Extending the Similarity Theory for Atmospheric Boundary Layers: Contribution from Background Stratification

Sylvain M. Joffre

Finnish Meteorological Institute
P.O. Box 503
FIN-00101 Helsinki, Finland

Abstract

The structure of the atmospheric boundary layer has traditionally been described using only surface fluxes and the Coriolis parameter. We review here the proposal of Kitaigorodskii (1988) and Kitaigorodskii and Joffre (1988) that the background stratification of the atmosphere can influence the overall properties of the boundary layer through the process of entrainment. Theoretical predictions have been successfully compared to various data sets.

Key words: Atmospheric boundary layer, similarity theory, turbulence, stratification effects

1 Introduction

The atmospheric boundary layer (ABL) is the layer where interactions take place between the surface, which captures most of the solar energy and redistributes it under different forms, and the large-scaled atmospheric flow which absorbs them in the end. This turbulent ABL is of great importance for transfer of momentum, heat and atmospheric constituents, dispersion of pollutants, for enhancing evaporation over the sea, in creating wind waves on water surface and producing drift currents in the upper layer of the water bodies. More recently, it has been recognized that transfers of many minor constituents (SO2, NOx, CO2, O3, C2H4) which often enter the atmosphere or are deposited to terrestrial and aquatic ecosystems via the ABL is one of the main link in the global biogeochemical cycles.

It is thus of primary importance to be able to understand, parametrize and simulate the structure and behaviour of the ABL. The classical way of describing its structure is through similarity theories (eg. Kazanski and Monin, 1960) where the only influencing agents are rotation and buoyancy. There are, however, other factors such as entrainment, the intensity of which is connected with background stratification. The height h of the ABL is the key parameter embedding all internal and external influences and having much importance for a host of applications is. Predictions of the ABL height has many theoretical and practical applications such as predictions of pollutant concentrations or of surface temperature. We will review in this paper new theoretical and empirical arguments proposed by Kitaigorodskii (1988) and Kitaigorodskii and Joffre (1988) for extending the similarity theory describing the height of the ABL.
2. Theoretical background

We assume an ABL structure where the external forcing parameters are the Coriolis parameter $f$, the surface roughness length $z_0$, and the background temperature lapse rate $\gamma = \partial \theta / \partial z$ above the ABL, and in which the internal turbulent parameters are the surface momentum flux (proportional to the friction velocity $u^*$) and the surface heat flux $Q_0$. The ABL profiles of mean quantities are not specified because we will only consider overall properties. The ABL profiles adjust to the overlying free stream values within an interfacial sublayer of thickness $\Delta h$.

Classically the ABL height is assumed to be a function of the length scale $L_E = u^*/f$, and $L_* = -u^3 / g \beta \kappa Q_0$, where $g \beta = g/T_0$ is the buoyancy parameter and $\kappa = 0.4$ the Karman constant.

A starting point for investigating the dynamics of the ABL is the equation of turbulent kinetic energy $E_K$ since it contains the basic physics driving the turbulence field. Since we are not interested here in the detail of the structure of the ABL but rather in the overall energetics we integrate the $E_K$-equation across the ABL and the interfacial layer yielding:

$$\langle E_K \rangle \partial h / \partial t = u_*^2 V_{SL} + \frac{1}{2} (\Delta u^2 + \Delta v^2) \frac{\partial h}{\partial t} + \frac{1}{2} h (g \beta Q_0 - g \beta \Delta \theta) - h <\varepsilon> - \varepsilon_h \Delta h$$

where $h <\varepsilon> = \int s(z)dz$ through the ABL from $z=0$ to $z=h$, $\varepsilon$ is the viscous dissipation and $\varepsilon_h$ its value integrated through the interfacial layer between $h$ and $h+\Delta h$. The terms $\Delta u$, $\Delta v$ and $\Delta \theta$ express the jumps of the wind components and temperature through the interfacial layer, respectively, while $V_{SL}$ is the horizontal wind velocity at the top of the surface layer. Assuming that the production of $E_K$ by shear in the interfacial layer (2nd rhs term) is balanced by the local viscous dissipation (last rhs term) and that the nonstationarity term (Ihs term) is small, we are left with:

$$-\frac{1}{2} h (g \beta Q_0 + g \beta Q_h) = C_f u_*^3 - h <\varepsilon>$$

where $Q_h = -\Delta \theta \partial h / \partial t$ is the entrainment heat flux at the top of the ABL and $C_f$ is a drag coefficient.

This equation can be solved using the concept of marginal Richardson number (Pollard et al. 1973; Joffre 1981) which leads to the result that the classical critical number $Ri_{crit} = 0.25$ fits the data in the case of small rotation effect ($hf/u^* < 0.1$) but increases as $hf/u^*$ increases (Joffre 1981).

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Another closure approach is to use the heat diffusion approach by which the term $Q_h$ is parametrized through an effective eddy diffusivity coefficient (Kitaigorodskii 1988; Kitaigorodskii and Joffre 1988), ie.:

$$Q_h = -\Delta \theta \partial h / \partial t = -K_h \partial \theta / \partial z = -K_h \gamma = -cw_m h \gamma$$

where $w_m = a_0 u^* + a_1 v^*$ is the velocity scale including the convective velocity scale $w^* = [g \beta h Q_0]^{1/3}$. Assuming that the dissipation term can be parameterized as $h <\varepsilon> = m_t u^* h^3$ with $m_t$ being a function of the Monin-Kazanski stability parameter $\mu_0 = L_E/L_*$. Thus, the final $E_K$-equation takes the nondimensional form:

$$a_0 k(h/L_N)^2 + (h/L_*) = 2\kappa C_f 2\kappa m_t$$

where has appeared the new length scale $L_N = u^*/N$, with $N = (g \beta)^{1/2}$ the Brunt-Väisälä frequency.
3. Comparisons with observations

This theoretical framework was tested by Kitaigorodskii and Joffre (1988) using several sets of data. They used data gathered during an intensive field campaign carried out over the frozen Northern Baltic Sea with a wide range of stable conditions and covering as well unstable conditions with a dual regime of thermal and mechanical production of turbulence (Joffre 1981, 1982). Additionally they use data from the Wangara campaign carried out in Australia (Clarke et al. 1971) and the North Atlantic JASIN data (Nicholls 1985).

a) Neutral conditions

In the asymptotic case of neutral conditions (i.e. \( |L^*| \to \infty \)) Eq. (3) tends towards the solution:

\[
\frac{h}{L^*} = \left( \frac{2C_f - m_0}{a_0} \right)^{1/2} L_N = bL_N \approx 4-13 L_N
\]

where \( m_i(0) = m_0 \). Note that if we use the factor \((2\pi)^{-1}\) to express \( N \), the factor \( b \) is of order 1. The data compiled by Kitaigorodskii and Joffre (1988) confirm this prediction.

b) Stable conditions

The velocity scale \( w_m \to u^* \) and \( m_i \to m_s \) such that Eq.(3) takes the form:

\[
a_0 \kappa \left( \frac{h/L^*}{(L^*/L^*)^2} \right) = 2 \kappa (C_f - m_s) = m'
\]

Thus \( h/L^* = F^+(L_N/L^*) \). According to Eq. (5), in the case of a strong capping lid (i.e., \( N \to \infty \)), \( h \) tends towards 14 \( L_N \). In the case of no background stratification (i.e., \( N \to 0 \)) but with a surface negative flux \( Q_0 \), the ABL height \( h \) is proportional to the Monin-Obukhov length scale, i.e. \( h \to m' L^* \) (with \( m' \approx 40 \)). This result was also derived by Kitaigorodskii (1960) for the surface layer of the ocean but with a much smaller coefficient.

The general solutions for stable conditions read:

\[
\frac{h}{L^*} = \left\{ -1 + \left[ 1 + 4 a' m' \left( L_N/L^* \right)^2 \right]^{1/2} \right\} \left\{ 2 a' \left( L_N/L^* \right)^2 \right\}^{-1}
\]

where \( a' = a_0 \kappa K_h (0.1-0.3) \). Comparison with atmospheric data show a good agreement up to values of \( L_N/L^* \approx 5 \) (Kitaigorodskii and Joffre, 1988). The scatter of the data can be explained for the most part by differentiating the data according to the remaining dimensionless parameter \( L_E/L_N \) (which has to be taken into account according to Buckingham's \( \pi \)-theorem). The degradation of the empirical fit under stronger stability conditions can be also due to problems connected to measuring turbulent fluxes and determining the ABL height under such regimes of intermitent weak turbulence.

c) Unstable conditions

Under unstable conditions we assume \( m_i \to m_u \) and the general \( E_K \)-equation (3) takes the form:

\[
a' + a'' |h/L^*|^{1/3} \left( h/L^* \right) (L_N/L^*)^2 - |h/L^*| = m''
\]

where \( a'' = a_0 \kappa^{2/3} K_h \) and \( m'' = 2 \kappa (C_f - m_u) \). Thus, in general \( h/L^* = F(L_N/L^*) \). In the special case of free convection (i.e., \( u^* \to 0 \)) the function \( F \) behaves asymptotically according to \( |L_N/L^*|^{3/2} \). Considering only conditions with \( m'' > 0 \), i.e. where both mechanical and thermal turbulence are active, we obtain the general solution:
\[ h/L_* = \left[ 1 + [1 + 4 a'* (L_N/L_*)^2]^{1/2} \right] \{ 2 a' (L_N/L_*)^2 \}^{-1} \]  \tag{8}

Once again the data compiled by Kitaigorodskii and Joffre (1988) fitted very well with these solutions and even the start of the convective regime with the \([L_N/L_*]^{3/2}\) -slope could be empirically observed. Here again the internal scatter of the data is described by the dimensionless parameter \(L_E/L_N = N/f\).

4. Discussion

Classical ground-based measurements of turbulent fluxes do not always provide a realistic description of the important contributions influencing the dynamics of the whole ABL. Rather, background stratification through the parameter \(N\) plays a very important role. This leads to the inclusion of the length scale \(L_N = u*/N\) into the similarity description of the ABL height, and the dimensionless ABL height \(h/L_*\) is a function of the dimensionless parameter \(L_N/L_*\) and \(L_E/L_N\). This approach was tested against very different sets of data by Kitaigorodskii and Joffre (1988) which leads to the conclusion that, in both stable and unstable conditions, this theory did not contradict the assumption that the balance between entrainment and surface heat flux can be parameterized in the simple form (1 -3).

More recently, Overland and Davidson (1992) also have found a clear dependence between the height of the inversion base height and the scale \(L_N\) defined at a fixed height of 500 m from measurements performed above the Arctic Ocean. Their proportionality coefficient was of the same order of magnitude (within the large scatter) as the one proposed by Kitaigorodskii and Joffre (1988).

Van Pul et al. (1994) also have found a good correlation between the height \(h\) of the stable ABL determined by lidar occultation and the scale \(L_N\) with a proportionality coefficient of 6. Vogelezang and Holtslag (1995) have derived an expression for stable and neutral ABL based on observations and large eddy simulation results showing that the background gradients are essential for describing \(h\). They show that their formulation reduced to our expression (4) with compatible numerical coefficients.

Kitaigorodskii (1992) has also shown that the present theory applies to the benthic boundary layer which was also confirmed by the modelling effort of Rahm and Svensson (1989) and by direct observations of the benthic boundary layer in the Norwegian Sea by Nabatov and Ozmidov (1987).

References


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