526. I

REMARKS CONCERNING THE PRESENT POSITION OF THE POLE

by

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Abstract

The distribution of continents and oceans largely determines the inertia of the Earth's crust. The axis about which the moment of inertia is a maximum pierces the surface 400 miles southwest of Hawaii. This is the position the north pole would eventually occupy provided (1) the equatorial bulge is not »frozen» but will adopt (in a time of order τ) the shape appropriate to diurnal rotation, (2) oceans and continents are isostatically balanced, (3) the distribution of ocean and continents remains fixed relative to one another, and (4) no other comparative asymmetries in the distribution of matter exists. The rate of polar migration depends on τ. BONDI and GOLD have interpreted the damping of the 14-month wobble in terms of plastic flow in the Earth's mantle, and obtain $\tau = 10$ years. If this value were applicable, then under the foregoing assumptions the travel time of the pole to Hawaii is 100,000 years, a rate of travel 1000 times the upper limits given by astronomical measurements during the last sixty years and by paleomagnetic evidence over the last 10 million years. For earlier times the paleomagnetic evidence indicates a direction opposite to that derived here. This leaves some questions to be answered.

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Introduction

The usual starting point of any discussion on polar wandering is to presume the Earth to be in equilibrium until suddenly disturbed by some implausible rearrangement of matter. The ensuing motion of the pole is then computed for an Earth made of material that can be modeled by appropriate combinations of springs and dashpots. Finally, the computed polar path is found to be in agreement with a bewildering array of paleoclimatic, and more recently paleomagnetic, evidence.

It may be pertinent to inquire whether for any prevailing Earth model the *present* distribution of matter is consistent with the *present* position of the pole. For example, as a consequence of GOLD's [4] model the pole ought to be south of Hawaii. There are ways out of this dilemma, as always, and these may provide some significant geophysical restraints.

The starting point of GOLD's theory is that the dimensions of the equatorial bulge are those appropriate to an equivalent rotating fluid. This suggests that the bulge adjusts itself to diurnal rotation no matter what the orientation of the axis of rotation may be. If the bulge does not offer any long term stability in the position of the pole, what does? The first thing that comes to mind is the distribution of continents and oceans. Compared to the equatorial bulge these are tiny markings on the surface, still they might be the determining factor if the continental distribution is permanent and the bulge appropriately plastic. A rapidly spinning rubber ball would orient itself relative to the rotational axis in accordance with tiny surface markings.

Principal axes of the crust

The problem then is to determine the principal axes of the Earth's crust, neglecting the equatorial eccentricity. If the continents formed a ring around the globe, then obviously one principal axis would be at right angles to this ring, and the continents would be on top of the equatorial bulge. The actual distribution is too complicated to obtain an answer by inspection, but we might guess that the pole should be in the Pacific. This turns out to be a good guess.

Let x,y,z be associated with the present geographic coordinates: x is drawn from the Earth's center through the equator at the meridian of Greenwich, y intersects the equator 90° east of Greenwich, and z is through the north pole. Moments A,B,C and products D,E,F of inertia are then given by

$$A = \Sigma m(y^2+z^2) \;\; etc.; \qquad D=\Sigma myz \;\; etc.$$

To evaluate $A,B,\ldots F$ it is convenient to switch to the spherical coordinates r,Θ,Φ, where Θ is the colatitude and Φ the east longitude. Now suppose that the variation of density with depth is uniform for all continents, to be designated by $\varrho'(r)$, similarly $\varrho(r)$ refers to oceanic sections. The excess of continental over oceanic inertia equals

$$A=\int\int\int(\varrho'-\varrho)r^4(\sin^2\Theta\,\sin^2\Phi+\cos^2\Theta)\,\sin\Theta\,dr\,d\Theta\,d\Phi=Ia$$
$$I=\int(\varrho'-\varrho)\,r^4\,dr, \qquad a=\int\int(\sin^2\Theta\,\sin^2\Phi+\cos^2\Theta)\sin\Theta\,d\Theta\,d\Phi \tag{1}$$

with a evaluated over continental parts of the Earth. Similarly $B=I\,b$, $\ldots F=I\,f$. We may visualize $A\ldots F$ as the inertia of continental veneers glued on the surface of a sphere.

The problem is to find the principal axis x',y',z', i.e., the axis for which the products of inertia

$$D'=\Sigma m\,y'\,z' \;\; \text{etc}.$$

vanish. The direction cosines of the x',y',z'-system relative to the x,y,z-system are defined by the scheme

	x	y	z
x'	α_1	β_1	γ_1
y'	α_2	β_2	γ_2
z'	α_3	β_3	γ_3

The solution is

$$\alpha_i \sim e(b-\lambda_i)+df, \quad \beta_i \sim d(a-\lambda_i)+ef, \quad \gamma_i \sim (a-\lambda_i)(b-\lambda_i)-f^2$$

with the proportionality factor determined by $\alpha_i^2+\beta_i^2+\gamma_i^2=1$. The three principal moments of inertia are $A'=I\,a'$, $B'=I\,b'$, $C'=I\,c'$, where $\lambda=a'$, $\lambda=b'$, $\lambda=c'$, are the three roots of the cubic equation

$$\begin{vmatrix} \lambda-a & f & e \\ f & \lambda-b & d \\ e & d & \lambda-c \end{vmatrix} =0$$

The moment of inertia about *any* axis with direction cosines $\alpha_1,\alpha_2,\alpha_3$ relative to the principal axes, is $I\,q$, where

$$q(\alpha_1,\alpha_2,\alpha_3)=a'\alpha_1^2+b'\alpha_2^2+c'\alpha_3^2\,. \tag{2}$$

The results of numerical calculations are given in Table 1. Revelle and I have previously evaluated the necessary integrals over all oceans using $10°\times 10°$ grids, and the continental values in column (1) were found by subtraction. Recent seismic work has shown that the dividing line between continental and oceanic structure corresponds roughly to the 1000 fathoms depth contour rather than the coastline. The values in the second column refer to these enlarged continents. After my calculation had been completed, my attention was drawn to the work of MILANKOVITCH [10], which contains values for a 20°×20° grid bounded at the coastline. Milankovitch's values are in column (3). I think his values for a,b,c are too large. The quantity $(a+b+c)/8\pi$ designates the fraction of the Earth's surface covered by continents. His values give 0.41. The accepted ratio is 0.29.

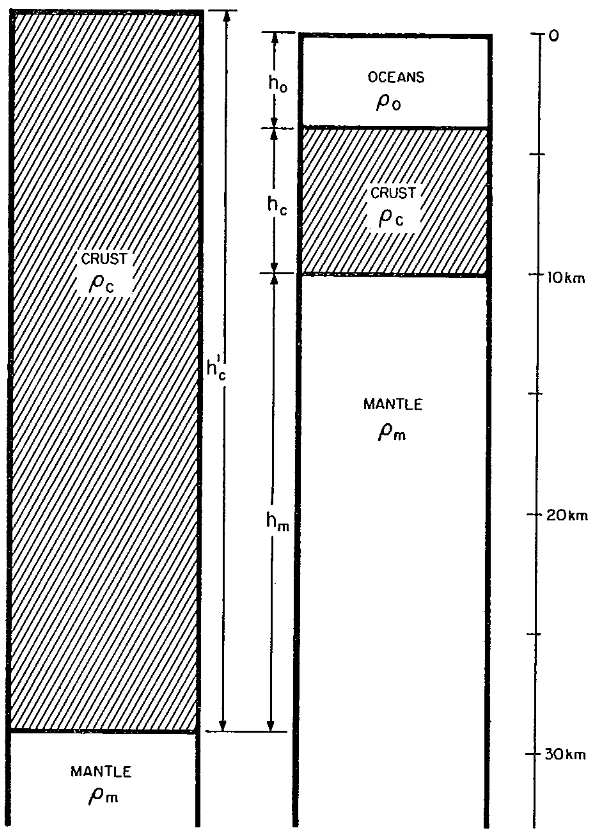

Fig. 1. Standard continental and oceanic sections. The continental section consists of crust material of density $\varrho_c = 2.75$ g cm^{-3} and thickness $h'_c = 30$ km. The oceanic section consists of ocean ($\varrho_o = 1.025$ g cm^{-3}, $h_o = 4$ km), crust ($\varrho_c = 2.75$ g cm^{-3}, $h_c = 6$ km). The density of the mantle is $\varrho_m = 3.25$ g cm^{-3}, and $h_m = 19.1$ km.

In all events there is an over-all agreement between the three calculations. The axis of maximum moment of inertia has a pole near Hawaii, and the axis of minimum moment has one near Archangel not too far from the actual pole. The results could hardly be in lesser accord with the present position of the pole.

MILANKOVITCH did not attempt to evaluate the integral I. On the basis of available seismic and gravity work it is now possible to do so. WORZEL and SHURBET [14] have proposed standard continental and oceanic sections. I have used a slightly simplified model (Fig. 1) by omitting the sediment layer. Continents and oceans are in isostatic equilibrium.

$$\varrho_c h'_c = \varrho_o h_o + \varrho_c h_c + \varrho_m h_m \,. \tag{3}$$

The elevation of continents above sea level turns out to be

$$z = h'_c - (h_o + h_c + h_m) = 0.9 \text{ km} \tag{4}$$

which is an acceptable value.

Table 1. Moments and products of inertia, $a, b, \ldots, e$, the three principal moments a', b', c', and their northern hemisphere poles for continents bounded by the coast line, the 1000 fathom line, and according to the calculation by MILANKOVITCH and HAUBRICH.

	coast line	1000 fathoms	Milankovitch	Haubrich
a	2.757	3.473	3.700	4.553
b	2.571	3.137	3.442	3.924
c	2.234	2.692	3.038	3.665
d	0.202	0.192	0.191	0.412
e	0.172	0.161	0.240	0.199
f	0.0725	−0.090	−0.002	−0.056
a'	2.809	3.547	3.790	4.569
Θ	75°	76°	70°	86°
	176°	199°	192°	178°
b'	2.665	3.152	3.491	4.252
Θ	66°	76°	76°	124°
	273°	293°	287°	73°
c'	2.088	2.603	2.899	3.321
Θ	29°	20°	24°	36°
	56°	66°	49°	69°

The evaluation of I (equation 1) involves third order approximations. First order terms cancel because the crust is thin compared to the radius of the Earth; second order terms cancel because of isostacy. Details are cumbersome and uninteresting. The result is

$$I = 2r_0^3(\varrho_c h'^2_c - \varrho_m h_m^2 - \varrho_c h_c^2 - \varrho_o h_o^2 - 2\,\varrho_c h_c h_m - 2\,\varrho_o h_o h_m - 2\,\varrho_o h_o h_c)$$
$$= 1.78 \times 10^{39} \text{ g cm}^2. \qquad (5)$$

where r_0 is the Earth's radius.

Figure 2 shows the moment of inertia of continents (the q-topography, Eq. 2) about any pole, in units of I. The topography has a maximum,

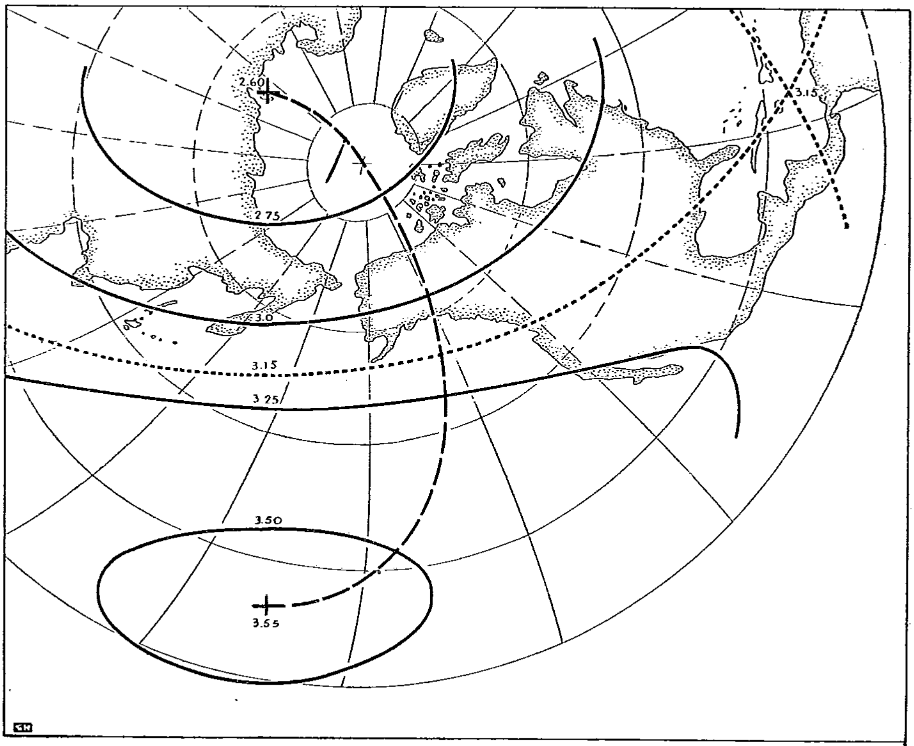

Fig. 2. Inertia of continents. The base chart is in polar coordinates with respect to the principal pole c' (near Archangel). Colatitudes Θ with respect to c' are equally spaced circles (not shown), with the equator ($\Theta = 90°$) through the Hawaii and West Indies poles. The light lines are co-ordinates relative to the present pole. The heavy lines give relative moment of inertia of the crust, q, about any axis; thus $q = 2.70$ about the present axis. The line through the present pole at right angles to the q-topography represents the probable path of polar wandering.

a', at the Hawaii pole, a minimum, c', at the Archangel pole, and a saddle point at the West Indies (value b'). For comparison, we have the following principal moments of inertia:

Earth	I_e	$=0.8\times10^{45}$ g cm^2
Equatorial bulge	I_b	$=3.0\times10^{42}$ g cm^2
Continents (isostatically balanced)	$a'I$	$=0.6\times10^{40}$ g cm^2

The calculations of Haubrich

After reading a rough draft of this manuscript, Mr. RICHARD HAUBRICH felt the need of treating the more general case of variable continents. He has permitted me to report his results here. Haubrich adapted the model of WORZEL and SHURBET [14], including the layer of oceanic sediments. When the continental elevation is at sea level ($z=0$) the density and thickness of the continental crust are 2.84 g cm^{-3} and 32.8 km respectively.

For an Airy-type compensation he solved equations (3) and (4) for h'_c and h_m as functions of continental elevation z, and evaluated $I(z)$ from (5):

z in km	0	.2	.5	1	2	4
I in 10^4 g cm^2	.19	.21	.25	.30	.43	.75

For a Pratt-type compensation h_o, h_c and h_m are fixed, and h'_c is determined from equation (4). The right side of equation (3) is now written $\varrho'_c h'_c$ instead of $\varrho_c h'_c$, and ϱ'_c is evaluated:

z in km	0	.2	.5	1	2	4
ϱ'_c in g cm^{-3}	2.84	2.82	2.80	2.76	2.68	2.52

For $z=0$ the models are identical; for $z=1$ km the Pratt-type compensation gives $I=.46\times10^{40}$ g cm^2 compared to $.30\times10^{40}$ for the Airy-type compensation, and 0.18×10^{40} from my calculations with $z=0.9$ km. Some of the discrepancy between HAUBRICH's and my calculations arises from the difference in the model; still the discrepancy is larger than I had suspected.

The tensor of inertia is evaluated from integrals such as

$$A=\iint I(\Theta,\Phi)(\sin^2\Theta\sin^2\Phi+\cos^2\Theta)\sin\Theta\, d\Theta d\Phi$$

taken over continental parts of the Earth. $I(\Theta,\Phi)$ is the value of I for a mean elevation $z(\Theta,\Phi)$ of a $10°\times10°$ square. It is no longer possible to factor A into a product of two integrals, $A = Ia$ as in the case of uniform continents (Eq. 2).

HAUBRICH's results for the Airy-type compensation are included in Table 1. The pole of maximum inertia is still in the central Pacific, but 10° further south than I found. The pole of minimum inertia is displaced towards the Himalaya complex in accordance with the larger moment of intertia accorded to high-lying regions. For a Pratt-type compensation the discrepancy between the uniform and non-uniform continent models would be larger still.

Further remarks concerning principal axes

The differences in the computed positions of the axes for various models give some indication of the limits placed by our present knowledge of the crust. Seismic evidence has not made a clear-cut decision between the two basic types of compensation. The shallow Moho under the Colorado and Mexican plateaus favors a Pratt-type compensation; the root beneath the Appalachians speaks for an Airy-type compensation. No allowance has been made in any of the models for the large shoal areas of the oceans (subcontinents) whose structure is intermediate between that of oceans and continents.

The following two examples serve to bring out explicitly the role played by major features. An icecap (density $\varrho_i = 0.9$ g cm^{-3}, thickness $h_i = 3$ km) covers Greenland. As a result of the ice load the crust is pushed downward into the mantle. Assuming Airy-type compensation, the products of inertia are

$$D_i = I'd_i, \quad E_i = I'e_i, \quad \text{where}$$

$$I' = \varrho_i h_i a^4 \iota, \quad \iota = 4\frac{h'_c}{a}\left(1 - \frac{\varrho_c}{\varrho_m}\right) + 2h_i\left(1 - \frac{\varrho_i}{\varrho_m}\right) = 0.003$$

with ι designating the isostatic factor. For Greenland bounded at the coastline $d_i = -0.010$, $e_i = 0.011$. The resultant products of inertia are $D_i = -1.33 \times 10^{34}$ g cm^2, $E_i = 1.47 \times 10^{34}$ g cm^2, compared to $D = Id = 3.4 \times 10^{38}$ g cm^2, $E = 2.9 \times 10^{38}$ g cm^2 for all continents. Clearly the inertia of the Greenland icecap is negligible, and no reasonable icecap can balance the torque exerted by continents. The Himalaya complex raises 3×10^{22}g of matter above normal continental height, and the Andes about 1×10^{22}g. The Alps are negligible. The products of inertia of the incremental masses, assumed isostatically compensated, are 1.3×10^{37}g

cm^2 and 0.4×10^{37}g cm^2 respectively. These values are sufficiently large compared to the products of inertia of the crust to account for the order of difference between HAUBRICH's calculation and mine (Table 1); still the over-all result as portrayed in Fig. 2 is substantially correct, the contour interval being of the order 10^{39}g cm^2.

GOLD has pointed out that a polar ocean provides a polar trap. If the pole were to move towards shore, additional ice is deposited there which would eventually drive the pole offshore. He suggests that the Arctic Ocean may account for the apparent stability of the pole over the last few million years. The foregoing calculations indicate that the effect of an icecap is relatively small.

The path of polar wandering

BURGERS [2] and INGLIS [6] have treated the dynamics of polar wandering for an elasto-viscous Earth: this corresponds to a spring and dashpot in series (Maxwell body; SCHEIDEGGER, [12]) so that for disturbances of short period the elastic properties dominate, whereas under prolonged stress the body yields like a fluid. For a sudden upheaval in the crust (say) the solution consists of oscillatory terms that are damped like $e^{-t/\tau}$, together with the velocity components

$$\dot{l} = -\frac{r_o}{\tau}\frac{E}{C-A}, \qquad \dot{m} = -\frac{r_o}{\tau}\frac{D}{C-A} \tag{6}$$

of the pole's wandering toward Greenwich, and 90° East of Greenwich, respectively. The moment of inertia of the equatorial bulge is $C-A$ and large compared to $B-A$. The principal pole of the Earth (not the crust) is at

$$l_p = -r_o E/(C-A), \qquad m_p = -r_o D/(C-A)$$

with respect to the coordinate system used to compute the inertial tensor. As the pole of rotation wanders, the position of the equatorial bulge changes with respect to the original coordinate system, and this affects the products of inertia D and E, and hence the position of the principal pole. Polar wandering continues until the principal axes of the Earth coincide once more with those of the crust. The reader is referred to BURGERS and INGLIS for a mathematical treatment, and to GOLD who anticipated these results by qualitative reasoning.

The derivation of (6) is based on the principle of conservation of angular momentum. Products of inertia D,E are associated with the difference in the inertia of equivalent continental and oceanic areas. In the case of isostatic compensation the difference is of second order; it arises only because continents are a bit further from the axis of rotation than oceans. It is instructive to point out that one can adopt an alternate point of view, which is to start with the difference in centrifugal force exerted on equivalent continental and oceanic areas. In the case of isostacy this again depends on a second order effect: the centrifugal force on continents is somewhat larger, these being further removed from the axis of rotation. This second order force is known as the Polfluchkraft of Eötvös. It has become so involved in speculations concerning continental drift, that it seems appropriate to emphasize that it enters here in a more subtle way. There is no need for this force to shift continents relative to one another, nor to slip the crust over the rest of the Earth; rather there is a torque on the Earth as a whole which will alter its orientation relative to the axis of rotation until the torque vanishes. This final position corresponds once more to rotation about the principal axis of the crust.

Polar wandering can be visualized as a slow wave-like propagation of the equatorial bulge with respect to the Earth, much like a tidal bulge. From the point of view of an observer in space the bulge remains fixed relative to the ecliptic plane, and the Earth slowly turns under it. This is not a *slippage* of the crust over the mantle as suggested by many authors. Slippage would require that the thickness of a viscous boundary layer be small compared to the Earth's radius, whereas just the reverse is true.[1]

Milankovitch reasoned that the pole must slide »downhill» toward the pole of maximum inertia, that is, along a line normal to the q-topography. This seems reasonable enough, yet I do not find it an obvious result. A derivation of the path, starting with (6), is given in the Appendix. It turns out that a path along the orthogonal is an adequate approximation provided the damping time τ is long compared to the period of free nutation. The observed ratio is 8:1.

The equation of the path is simple in terms of the geographical coodinates with respect to the principal continental axis: Let Θ' be colatitude south of the Archangel pole, and Φ' longitude east of the Hawaii pole. Eq. (2) can be written

[1] For a viscosity $\nu = 10^{20}$ cm^2 sec^{-1} and a period T of only 10 years, the thickness of the layer is $\sqrt{\nu T/\pi} = 10^{14}$ cm $= 10^5$ times the Earth's radius.

$$\sin^2\Theta'(k-\sin^2\Phi')=(q-c')/(a'-b'), \qquad k=(a'-c')/(a'-b').$$

The family of lines at right angles to the q-contours is then

$$\tan\Theta' \cos\Phi' \tan^k\Phi' = \text{constant}, \tag{7}$$

an equation given by MILANKOVITCH. The orthogonal line through the present pole is drawn in Figure 2.

The direction of polar wandering

The computed direction of polar wandering is toward the principal pole near Hawaii. On the basis of paleoclimatic evidence which he considered as »unzweideutig», MILANKOVITCH drew the arrow along his orthogonal (which does not differ too much from the orthogonal in Figure 2) pointing *away* from the Pacific. JARDETSKY [7] seems to have gone along with this interpretation. Recent paleomagnetic evidence also indicates wandering away from the Pacific, but the evidence is far from *unzweideutig*, I think, and the time scales of the climatic and magnetic evidence do not agree at all well. The interpretation of Milankovitch and Jardetzky implies a reverse polarity, or a negative value of the integral I. This means that the inertia of continents is *less* than that of oceans in contradiction with the present isostatic model. On this basis GUTENBERG ([5], p. 203) has criticized Milankovitch's paper.

However the suggestion of a reverse polarity desires further consideration. In the isostatic model the inertia of the crust depends on the second order term resulting from the slightly larger radial distance of continents than oceans. Suppose that isostatic balance does not hold precisely, but that there is a slight erosion of continental matter and sedimentation on the ocean floor which is not compensated. This represents a first order effect which might reverse the sign of I. This is an intriguing possibility. It would place the pole of maximum inertia at Archangel, where it is a relatively small distance from the actual pole (though HAUBRICH's calculation places it further south); whereas for the isostatic model the pole is about as far removed from the principal pole as it can be.

For the isostatic model $I=1.78\times10^{39}$g cm^2. How much material would have to be eroded from continents and deposited on the sea floor to make $I=0$, presuming the erosion process to be uncompensated? The oceans occupy 2.5 times the area occupied by continents. For $\varrho_c\Delta h$ g cm^{-2} removed from continents, 0.4 $\varrho_c\Delta h$ g cm^{-2} are deposited as sediments, and I is diminished by $1.4\ \varrho_c \Delta h\, r_0^4$. Uncompensated erosion by $\Delta h = 28$ meters will make $I = 0$. Hence if the elevation of continents above sea level by 900 meters represents an isostatic elevation of 930 together with an uncompensated lowering by 30 meters, the sign of I would indeed be negative.

What do these models imply with regard to gravity anomalies? The simplest procedure is to take continents as elevated strips on an infinite plane. First take the absurd example of complete lack of compensation. If the oceans are condensed from a density ϱ_o to ϱ_c, sea level is lowered by $h_o(1 - \varrho_o/\varrho_c) = 2.5$ km, and the elevation of continents above the condensed sea is $\Delta h = 0.9 + 2.5 = 3.4$ km. The gravity anomaly over the central portion of the continent is

$$\Delta g = 2\pi\, G\, \varrho_c \Delta h = 0.392 \text{ cm sec}^{-2},$$

where G is the gravitational constant. For the compensated model the gravity anomaly is found by an extension of the formula given by JEFFREYS ([8], p. 175). The strip extends from $\xi = -L$ to $\xi = +L$. Jeffreys has shown that Δg is remarkably uniform except near the continental edges, and we might as well restrict ourselves to the center, $\xi = 0$. Writing $\Delta\varrho = \varrho'(\zeta) - \varrho(\zeta)$ for the excess of continental over oceanic density at depth ζ, we have

$$\begin{aligned} \Delta g &= 4G \int\int \Delta\varrho \sin \varkappa L\, e^{-\varkappa L} \varkappa^{-1}\, d\varkappa\, d\zeta\,, \\ &= 4G \int \Delta\varrho \cot^{-1}(\zeta/L)\, d\zeta\,, \\ &= 2\pi G \int \Delta\varrho\, d\zeta - \frac{4G}{L} \int \Delta\varrho\, d\zeta\,. \end{aligned}$$

Δg gives the excess of free-air gravity anomalies on continents over that on oceans. In the case of isostasy, only the second term remains. For the model in Figure 1 and for $2L = 6000$ km this gives $\Delta g = +0.8$ mgals. In the case of an uncompensated lowering of continents by $\Delta h = 4.8$ meters, the positive anomaly is annulled. Uncompensated erosion by 28 meters gives $\Delta g = -4.5$ mgals. There may be some concern about the thin plate model of continents, but it can be shown that little is changed by allowing for the spherical shape. The calculations are given by JEFFREYS ([8], 5.06) and need not be repeated here. The results are sommarized in Table 3.

If gravity anomalies on continents were consistently higher, or lower than over oceans, then the question of whether I is positive or negative could be decided. Unfortunately relatively large local anomalies make it impossible to make this decision. Certainly the assumption of no compensation is wildly wrong, but it is impossible to tell whether compensation is by 100% or 99%, and this is what we need to know. JEFFREYS ([8], p. 175) gives the following average values of free-air anomalies: Continents 0°—60° East, +8.4±3.6 mgals; Atlantic Ocean +6.4±4.7; North and South America, —1.6±5.7; these averages do not suggest that the oceans have consistently lower anomalies than the continents.

There is a fair chance that this problem can be resolved from the observed orbit of the IGY satellites.

The following hypothetical train of events may serve to illustrate the present considerations. Suppose that initially continents and oceans are

Table 3. Horizontal pressure difference in the mantle between continents and oceans, free air gravity anomalies on continents and products of inertia of continents for various models

		ΔP dynes cm^{-2}	Δg milligals	I g cm^2
(1)	No compensation	92×10^7	392	154×10^{39}
(2)	Isostasy	0	0.8	1.78×10^{39}
(3)	No gravity anomalies	-0.2×10^7	0	1.48×10^{39}
(4)	No continental torque	-1.1×10^7	-4.5	0

compensated. The inertia of the continents exceeds that of the oceans (I is positive) and the pole is south of Hawaii. The free-air gravity anomaly over continents is about +1 mgal. Subsequently erosion takes place, and the continents lose mass, the sea bottom gains mass. The erosion is too rapid for isostatic compensation to be effective or perhaps the stresses involved are below some critical values required for the initiation of compensating currents in the mantle. As a result, the excess inertia of the continents dwindles, and by the time the continents have been eroded by 30 meters and the sedimentary blanket thickened by something like 15 meters, the inertia per unit area is the same for continents and oceans. Subsequent erosion reverses the polarity of the principal axis and the pole of rotation moves rapidly towards the present position. At the time of reversal the gravity anomaly over continents is about —5 mgals. Further erosion will stabilize the pole in its new position. Uncompensated erosion by an additional 30 meters would assure that the pole remain essentially in its present position without being materially affected by icecaps and mountain formations.

At the end of the erosional epoch, the isostatic state is again approached, and the pole switches back to Hawaii.

The gravity data are not unfavorable to this hypothesis, but there is some difficulty in reconciling the required departures from isostasy with fluidity indicated by the damping of the wobble. A surface load leads to an adjustment in the shape of the Earth. This involves local deformation (high order spherical harmonics) as well as global deformation (low order harmonics). The local adjustments take place relatively slowly, the global adjustments occur rapidly. The products of inertia involve deformations of order 2, and so does the equatorial bulge. Hence the time constant of isostatic adjustment of the inertia terms turns out to be the same as the time constant τ of polar wandering: 10 years according to Bondi and Gold (see next section). In the case of a uniform sedimentation it follows that the departure from isostasy is due to sediments deposited in 10 years. This is of the order 5 cm, and not 15 meters as required. There is still the possibility that the uncompensated load is maintained until some critical stress is exceeded. The elasto-viscous model is then wrong. Bondi and Gold's theory of the damping of the wobble becomes doubtful, for certainly the stresses involved are then far below the critical value.

But even if the Earth can maintain sufficient anisostasy to reverse I, one would expect polar wandering towards Archangel at a rate of a few tens of meters per year, unless by chance I is very near zero. This is still too fast. To say that the distance between the present pole and Archangel is too small to be significant is to imply that the values of d and e in Table 1 do not differ significantly from zero. I think they do.

The rate of polar wandering

The observed damping time of the 14-month nutation (Chandler wobble) has been estimated by Rudnick [11] as 10 years and by Walker and Young [13] as 10 to 30 years. Bondi and Gold [1] have presented arguments as to why the damping must be due to dissipation in the Earth's mantle; if so, then according to the elasto-viscous model the damping time and the time constant τ of polar wandering are one and the same, and accordingly the rate of polar wandering can be computed for a given disturbance from equation (6). Using the value $\tau=10$ years it is embarrassingly simple to turn the Earth around. The Himalayas alone could do it in a 100 million years, and so could reasonable climatic fluctuation in sea level. Even winds would have a non-negligible effect.

Numerical values appropriate to the 1000 fathom uniform continents give a movement by 100 meters per year toward Vancouver. Astronomical observations place an upper limit of 5 meters during the last 50 years. The rate would slowly increase, being largest when the pole is in the Gulf of Alaska, midway between the two principal poles; from then on polar wandering diminishes. The actual rate can be found from equations (7) and (10). The time required to travel most of the way to the Hawaii pole is of the order (Appendix, Eq. 10)

$$\frac{I_b}{(a'-b')I}\,\tau$$

or 100,000 years. Paleomagnetic evidence places an upper limit of about 10° latitude during the last ten million years. In both instances the limits are 1/1000 the derived values.

Inhomogeneities in the mantle

The calculations so far have been made under the assumption that the distribution of continents and oceans are the features most likely to be responsible for the position of the pole.

Irregularities in the distribution of matter in the *mantle* could be an important factor. JEFFREYS ([8], p. 177) has compiled a free-air anomaly chart for the world. These anomalies represent departures from the *normal gravity*, which contains terms in $\cos^2\Theta$ and $\cos^2 2\Theta$. There are large areas for which gravity observations are lacking, and when these become available the coefficient of $\cos^2\Theta$ and $\cos^2 2\Theta$ in the formula for normal gravity will be altered. This means that the plotted free-air pattern is subject to uncertainty not only in regions where observations are lacking, but in other regions as well.

The first thing one notices from Jeffreys's chart is that the free-air gravity anomalies bear no obvious resemblance to the continental structure. If the continental structure were badly out of isostatic adjustment, then there would be such a resemblance. The least forced interpretation is that the continental structure is nearly in isostatic adjustment (though small but important departures of the kind discussed in the previous section can by no means be ruled out), and that the observed gravity anomalies have their source in the mantle. They might be an indication of bubbles of heavy and light material supported by viscous forces in convection currents. The pressure along the bottom of a boiling pot of porridge is not constant, nor is the gravity along the surface. The observation that the pole is not now moving quickly implies then that the distribution in the mantle has been essentially stationary for at least 100,000 years.

The observed gravity pattern does not provide a unique solution concerning the depth of the mass anomalies. Assume that they occur at one fixed distance r_1 from the Earth's center, and let σ_2 designate that part of the density anomaly per unit area that can be represented by spherical harmonics of degree two. These are the only terms in the density distribution that contribute to the products of inertia. Thus

$$D=\iiint r^4 \varrho(r) \sin^2\Theta \cos\Theta \sin\Phi \, dr \, d\Theta \, d\Phi = r_1^4 \sigma_2$$

where

$$\sigma_2=\iint \sigma(\Theta,\Phi) \sin^2\Theta \cos\Theta \sin\Phi \, d\Theta \, d\Phi \, .$$

Define

$$g_2=\iint g(\Theta,\Phi) \sin^2\Theta \cos\Theta \sin\Phi \, d\Theta \, d\Phi$$

and similarly g_2' by replacing $\sin\Phi$ with $\cos\Phi$. The free-airanomalies at the surface due to σ_2 are (JEFFREYS, [8], p. 171)

$$g_2=\frac{12\pi G}{5}\left(\frac{r_1}{r_0}\right)^3 \sigma_2$$

and we can express D in terms of g_2. It is convenient to introduce the Earth's moment of inertia, $I_e=\frac{1}{3} M a^2$, its mass $M=\frac{4}{3}\pi \varrho_e a^3$ and the mean gravity at the surface, $\bar{g}=\frac{4}{3}\pi G r_0 \varrho_e$. Then

$$D=\frac{5}{4\pi} I_e \frac{r_1}{r_0} \frac{g_2}{\bar{g}}, \qquad E=\frac{5}{4\pi} I_e \frac{r_1}{r_0} \frac{g_2'}{\bar{g}}$$

Harmonic analysis of Jeffreys's chart gives

$$g_2=-0.41 \text{ mgals}, \qquad g_2'=2.10 \text{ mgals}$$

so that

$$D=-1.4(r_1/r_0)10^{38} \text{ g cm}^2, \qquad E=6.9(r_1/r_0)10^{38} \text{ g cm}^2$$

compared to

$$D=3.4\times 10^{38} \text{ g cm}^2 \qquad E=2.9\times 10^{38} \text{ g cm}^2$$

for the isostatically compensated crust. The ratio r_1/r_0 varies from 1/2 at the bottom of the mantle to 1 at the top. The first thing to notice is that the products of inertia of the mantle, as inferred from free-air gravity anomalies, are of the same order as those of the crust; the second thing is that they are not properly distributed in longitude to balance those of the crust. It will be recalled that the isostatic balanced continents tend to move the pole toward Vancouver. What is required for balance are gravity anomalies with the extreme values of the second degree harmonic distributed as follows:

	130°W	50°E
Northern Hemisphere	+5 mgals	−5 mgals
Southern Hemisphere	−5 mgals	+5 mgals

Such distribution cannot be ruled out on the basis of our present knowledge of gravity.

It should be pointed out that, according to the present hypothesis, it is not accidental that the inertial terms in the mantle should be of equal magnitude to those in the crust. Suppose for the moment those in the mantle were much larger. Then the pole would rapidly adjust so as to reduce their magnitude, and in the final position only a small residue would be left, just large enough to balance the crust.

If this explanation of the present position of the pole is the correct one, then there are three sets of principal axes, that of the crust, that of the mantle, and the combined set. The combined set then presumably coincides with the true axis. Crust and mantle would twist relative to one another, until eventually the axes of crust and mantle coincide with each other and with the axis of rotation. The twisting motion might be from 1000 to 10,000 times slower than polar wandering. First of all the shear is larger because only a thin outer shell is involved, and secondly the viscosity of the outer mantle may be larger than the average viscosity of the Earth.

Discussion

For an elasto-viscous Earth with isostatically compensated continents and oceans the stable position of the pole is near Hawaii; if it were elsewhere it would reach Hawaii probably in 100,000 years. The following alternatives might account for the fact that the pole is not near Hawaii. (1) The equatorial bulge is frozen. The pole then remains practically where it is regardless of the distribution of continents and oceans; BONDI and GOLD's treatment of the damping of the Chandler wobble is not correct, and the problem remains to be solved. (2) The bulge is fluid, and the pole is determined by the distribution of continents. These are not balanced isostatically, but more than 30 meters have been eroded from continents and deposited on the sea floor without compensation. This scheme would place the stable pole much nearer the observed position (but I think still too far away). There is no conflict with observed gravity values, but it is difficult to assign sufficient fluidity to account for the damping of the Chandler wobble and yet achieve the required degree of anisostacy to reverse the polarity of the principal axes. (3) There are mass anomalies in the mantle which together with the distribution of land and sea determine the axis of rotation. The associated gravity anomalies would be of the same order as those observed, but the required pattern is quite different from the one drawn by JEFFREYS. (4) I favor a still different alternative: the Earth is anelastic in the sense that it behaves as an elastic solid for short-period disturbances, and, given sufficient time, will adopt a shape appropriate to an equivalent rotating fluid. This much it has in common with the elasto-viscous model; but there are any number of laws which have such an asymptotic behavior without requiring that the time constant of nutational damping be identified with the time constant of polar wandering. SCHEIDEGGER [12], for example, suggests that for time intervals of hours to 15,000 years the Earth behaves as a »Kelvin body« (spring and dashpot in series) with a »viscosity« of 10^{17} cm^2 sec^{-1}, whereas for time intervals larger than 15,000 years it behaves as a »Bingham body« (finite yield strength) with viscosity of 2×10^{21} cm^2 sec^{-1}. For a 10 years damping time then the pole requires $10^5 \times (2 \times 10^{21}/10^{17}) = 2 \times 10^9$ years to migrate to Hawaii; there is no discrepancy with astronomic and paleo-magnetic observation, in fact there is no polar wandering in the usual sense. — In all events some very serious doubts concerning the elasto-viscous model have been raised by CARL ECKART during a symposium at the Massachusetts Institute of Technology in September 1956, and these have their roots in the distinction between »deformation« and »strain« (ECKART, [3]). More recently KNOPOFF and MACDONALD (in press) have proved that no possible combination of springs and dashpots can adequately portray the damping of seismic waves. It appears that the extrapolation from nutational damping to polar wandering is on shaky ground. Our dilemma is resolved if the time constant of polar wandering is increased by a factor 10^3 over what was used in this paper; if the factor is larger than 10^5 there can be no polar wandering.

Appendix

At time 0 the pole of rotation is at

$$x=0, \qquad y=0, \qquad z=r_0.$$
$$x'=\gamma_1 r_0, \qquad y'=\gamma_2 r_0, \qquad z'=\gamma_3 r_0$$

in terms of present coordinates and principal axes respectively. After a time δt the pole is at

$$x=\delta x, \qquad y=\delta y, \qquad z=r_0,$$

and

$$x'=\gamma_1 r_0+\alpha_1\,\delta x+\beta_1\,\delta y,$$

with similar expressions for y', z'. The direction cosines of the new axis are of the form $\gamma_1+\delta\gamma_1$, so that

$$x'=(\gamma_1+\delta\gamma_1)\,r_0.$$

Hence

$$\alpha_1\,\delta x+\beta_1\,\delta y=r_0\,\delta\gamma_1$$
$$\alpha_2\,\delta x+\beta_2\,\delta y=r_0\,\delta\gamma_2$$
$$\alpha_3\,\delta x+\beta_3\,\delta y=r_0\,\delta\gamma_3$$

We shall need to express the products of inertia in terms of the principal system

$$\begin{aligned} D&=\sum m\,y\,z=\sum m(\beta_1 x'+\beta_2 y'+\beta_3 z')\,(\gamma_1 x'+\gamma_2 y'+\gamma_3 z')\\ &=\sum m\,\beta_1\gamma_1 x'^2+\sum m\,\beta_2\gamma_2 y'^2+\sum m\,\beta_3\gamma_3 z'^2\\ &=\tfrac{1}{2}[\beta_1\gamma_1(B'+C'-A')+\beta_2\gamma_2(C'+A'-B')+\beta_3\gamma_3(A'+B'-C')]\\ &=\beta_1\gamma_1\,(C'-A')+\beta_2\gamma_2\,(C'-B'). \end{aligned} \tag{9a}$$

Similarly

$$E=\alpha_1\beta_1\,(C'-A')+\alpha_2\gamma_2\,(C'-B') \tag{9b}$$

Combining (6) (8) and (9) and setting $K=I/(C_0-A_0)$ gives

$$\frac{\delta\gamma_1}{\delta t}=-\frac{K\alpha_1}{\tau}e-\frac{K\beta_1}{\tau}d$$

$$= -\frac{K}{\tau}[\gamma_1(\alpha_1^2+\beta_1^2)(c'-a')+\gamma_2(\alpha_1\alpha_2+\beta_1\beta_2)(c'-b')]$$

$$= -\frac{K}{\tau}[\gamma_1(1-\gamma_1^2)(c'-a')-\gamma_2(\gamma_1\cdot\gamma_2)(c'-b')]$$

$$\frac{\delta\, ln\, \gamma_1}{\delta t} = \frac{K}{\tau}[(\gamma_1^2-1)(c'-a')+\gamma_2^2(c'-b')]$$

Similarly

$$\frac{\delta\, ln\, \gamma_2}{\delta t} = \frac{K}{\tau}[\gamma_1^2(c'-a')+(\gamma_2^2-1)(c'-b')]$$

$$\frac{\delta\, ln\, \gamma_3}{\delta t} = \frac{K}{\tau}[\gamma_1^2(c'-a')+\gamma_2^2(c'-b')]$$

and

$$\frac{\delta}{\delta t}(ln\,\gamma_2 - ln\,\gamma_1) = \frac{K}{\tau}[(c'-a')-(c'-b')] = -\frac{K}{\tau}(a'-b')\,. \qquad (10)$$

But $\gamma_2/\gamma_1 = \tan\Phi'$, so that

$$\tan\Phi' = \tan\Phi_0'\, e^{-K(a'-b')t/\tau} \qquad (11)$$

where Φ_0' designates the east longitude of the pole (referred to principal axes) at the present time, $t=0$. From the last two equations

$$\frac{\delta}{\delta t}\, ln\,(\gamma_3/\gamma_2) = \frac{K}{\tau}(c'-b')$$

Dividing by (10) yields

$$\frac{\delta\, ln\,(\gamma_3/\gamma_2)}{\delta\, ln\,(\gamma_2/\gamma_1)} = \frac{b'-c'}{a'-b'} = k-1$$

which reduces to equation (7) in the text.

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REFERENCES

1. BONDI, H. and T. GOLD, 1955: On the damping of the free nutation of the earth, *Mon. Not. Roy. Ast. Soc.*, **115**, 41–46.
2. BURGERS, J. M., 1955: Rotational motion of a sphere subject to visco-elastic deformation, *Proc. Konikl. Ned. Akad. Wetenshap. B*, **58**, 219–237.
3. ECKART, C., 1948: The theory of elasticity and anelasticity, *Phys. Rev.*, **73**, 373–382.
4. GOLD, T., 1955: Instability of the Earth's axis of rotation, *Nature*, **175**, 526–534.
5. GUTENBERG, B., 1951: *Internal constitution of the Earth*, 2nd Ed., Dover.
6. INGLIS, DAVID R., 1957: Shifting of the Earth's axis of rotation, *Rev. of Mod. Physics*, **29**, 9–19.
7. JARDETZKY, W., 1949: On the rotation of the Earth during its evolution, *Trans. Amer. Geophys. Union*, **30**, 797–817.
8. JEFFREYS, SIR HAROLD, 1952: *The Earth*, Cambridge Press.
9. KNOPOFF, L. and G. MACDONALD: The attenuation of small amplitude stress waves in solids, *Rev. of Mod. Physics*, (in press).
10. MILANKOVITCH, M., 1934: Der Mechanismus der Polverlagerungen und die daraus sich ergebenden Polbahnkurven, *Gerlands Beitr. z. Geoph.*, **42**, 70–97.
11. RUDNICK, P., 1956: The spectrum of the variation in latitude, *Trans. Amer. Geophys. Union*, **37**, 137–142.
12. SCHEIDEGGER, A. E., 1957: Rheology of the Earth, *Canadian J. of Physics*, **35**, 383–397.
13. WALKER, A. M. and A. YOUNG, 1957: Further results on the analysis of the variation of latitude, *Mon. Not. Roy. Astr. Soc.*, **117**, 119–141.
14. WORZEL, J. L. and G. L. SHURBET, 1955: Gravity interpretations from standard oceanic and continental section, *Geol. Soc. Amer. Spec. Paper* **62**, 87–100.