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ON SEICHES IN SOME LAKES IN FINLAND

by

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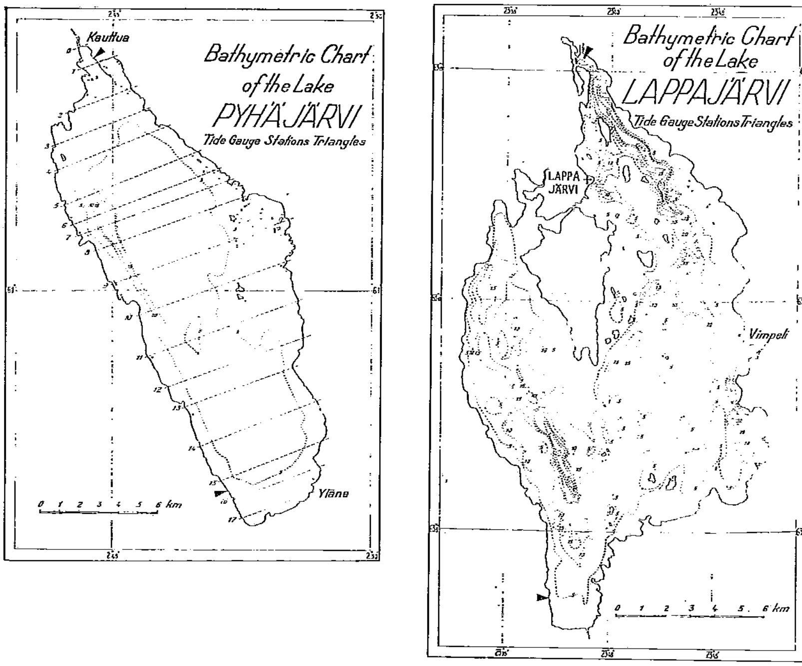
Abstract

In order to investigate seiches in lakes in Finland, special water level observations were arranged in lake Pyhäjärvi and lake Lappajärvi. The lengths of the periods were determined from limnograms. The result was checked theoretically.

Seiche in water level fluctuations in Finnish lakes is in general not a particularly distinct phenomenon. In limnograms of lakes some oscillation is observed at times, and in some lakes even regular ones, which may be presumed to reflect seiches, although the amplitudes of oscillation are rather small. The lake basins in Finland are usually rather irregular in shape and hence in the same lake several different standing waves may occur simultaneously, the resulting wave being complex in form.

In order to obtain a more distinct idea of the seiches occurring in Finnish lakes, special water level observations were arranged in two regular shaped lakes, namely, in lake Pyhäjärvi in October 1959 and in lake Lappajärvi in August and September 1960 (Fig. 1). From bathymetric charts of the lakes it is seen that the shape of the basin of lake Pyhäjärvi is more or less regular, the lake bottom even and the surface open and almost free of islands. The mean depth of the lake is 5.5 meters, its greatest depth 25 meters and its greatest length 25.5 kilometers. Lake Lappajärvi is not as regular in shape. The island of Kärnä divides the northern part of the lake into two different parts. The lake bottom is



Figs. 1. Maps of lake Pyhäjärvi and lake Lappajärvi.

not as even as in lake Pyhäjärvi. The mean depth of lake Lappajärvi is 7.5 meters, its greatest depth 38 meters and its greatest length about 21 kilometers.

At both ends of both lake Pyhäjärvi and lake Lappajärvi water level recorders »Ott-Schreibpegel Type XV» with recording scale 1:1 were se up (Fig. 1). The reading accuracy of the limnogram was 1 millimeter fo water level and 5 minutes for time. The floater of the recorder was situated in a sheltered tank. The tank was connected with the lake by a pipe. The relation between the cross sections of the pipe and the tank was 1:150.

As expected, seiche was distinctly demonstrated, in the observations made on lake Pyhäjärvi and lake Lappajärvi (Figs. 2). In these water level records, it is characteristic that wave groups appear with a length of period remaining nearly unchanged for some time. In Fig. 2a it is seen that the Kauttua recorder has recorded two wave groups of different types, and in lake Lappajärvi a wave of similar kind has lasted relatively long.

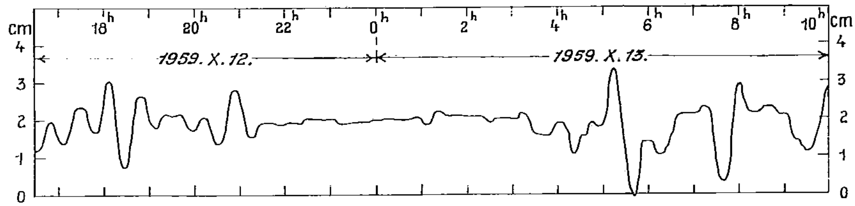


Fig. 2 a. Water level records of lake Pyhäjärvi at Kauttua.

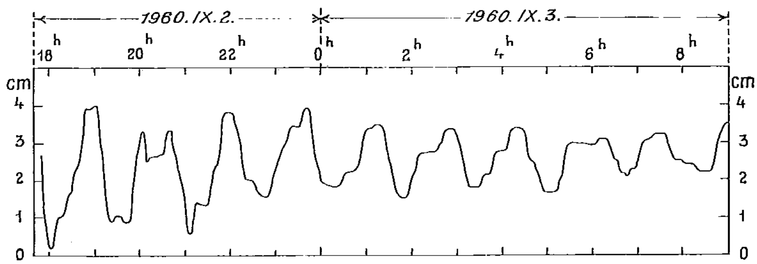


Fig. 2 b. Water level records of lake Lappajärvi at the northern end.

The lengths of the periods were determined by using groups of waves of a similar kind. The time interval between occurrences of groups of waves of a similar kind was divided by the number of waves included in the group, the number being at least 4, with a possible increase to 15. From these individual cases the final periods were computed as weighted means.

Analysis of the limnograms of lake Pyhäjärvi gives the same average length, 101 minutes for uninodal oscillation for both Kauttua and Yläne. In Kauttua's limnogram, in addition, a period with an average length of 42 minutes is seen rather clearly as well as a period occurring less frequently, with a length of 57 minutes. The latter period may represent a binodal and the former a trinodal wave. According to CHRYSTAL [3], the theoretical times of oscillation are related to a basin with a convex parabola-shaped bottom as the figures 100:57.7:40.8. In Fig. 3 the simultaneous uninodal seiche in both ends of lake Pyhäjärvi is seen.

The average amplitude of the uninodal wave was 9 millimeters, but in individual cases 2—3 centimeters. As no specially strong winds occurred during the observation period it is possible that in an exceptional case the amplitude in lake Pyhäjärvi may increase. Computations of

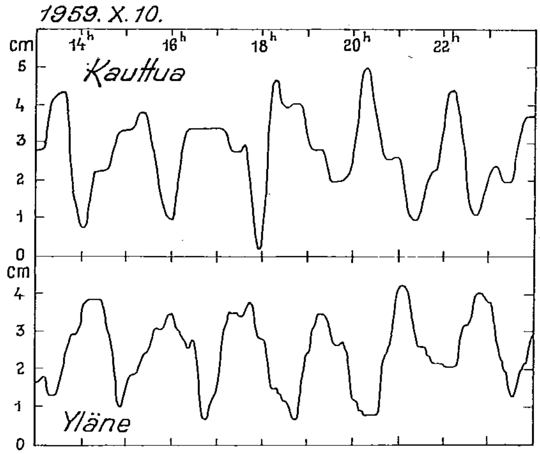


Fig. 3. Simultaneous seiches in lake Pyhäjärvi.

the dispersion of the water level values, read every 10 minutes, give a result of 10 millimeters.

From observations in lake Lappajärvi it appears that in the northern end of the lake a rather distinct seiche occurs with a period of 93 minutes and another, less confidently determined, with a period of about 46 minutes. In the southern end of the lake a distinct seiche is observed with a period of 48 minutes and occasionally a wave with a period of 94 minutes. From these facts as well as from some other observations, it may be concluded that the period of the uninodal wave is about 93 minutes and that of the binodal wave about 48 minutes. Fig. 4 illustrates the uninodal oscillation in lake Lappajärvi.

Seiche does not seem to be as distinct in lake Lappajärvi as in lake Pyhäjärvi. The basin of lake Lappajärvi is in two parts and the lake in itself rather wide. Hence the oscillation here may be rather complex.

The results obtained above have been checked by computing the theoretical oscillation times for lake Pyhäjärvi. Computations have been carried out by using the method of Defant [1] in the same way as Neumann [4] has done in relation to the Baltic Sea.

In a rectangular co-ordinate system, where the x and y axes are on the undisturbed surface of the lake with the x axis lying in the longitudinal direction of the lake and the positive direction of the z axis is perpendicular to the lake surface, the equation of motion, disregarding the earth's rotation, may be written

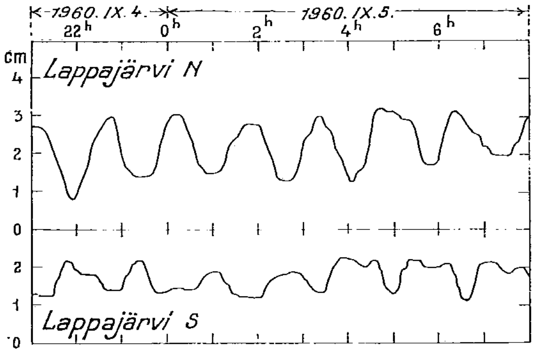


Fig. 4. Simultaneous seiches in lake Lappajärvi.

$$\frac{\partial^2 \xi}{\partial t^2} = - g \frac{\partial \eta}{\partial x} \tag{1}$$

and the equation of continuity

$$\eta = - \frac{1}{b(x)} \cdot \frac{\partial [S(x) \cdot \xi]}{\partial x} \tag{2}$$

In these equations η indicates the vertical and ξ the horizontal oscillation of the water particle, $b(x)$ the changing breadth of the lake and $S(x)$ the cross-sectional area of the lake perpendicular to the x axis.

Fitting in equations 1 and 2, according to DEFANT, the expressions

$$\begin{aligned} \xi &= \xi_0(x) \cos\left(\frac{2\pi}{T} t - \tau\right) \\ \eta &= \eta_0(x) \cos\left(\frac{2\pi}{T} t - \tau\right) \end{aligned} \tag{3}$$

where $\xi_0(x)$ and $\eta_0(x)$ indicate the maxima values of ξ and η independent of time and T the free oscillation period, the following equations

$$\begin{aligned} 2\Delta\eta_0(x) &= \frac{4\pi^2}{gT^2} \cdot 2\xi_0(x)\Delta x \\ 2\xi_0(x) &= - \frac{1}{S(x)} \int_0^x 2\eta_0(x)\, b(x)\, dx \end{aligned} \tag{4}$$

are obtained. On account of the character of the problem, $\xi = 0$ in both ends of the lake when $q = \int_0^x 2\eta_0(x) b(x) dx = 0$. From this fact T

can be determined. Using the approximate value of 116 minutes given in MERIAN's formula, the approximate values of T can be determined by iteration. For lake Pyhäjärvi, which is divided into elements as shown in Fig. 1, the following results were obtained

$T =$	120	110	100	98	minutes
$q =$	25.08	12.87	1.26	−0.88	10^{-3} km^3

From the foregoing, 98.8 min will be obtained by straight line interpolation for the uninodal oscillation time and 101 minutes from observations, as mentioned before. Calculations indicate that the nodal line is half way between sections No 9 and No 10 (Fig. 1). Further, from the same calculations the theoretical water flow which within a time period $\frac{1}{4}T$ goes through any one controlled section can be obtained. Assuming that $2\xi_0 = 10$ centimeters, which seems possible in certain cases, the volume of water which in the period $\frac{99}{4} \approx 25$ minutes goes through the nodal line section will be about $4 \cdot 10^{-3}$ cubic kilometers, which is about 0.5 per cent of the whole amount of water in lake Pyhäjärvi. The mean velocity of this streamflow is 0.4 cm/sec in cross section.

The theoretic determination of the oscillation time of lake Lappajärvi proved difficult. The island in the northern end of the lake as well as the relative breadth of the lake cause some uncertainty in dividing the lake into appropriate elements. MERIAN's formula gives the result as 86 minutes and the observations 93 minutes, as has been mentioned above.

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