolt-Börnstein, 1952: Zahlenwerte und Funktionen aus Physik, Chemie, Astronomie, Geophysik und Technik. III Band, p. 440.
8. Palmén, E., 1930: Untersuchungen über die Strömungen in den Finnland umgebenden Meeren. *Soc. Scient. Fennica.* Comm. V 12. p. 10.
9. Simojoki, Heikki, 1946: On the temperature and salinity of the Sea in the vicinity of the Bogskär lighthouse in the northern Baltic. *Comm. Soc. Scient. Fennica.* XIII. 7. p. 15.
10. Wallerius, Daniel, 1940: Östersjövattnets värmeinnehåll. *Svenska Hydrografiska-Biologiska Kommissionens Fyrskeppsundersökning* 1932. Göteborg.