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Self-Similarity Concept in Marine System Modelling

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Abstract

The self-similarity concept for sea temperature was firstly introduced by Kitaigorodskii and Miropolski (1970). They found that a non-dimensional temperature is only dependent on a non-dimensional vertical coordinate. In our paper it is shown that also the flux quantities of marine system variables (salinity, temperature, buoyancy, oxygen, nutrients etc.) have a self-similar vertical structure. The derivation of the equations for the fluxes allows us to present the fully 3-dimensional problem in the sea by using the self-similarity concept. The marine system variables mentioned above seem to create self-similar structure in the sea: we suppose that the currents destroy it, because no self-similarity profiles for currents have been found.

Key words: self-similarity, modelling, turbulence, Baltic Sea

1 . Introduction

The dynamics of the marine system is characterized by processes covering a wide range of spatial and temporal time scales. *Nihoul* and *Djenidi* (1987) presented that the marine system can be described by fairly well-defined *"spectral windows"* i.e. domains of length scales and time scales associated with identified phenomena. The transfer of energy between windows is effected by non-linear interactions. The variability in the sea is described by three main categories: **small-scale, marine weather and long-term processes.** The small-scale processes are including microscale, mesialscale and mesoscale processes.

The microscale processes include 3-dimensional turbulence and surface waves. The time scale is from seconds to minutes. The mesialscale processes are formed by internal waves and by microstructure "bliny" turbulence. The time scale of these processes is from minutes to hours. The mesoscale processes include inertial oscillations, which have for example a time scale of 13-14 hours in the Baltic Sea, tides and storm surges. The ***marine weather*** describes *diurnal* and *synoptic variations.* These processes have a characteristic time scale from a day up to some weeks. The **long-term** processes can be separated to **seasonal** and **global** variations. The seasonal variations are formed by the seasonal changes of the atmospheric motion and the global variations are characterized by climate change processes.

The marine weather and the seasonal variability form a two-layer vertical structure; a quasihomogeneous layer with intense turbulence and a stratified layer with intermittent turbulence. The vertical structure of marine weather and long-term processes is characterized by a so-called self-similarity profile. It means that a self-similarity structure can be found from measurements of different marine system variables only if an averaging over small-scale oscillations is carried out.

The background of self-similarity concept comes from nondimensional analysis. A classical scientist as von Karman has dealt with such an analysis in studies of flows in a pipe. A good review of nondimensional analysis and self-similarity concept in general has been given by *Barrenblatt* (1975).

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The self-similarity of an unknown variable (for example temperature T in the pycnocline layer) in a two-dimensional coordinate system will be described in a non-dimensional form:

$$\theta = \frac{T_s(t) - T(z,t)}{T_s(t) - T_H} \tag{1}$$

by a non-dimensional coordinate:

$$\xi = \frac{z - h(t)}{H - h(t)} \tag{2}$$

So,

$$\theta = f(\xi) \tag{3}$$

where:

$T_s(t)$- temperature in the upper mixed layer, $T(z,t)$ -temperature profile in the vertical direction, T_H -temperature at the lower boundary of the ocean active layer, which is approximated to be constant, z - the vertical coordinate, $h(t)$ - the thickness of the upper mixed layer, H - the depth of the ocean active layer, mime.

In this way a two-dimensional problem becomes as a one-dimensional problem. So, by using the self-similarity approach we can reduce the dimension number of the problem. The self-similarity in data analysis means that the data measurements in nondimensional coordinates can be described by a single curve.

2. *The birth and development of self-similarity concept*

More than two decades ago *Kitaigorodskii* and *Miropolski* (1970) published the first paper, where the vertical structure of the ocean temperature was solved in terms of the self-similarity concept. By using this concept they were able to calculate the seasonal variations of the thickness of quasi-homogeneous layer and of the vertical temperature profile in the seasonal thermocline.

Kitaigorodskii and *Miropolski* (1970) summarized their new founding in the following way. The depth of the ocean active layer is about 200-250 m. There is at first the quasihomogeneous upper layer, thickness of which has a great variability in the function of time. Below the quasihomogenous layer, a thermocline layer exists, where the temperature falls sharply. According to the abovementioned vertical structure *Kitaigorodskii* and *Miropolski* (1970) concluded: in the upper mixed layer the temperature cannot change with depth and it is equal to the surface temperature. The vertical water temperature profile in the thennocline can be described by a nondimensional temperature 0, which only depends on a non-dimensional vertical coordinate ξ (see equation 4).

Verification of this hypothesis and determination of the function $\theta=\theta(\xi)$ can be based on measurements of the temperature profile over a long time interval. An approximate analytical expression for $\theta(\xi)$ can be found by using a method similar to that of Karman and Polhausen in boundary layer theory.

Kitaigorodskii and *Miropolski* (1970) got:

$$\theta(\xi) = 8/3\xi - 2\xi^2 + 1/3\xi^4 \tag{4}$$

They studied the turbulent exchange through the thermocline in two separate cases: firstly, when the thermocline "locks" the heat flux coming from above. In that case the turbulence in the thermocline is intermittent from its origin and the principal source of turbulence comes from the breakdown of internal waves. Secondly, they studied the case, where the heat flux is continuos across the boundary of the upper mixed layer and the thermocline. By using these two alternatives, different evolution equations forthe thickness of the upper mixed layer were derived.

Miropolski et al. (1970) found average monthly dimensionless temperature profiles for two ocean stations "Papa" and "Tango". They concluded that the internal consistency of the universal structures is mostdistinct in July-September, when the seasonal thermocline is developed. The great scatter of points in the profile in winter can be explained by the great thickness of the quasi-homogeneous layer and by the weak seasonal thermocline. *Kharkov* (1977) developed a parameterization for the two-layer structure of the upper ocean layer. He proposed a relationship between the homogeneous layer thickness and the thermocline thickness on the basis of laboratory and observational data on the ocean temperature field. The study by *Kharkov* (1977) confirmed the universal nature of the temperature distribution in the upper thermocline. *Kamenkovich* and *Kharkov* (1975) studied the parameterization of the vertical eddy flux during the seasonal changes of the thermal structure of the ocean. They used a three-layer model which consisted of the upper homogeneous layer, the seasonal thennocline and the main deep ocean. The model was a one-dimensional one. Two separate cases were investigated: the mixed layer is increasing (entrainment) and the mixed layer is decreasing (detrainment). The computations of temperature, turbulent heat fluxes and the thickness of the upper mixed layer were compared with measurements with promising results. *Arsenejev* and *Felsenbautn* (1977) found a simple polynomial expression for self-similarity:

$$\theta = 1 - (1-\xi)^3 \qquad (5)$$

Reshetova and *Chalikov* (1977) extended the self-similarity hypothesis for the first time to salinity. They calculated self-similarity profiles for salinity according to measurements carried out in the Pacific Ocean. According to *Reshetova* and *Chalikov* (1977) the results indicate that the dimensionless salinity and density have a tendency to group along the universal profile with a variance which is appreciable less than the characteristic gradient within the entire active ocean layer. However, the results were re-examined later on and it became out that the scatter of points on the empirical curves was too large. The idea of self-similarity became doubtful.

Linden (1975) carried out laboratory investigations by using a rectangular tank. There was initially a two-layer vertical structure in the tank. The upper layer was well-mixed and below it there was a layer with a constant density gradient. Turbulence was produced in the tank by oscillating a horizontal grid with a stroke. After several experiments an average, non-dimensional vertical structure for density was found. *Linden (1975)* found out that it had a very similar appearance as those structures for temperature calculated by *Kitaigorodskii* and *Miropolski* (1970). Actually, the profile (4), which was found by *Kitaigorodskii* and *Miropolski* (1970), is valid only for the case when the mixed layer is increasing (entrainment). The physical conditions in the laboratory experiments carried out by *Linden* (1975) presented not the case of entrainment. The profile found by him has a resemblance to the new profile (7) found by *Tamsalu* (1982) and by *Mälkki* and *Tamsalu* (1985) described later in this section.

The physical background of self-similarity has been investigated by several scientists. In 1970s *Barrenblatt* (1978) and *Turner* (1978) made the first studies. *Barrenblatt* (1978) concluded that in the case of mixed layer increasing, the thermocline is treated as a quasistationary thermal and diffusion wave. According to *Barrenblatt* (1978) and *Turner* (1978) it is likely that the energy needed to prevail the sharp gradient below the surface layer in the upper thermocline will be supplied by the breaking of the internal waves. *Zilitinkevich* and *Rumjantsev* (1990) concluded as follows. Effective heat conductivity K in thermocline is much higher than the molecular conductivity and that K increases when $-\partial T/\partial z$ increases (not the well-known inversely dependence of K on $-\partial T/\partial z$. The direct dependence between K and $-\partial T/\partial z$ can be explained in the following way: The disturbances at the lower boundary of mixed layer generate internal waves which propagate downwards and therefore transfer kinetic energy downwards. The occurrence of breaking is more likely when high temperature gradients exist. In this way, turbulence "spots" are generated i.e. the waves expend a part of their energy for the generation of intermittent turbulence. The theory is similar to *Turner's* (1978) but *Zilitinkevich* and *Runtjantsev* (1990) expressed that the role of buoyancy should also be taken into account, so that the mechanism would work.

Zilitinkevich and *Rutnjantsev* (1990) and *Mironov* et. al (1991) pointed out that processing of oceanic data *(Miropolski et al.,* 1970, *Reshetova* and *Chalikov,* 1977) revealed so great scatter of points on the empirical Curves θ(ξ) that concept of self-similarity of the thermocline became doubtful. *Tamsalu* (1982), *Mälkki* and *Tamsalu* (1985), by using the measured data of *Nõmm* (1988), found that the self-similarity profile strongly depends on the evolution of the mixed layer thickness. There are two different self-similarity structures: firstly, the case of entrainment when the homogeneous layer is deepening (storm) and secondly, the case when the mixed layer is decreasing (storm subside). Thus, the similarity function in the thermocline (more generally in pycnocline) has the following forms:

$$\theta_1(\xi) = 1 - (1-\xi)^3 \qquad (6)$$

when the mixed layer is increasing

$$\theta_2(\xi) = 1 - 4(1-\xi)^3 + 3(1-\xi)^4 \qquad (7)$$

when the mixed layer is decreasing (see Figure 1).

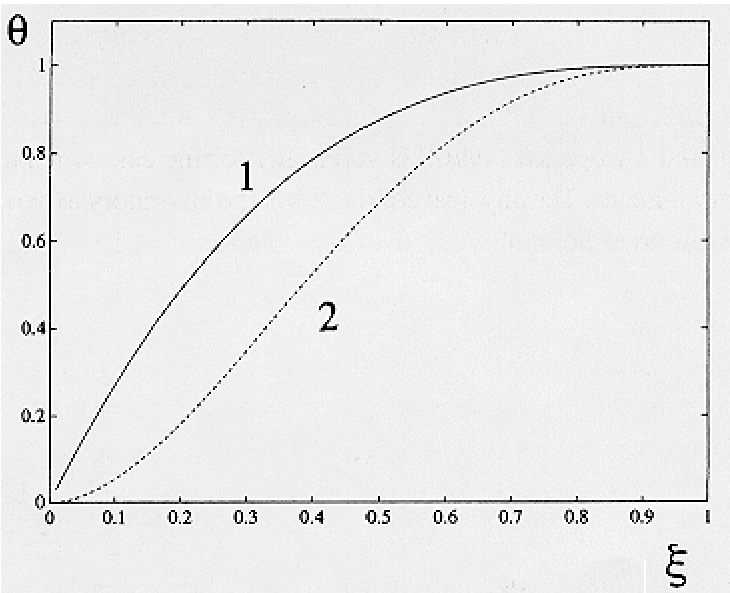

Fig. 1. Self-similarity structure for buoyancy θ as a function of ξ. Curve 1 - mixed 1ayer depth is increasing; curve 2 - mixed layer depth is decreasing.

In the real situation these two profiles are mixed, so the observations situate between these two curves.

We can suppose that in three-dimensional concept self-similarity of vertical fluxes is of primarily importance and that a self-similarity profile for vertical fluxes can be found at short time and space scales; i.e. in the turbulent scale of motion. So, the self-similarity of different marine system variables is only a product of the flux -self-similarity. On the other hand, self-similarity profiles have not been found for currents. The role of currents seems to be to destroy self-similarity structure. That's probably why the self-similarity can be found for marine system variables only if an averaging over the small scale is carried out. Thus, the destroying effect of currents is smoothed out.

The heat (buoyancy) flux self-similarity concept was proposed by. *Leonov* and *Miropolski* (1977) and by *Tamsalu* (1982). In terms of traditional self-similarity concept and heat transfer equation *Zilitinkevich* and *Mironov* (1992) studied vertical fluxes through the thermocline. They developed a model of heat transfer in thermocline from considerations of turbulent energy budget and from expressions for effective heat conductivity, which is based on dimensional arguments using buoyancy parameter, temperature gradient and turbulent length scale as governing parameters. This energy balance model is applicable to the mixed layer deepening as well as for its steady state and collapse. In the next section, a new approach in self-similarity theory will be shown; the self-similarity profiles for vertical turbulent fluxes are derived.

3. *Derivation of self-similarity profile for flux quantities*

With the linear version of the equation of state, the buoyancy can be written as follows.

$$\frac{\partial \bar{b}}{\partial t} + u\frac{\partial \bar{b}}{\partial x} + v\frac{\partial \bar{b}}{\partial y} + w\frac{\partial \bar{b}}{\partial z} + \frac{\partial}{\partial x}\langle u'b'\rangle + \frac{\partial}{\partial y}\langle v'b'\rangle + \frac{\partial}{\partial z}\langle w'b'\rangle = \frac{\alpha_T}{c_p\rho_0}\frac{\partial I}{\partial z} \quad (8)$$

The equation of continuity has the following form:

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \quad (9)$$

where:

u, v, z are velocity components in x, y and z-directions, $b = g\frac{\rho - \rho_0}{\rho_0} - b$ - buoyancy, $\langle u'b'\rangle$ - x-component of macroturbulence, $\langle v'b'\rangle$ - y-component of macroturbulence, $\langle w'b'\rangle$ - microturbulence, α_T - constant, ρ_0 - mean density, I - penetrative component of solar radiation.

Because the vertical structure of the sea has a clearly formed two-layer system, we write the equations (8) and (9) in a new coordinate system:

$$t = t', x = x', y = y', \xi_k = \frac{z_k - h_k}{D_k}; k = 1,2 \quad (10)$$

$h_1 \le z_1 \le h_2$; $h_2 \le z_2 \le H$

where:
h_1 - sea-level deviation
h_2- mixed layer thickness
H - sea depth (thickness of active or seasonal thermocline)
$D_1 = h_2 - h_1$; $D_2 = H - h_2$
By using (10) we can write equations (8) and (9) as follows:

$$D_k\left[\frac{\partial b_k}{\partial t} + u_k\frac{\partial b_k}{\partial x} + v_k\frac{\partial b_k}{\partial y}\right] + \omega_k\frac{\partial b_k}{\partial \xi_k} +$$

$$\frac{\partial}{\partial x}\langle u_k' b_k' D_k\rangle + \frac{\partial}{\partial y}\langle v_k' b_k' D_k\rangle + \frac{\partial q_k}{\partial \xi_k} - \frac{\partial \delta_k \partial b_k}{\partial t \partial \xi_k} = \frac{\partial T}{c_p\rho_0}\frac{\partial I}{\partial \xi_k} \quad (11)$$

where:

$$q_k = \langle w_k' b_k'\rangle - \langle u_k' b_k'\rangle\frac{\partial \xi_k}{\partial x} - \langle v_k' b_k'\rangle\frac{\partial \xi_k}{\partial y}$$

$$\omega_k = w_k - u_k\frac{\partial \xi_k}{\partial x} - v_k\frac{\partial \xi_k}{\partial y}$$

$\delta_k = h_k + \xi_k D_k$

$k = 1,2$

For continuity we have:

$$\frac{\partial u_k D_k}{\partial x} + \frac{\partial v_k D_k}{\partial y} + \frac{\partial \omega_k}{\partial \xi_k} = 0 \qquad (12)$$

We will solve equation (11) by the split-up method *(Marchuk,* 1975). In the first-order accuracy in time $t_i \leq t \leq t_{i+1/2}$ we will solve the equation:

$$D_k\left[\frac{\partial b_k}{\partial t} + u_k\frac{\partial b_k}{\partial x} + v_k\frac{\partial b_k}{\partial y}\right] + \omega_k\frac{\partial b_k}{\partial \xi_k} + \frac{\partial}{\partial x}\left\langle u_k' b_k' D_k\right\rangle + \frac{\partial}{\partial y}\left\langle v_k' b_k' D_k\right\rangle = 0 \qquad (13)$$

In the second order accuracy in time $t_{i+1/2} \leq t \leq t_{i+1}$ we will solve equation:

$$D_k\frac{\partial b_k}{\partial t} + \frac{\partial q_k}{\partial \xi_k} - \frac{\partial \delta_k}{\partial t}\frac{\partial b_k}{\partial \xi_k} = \frac{\alpha_T}{c_p\rho_0}\frac{\partial I}{\partial \xi_k} \qquad (14)$$

We can suppose that equation (14) develops self-similarity structure and equation (13) destroys it. Thus, for determination of the flux structure we will concentrate our interest in to equation (14).

We can write equation (14) for the upper mixed layer and for the pycnocline layer.

For the mixed layer we have:

$$D_1\frac{\partial b_1}{\partial t} + \frac{\partial q_1}{\partial \xi_1} = \frac{\alpha_T}{c_p\rho_0}\frac{\partial I}{\partial \xi_1}; \frac{\partial b_1}{\partial \xi} \equiv 0 \qquad (15)$$

For the pycnocline layer we have:

$$D_2\frac{\partial b_2}{\partial t} - \frac{\partial \delta_2}{\partial t}\frac{\partial b_2}{\partial \xi_2} + \frac{\partial q_2}{\partial \xi_2} = 0; \frac{\partial I}{\partial \xi} \equiv 0 \qquad (16)$$

We have the following conditions:

$$\begin{aligned}
&\xi_1 = 0 \quad q_1 = \alpha_T\langle w'T'\rangle - \beta_s\langle w'S'\rangle = q^0 \\
&\xi_1 = 1 \text{ and } \xi_2 = 0 \quad q_1 = q_2 = q^h \qquad (17) \\
&\xi 2 = 1 \qquad q_2 = 0
\end{aligned}$$

$$D_1\int_0^1 q_1 d\xi_1 = m_1 u_*^3; q^h = \alpha_b(b_1 - b_H)\frac{\partial h_2}{\partial t} \quad \text{if } \frac{\partial h_2}{\partial t} > 0 \qquad (18)$$

$$q_h = 0 \qquad \int_0^1 q_2 d\xi_2 = \bar{q} \qquad \text{if } \frac{\partial h2}{\partial t} \leq 0 \qquad (19)$$

where T' is a fluctuation of temperature, S' is a fluctuation of salinity, βs is a constant, u_* is the friction velocity of wind and m_1 is a experimental coefficient (m_1=1.06, *Niiler* and *Kraus, 1977).* Buoyancy *flux* q_k will be written by using (17) as follows:

$$q_1 = q^0\left[1 - Q_1(\xi_1)\right] + Q_1(\xi_1)q^h \qquad (20)$$

$$q_2 = q_h\left(1 - Q_2(\xi_2)\right), \text{ if } \frac{\partial h_2}{\partial t} > 0; q_2 = -\bar{q}\, Q_2(\xi_2) \text{ if} \frac{\partial h2}{\partial t} < 0 \quad (21)$$

When the depth of the mixed layer is decreasing, the buoyancy flux between the mixed layer and the pycnocline becomes to zero and turbulence in the pycnocline layer is formed by breakdown of internal waves. We propose here that the internal waves in the stratified layer are formed by the dynamics of the mixed layer:

$$\bar{q} = D_2 \frac{\partial b_1}{\partial t} \qquad (22)$$

The main problem in the approximations (20)-(21) is the nondimensional function $Q_k(\xi_k)$. In the upper quasihomogeneous layer $Q_1(\xi_1)$ is a linear function of coordinate (see for example *Nifler,* 1975).

$$Q_1(\xi_1) = \frac{\delta_1}{D_1} = (h_1 + \xi_1 D_1)/D_1 \qquad (23)$$

The determination of $Q_2(\xi_2)$ is still an unknown function and thus it is a main topic of this article. After integration of (15) and (16) in vertical direction (see appendix 1) we get by using (17)-(21)

$$\frac{\partial b_1}{\partial t} = \frac{R_b}{D_1}; \frac{\partial b_H}{\partial t} = -\varepsilon_H \frac{R_b}{D_1}; \frac{\partial h_2}{\partial t} = -\varepsilon_h \frac{D_2}{b_1 - b_H}\frac{R_b}{D_1} \qquad (24)$$

$$q_h = -\varepsilon_q D_2 \frac{R_b}{D_1} if \frac{\partial h_2}{\partial t} > 0; \bar{q} = -\varepsilon_q D_2 \frac{R_b}{D_1} if \frac{\partial h_2}{\partial t} \le 0$$

where:

$$R_b = 2\left(q_0 - \frac{m_1 u_*^3}{D_1} + \frac{\alpha_T I_0}{c_p \rho_0}\left(1 + \frac{1}{\gamma D_1} e^{-\gamma D_1}\right)\right) if \frac{\partial h2}{\partial t} > 0$$

$$R_b = \left(q_0 + \frac{\alpha_T I_0}{c_p \rho_0}\right) \qquad \text{if } \frac{\partial h2}{\partial t} \le 0$$

g is the attenuation coefficient of solar radiation.

In the calculation of the depth of the mixed layer the equation of turbulent energy has been used in rather simple forms of the turbulent energy equation has been developed *(Garnich* and *Kitaigorodskii,* 1977, 1978; *Kitaigorodskii,* 1979) where e.g. the effects of the breaking of surface wind waves in the mixed layer has been taken into account.

By substituting (23) and (A 1.17) and (A 1.23) to (21) we find the following equation for $q(\xi)$:

$$q_2 = -2D_2 \frac{R_b}{D_1} \varphi_1(\xi_2) \qquad \text{if } R_b < 0\left(\frac{\partial h_2}{\partial t} > 0 \right) \qquad (25)$$

$$q_2 = D_2 \frac{R_b}{D_1} \varphi_1(\xi_2) \qquad \text{if } R_b > 0\left(\frac{\partial h}{\partial t} \le 0 \right) \qquad (26)$$

where:

$$\varphi_1(\xi_2)=2/3(1-\xi_2) + 1/3(1-\xi_2)^4 \qquad \text{if } R_b < 0$$

$$\varphi_1(\xi_2)=1/12\xi_2(1-\xi_2)^4 \qquad \text{if } R_b \ge 0$$

For calculation *of* the buoyancy b_k we get in the second-order accuracy in time:

$$\frac{\partial b_2}{\partial t} + \frac{\sigma_2}{D_2} \frac{\partial b_2}{\partial \xi_2} = \varphi_2(\xi_2) \frac{R_b}{D_1} \qquad (27)$$

where:

$$\sigma_2 = \frac{2}{3} \frac{1-\xi_2}{b_h - b_H} D_2 \frac{R_b}{D_1}; \varphi_2(\xi_2) = -\frac{1}{3}\left[1 + 2(1-\xi_2)^3\right] \qquad \text{if} \frac{\partial h_2}{\partial t} > 0$$

$$b_h = b_{1_{\xi_1 = 1}} \quad \varphi_2(\xi_2) = -4(1-\xi)^3 + 5(1-\xi)^4 \qquad \text{if} \frac{\partial h2}{\partial t} \le 0$$

Substituting (27) and (28) to (13) and (14) we get the following equation for determination of buoyancy without the splitting-up method; where the destroying effect of self-similarity by dynamics will be included:

$$D_1\left[\frac{\partial b_1}{\partial t} + u_1 \frac{\partial b_1}{\partial x} + v_1 \frac{\partial b_1}{\partial y} \right] + (\omega_1 + \sigma_1) \frac{\partial b_1}{\partial \xi_1} - \frac{\partial}{\partial x} \mu D_1 \frac{\partial b_1}{\partial x} - \frac{\partial}{\partial y} \mu D_1 \frac{\partial b_1}{\partial y} =$$

$$q^0 - q^h + \frac{\alpha_T}{c_p \rho_0} \frac{\partial I}{\partial \xi_1} \qquad (28)$$

$$D_2\left[\frac{\partial b_2}{\partial t} + u_2 \frac{\partial b_2}{\partial x} + v_2 \frac{\partial b_2}{\partial y} \right] + (\omega_2 + \sigma_2) \frac{\partial b_2}{\partial \xi_2} - \frac{\partial}{\partial x} \mu D_2 \frac{\partial b}{\partial x} - \frac{\partial}{\partial y} \mu D_2 \frac{\partial b}{\partial y} =$$

$$\varphi_2(\xi_2) \frac{R_b}{D_1} + \frac{\alpha_T}{c_p \rho_0} \frac{\partial I_2}{\partial \xi_2} \qquad (29)$$

where:

$$\sigma_1 = \frac{2}{3} \xi_1 \frac{D_2}{b_h - b_H} \frac{R_b}{D_1} - (1-\xi_1) \frac{\partial h_1}{\partial t}$$

$$q^h = -2D_2 \frac{R_b}{D_1} \text{ if } \frac{\partial h_2}{\partial t} > 0 \text{ and } q^h = 0 \text{ if } \frac{\partial h2}{\partial t} \le 0$$

Where the traditional way for parameterization of macroturbulent mixing is used:

$$\langle u'_k b'_k \rangle = -\mu \frac{\partial b_k}{\partial x}; \qquad \langle v'_k b'_k \rangle = -\mu \frac{\partial h_k}{\partial y} \tag{30}$$

where μ is the macroturbulent coefficient.

By the same way it is possible to describe the equations for temperature, salinity and for the ecosystem components.

Substance equation for ecosystem component c_k, for example, has the following form:

$$D_k\left[\frac{\partial c_k}{\partial t} + u_k \frac{\partial c_k}{\partial x} + v_k \frac{\partial c_k}{\partial y}\right] + (\omega_k + \sigma_k)\frac{\partial c_k}{\partial \xi_k} - \frac{\partial}{\partial x}\mu D_k \frac{\partial c_k}{\partial x} + \frac{\partial}{\partial y}\mu D_k \frac{\partial c_k}{\partial y} = \tag{31}$$

$$\varphi_k(\xi_k)\frac{c_h - c_H}{b_h - b_H}\frac{R_b}{D_1} + G_k$$

where G_k describes biochemical reactions.

The whole problem will be calculated together with equations of motion, equations of salinity and temperature, equation of state, equation of continuity and equation of the mixed layer thickness (h_2).

4. *Conclusions*

The *25* years history of self-similarity concept in marine science includes many important milestones. In the work presented by *Kitaigorodskii* and *Miropolski* (1970) the existence of self-similarity of temperature in the Ocean was shown for the first time. However, some further studies of self-similarity concept showed that the measured profiles and those ones produced by using the self-similarity concept did not fit too well. The self-similarity concept became doubtful. The works by *Tanisalu* (1982) and by *Mälkki* and *Tantsalu* (1985) showed that the self-similarity profiles depend on the time evolution of the upper mixed layer.

In this paper we have derived a self-similarity profile for the vertical fluxes of turbulence. So, by finding out the self-similarity structure also for the turbulent fluxes, not only for single marine system variables, it becomes possible to solve the 3-dimensional fields for marine system variables.

Further on, we can suppose that the flux-self-similarity is actually of primarily importance and that the self-similarity of different marine system variables is only a product of the flux - self-similarity in the turbulent scale of motion. However, self-similarity structure has not been found for the vertical profiles of currents. It is most likely that currents destroy the self-similarity structure. This is an explanation why self-similarity profiles for marine system variables can be found only after averaging over the inertial period. Thus, the destroying effects of currents will be smoothed out.

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APPENDIX 1

By using the self-similarity structure for buoyancy and for buoyancy flux equation (16) has the following form, if H is not a function of time *t*:

$$D_2\left[(1-\theta)\frac{\partial b_1}{\partial t}+\theta\frac{\partial b_H}{\partial t}\right]+(b_1-b_H)(1-\xi_2)\frac{d\theta}{d\xi}\frac{\partial h_2}{\partial t}=q^*\frac{dQ}{d\xi_2} \quad \text{(A1.1)}$$

where $q^* = q^h$ if $\frac{\bar{\partial} h_2}{\bar{\partial} t} > 0$ and $q^* = \bar{q}$ if $\frac{\bar{\partial} h2}{\bar{\partial} t} \le 0$

We have the following boundary conditions:

$\xi_2=0 \quad \theta=0 \quad Q=0$

$\xi_2=1 \quad \theta=1 \quad Q = 1 \quad$ if $\frac{\bar{\partial} h_2}{\bar{\partial} t} > 0$ (A1.2)

$\xi_2=0 \quad \theta=0 \quad Q=1$

$\xi_2=1 \quad \theta=1 \quad Q = 1 \quad$ if $\frac{\bar{\partial} h2}{\bar{\partial} t} \le 0$ (A1.3)

Integrating equation (A.1.1) with respect to ξ_2 between the limits 0 and 1 and by using the boundary conditions (A.1.2) and (A.1.3) we obtain:

$$D_2\left[(1-\kappa_1)\frac{\partial b_1}{\partial t}+\kappa_1\frac{\partial b_H}{\partial t}\right]+\kappa_1(b_1-b_H)\frac{\partial h_2}{\partial t}=q_h \text{ if } \frac{\bar{\partial} h_2}{\bar{\partial} t}>0 \quad \text{(A1.4)}$$

$$D_2\left[(1-\kappa_1)\frac{\partial b_1}{\partial t}+\kappa_1\frac{\partial b_H}{\partial t}\right]+\kappa_1(b_1-b_H)\frac{\partial h_2}{\partial t}=0 \text{ if } \frac{\bar{\partial} h2}{\bar{\partial} t}\le 0 \quad \text{(A1.5)}$$

Double-integration (A1.1), first from 0 to ξ_2, then from 0 to 1, yields:

$$D_2\left[(1/2-\kappa_2)\frac{\partial b_1}{\partial t}+\kappa_2\frac{\partial b_H}{\partial t}\right]+2\kappa_2(b_1-b_H)\frac{\partial h_2}{\partial t}=m_2 q_h \text{ if } \frac{\bar{\partial} h_2}{\bar{\partial} t}>0 \quad \text{(A1.6)}$$

$$D_2\left[(1/2-\kappa_2)\frac{\partial b_1}{\partial t}+\kappa_2\frac{\partial b_H}{\partial t}\right]+2\kappa_2(b_1-b_H)\frac{\partial h_2}{\partial t}=\bar{q} \quad \text{if } \frac{\bar{\partial} h2}{\bar{\partial} t}\le 0 \quad \text{(A1.7)}$$

where:

$$\kappa 1=\int_0^1 \theta d\xi;\kappa 2=\int_0^1\int_0^\xi \theta d\xi d\xi; m_2=\int_0^1 Q d\xi$$

From *Mälkki* and *Tamsalu* (1985) we get:

$\kappa_1 = 0.75$; $\kappa_2 = 0.3$ and $m_2 = 0.6$ if $\frac{\bar{\partial} h_2}{\bar{\partial} t} > 0$ (A1.8)

$$\kappa_1 = 0.6; \quad \kappa_2 = 0.2 \qquad\qquad \text{if } \frac{\partial h2}{\partial t} \le 0 \qquad (A1.9)$$

By using condition (18) and equation (20) for the upper layer

$$\frac{\partial b_1}{\partial t} = \frac{R_b}{D_1} \qquad (A1.10)$$

we have the following equation for b_H, h_2 and q_h if $\frac{\partial h_2}{\partial t} > 0$ ($R_b < 0$).

$$\frac{\partial b_H}{\partial t} = -\varepsilon_H \frac{R_b}{D_1}; \frac{\partial h_2}{\partial t} = -\varepsilon_h \frac{D_2}{b_1 - b_H} \frac{R_b}{D_1}; q_h = -\varepsilon_q D_2 \frac{R_b}{D_1} \qquad (Al.11)$$

where:

$$\varepsilon_h = \left[\frac{\kappa_1 / 2 - \kappa_2}{\kappa_1 \kappa_2 - \alpha_b (\kappa_1 m_2 - \kappa_2)} \right]; \varepsilon_H = \frac{\alpha_b (1/2 - \kappa_2 - m_2 (1 - \kappa_1))}{\kappa_1 \kappa_2 - \alpha_b (\kappa_1 m_2 - \kappa_2)} \varepsilon_H = \alpha_b \varepsilon_h$$

The proportional coefficient α_b is still unknown. Let's write Eq. (A1.4) in an other form.

$$D_2 \frac{\partial \bar{b}_2}{\partial t} + (b_1 - \bar{b}_2) \frac{\partial h_2}{\partial t} = \frac{\alpha_b}{\kappa_1} (b_1 - \bar{b}_2) \frac{\partial h_2}{\partial t} \qquad (A1.12)$$

We got from Krauss (1981) that before and after a storm, during which the temperature decreased about 4 degrees and the upper mixed layer deepened more that 10 meters, the mean temperature of the pycnocline layer $\bar{T}_2$ retained nearly as constant (see Figure 2).

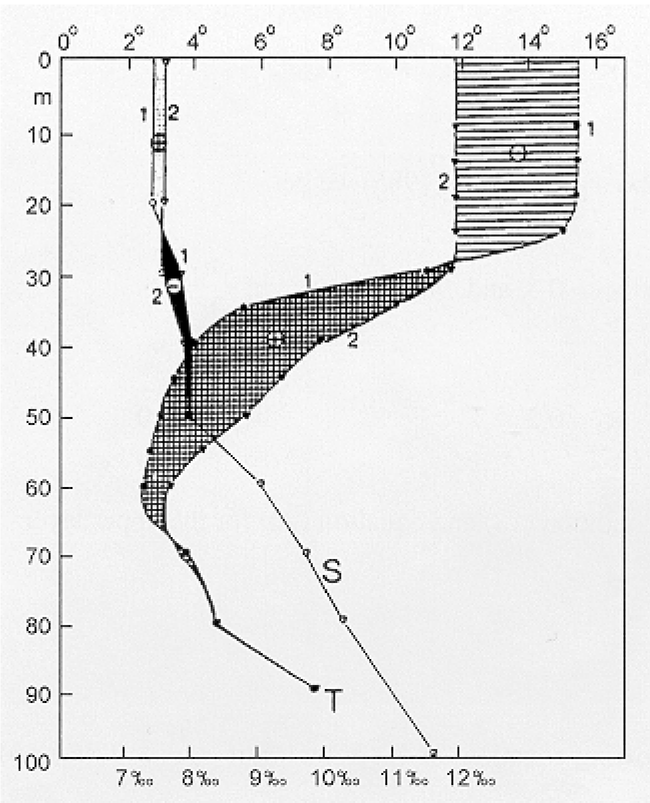

Fig. 2. Temperature and salinity profiles: (1)-before a storm and (2)-after a storm (from *Krauss*, 1981).

$$\frac{\partial \bar{b}_2}{\partial t} \approx 0 \qquad \text{(A1.13)}$$

It means, that

$$a_b = \kappa_1 \qquad \text{(A1.14)}$$

By using (A1.8) and (A1.10) we have in the case that the mixed layer is increasing $\left(\frac{\partial h_2}{\partial t} > 0\right)$:

$$\varepsilon_h = 2/3 \quad \varepsilon_H = 1/3 \quad \varepsilon_q = 1/2 \qquad \text{(A1.15)}$$

Substituting (A1.11) to (16) and by using (A1.15) we have the following equation for the determination of Q:

$$1-(1+\varepsilon_H)\theta - \varepsilon_h(1-\xi_2)\frac{d\theta}{d\xi_2} = -\varepsilon_q \frac{dQ}{d\xi_2} \qquad \text{(A1.16)}$$

After integration of (A1.16) with respect to ξ_2, and by using condition (A1.2), we get:

$$Q(\xi_2)=1-2/3(1-\xi)-1/3(1-\xi)^4 \qquad \text{(Al.17)}$$

In case, when the mixed layer is decreasing, ($\partial h/\partial t \leq 0$ and $R_b > 0$) we propose that:

$$\bar{q} = D_2 \frac{R_b}{D_1} = \alpha_q D_2 \frac{R_b}{D_1} \qquad \text{(A1.18)}$$

where α_q is a coefficient of proportionality.

Then, by using conditions (19) and (A.10), we get the following equations for b_H and h_2:

$$\frac{\partial \bar{b}_H}{\partial t} = -\varepsilon_H \frac{R_b}{D_1}; \frac{\partial \bar{h}_2}{\partial t} = -\varepsilon_h \frac{D_2}{b_1 - b_H}\frac{R_b}{D_1} \qquad \text{(Al.19)}$$

where:

$$\varepsilon_h = \frac{\kappa_1/2 - \kappa_2 - \alpha_q \kappa 1}{\kappa_1\kappa_2}; \varepsilon_H = \frac{2\kappa_2 - \kappa_1/2 - \kappa_1\kappa_2 + \alpha_q\kappa_1}{\kappa_1\kappa_2}$$

if $\kappa_1 = 0.6$; $\kappa_2 = 0.2$ (see *Mälkki* and *Tamsalu*, 1985) then:

$$\varepsilon_h = 5/6-5\alpha_q; \quad \varepsilon_H = -1/6 + 5\alpha_q \qquad \text{(A1.20)}$$

In case of stable stratification and Ri numbers above critical, the principal source of turbulence is the process of breaking of internal waves. This is called as the locking effect of turbulence. The internal waves have a small amplitude in the bottom layer. Thus, the changes of the bottom temperature (buoyancy) are negligible. In the first approximation we take, that:

$$e_H = 0 \qquad \text{(A1.21)}$$

Then we have, that:

α_q = - 1/30 and ε_hm = 2/3 (A1.22)

By using (A1.21) and (A1.22) we have the following equation for Q, in the case when the mixed layer is decreasing:

$$Q = 12\xi\,(1-\xi)^4 \qquad \text{(A1.23)}$$

The distribution of non-dimensional turbulent flux Q is presented in Figure 3 in cases of increasing (A1.17) and decreasing (A1.23) of the mixed layer depth.

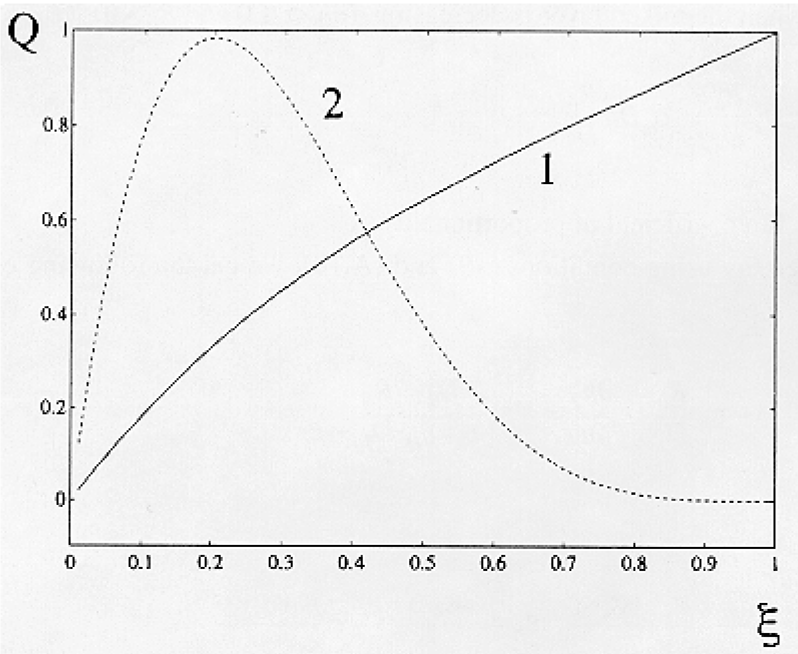

Fig. 3. Self-similarity structure for buoyancy flux Q as a function of ξ. Curve 1-mixed layer depth is increasing; curve 2 - mixed layer depth is decreasing.