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Further Work on the Kitaigorodskii Roughness Length Model: A New Derivation Using Lettau's Expression on Steep Waves[1]

Carsten Hansen[1] and Søren E. Larsen[2]

[1]Danish Meteorological Institute DK-2100 Copenhagen Ø, Denmark
[2]Risø National Laboratory DK-4000 Roskilde, Denmark

Abstract

A model for the roughness of the sea surface is derived by a combination of two ideas: a) Kitaigorodskii's (1973) about wavelets being roughness elements that move with their associated phase speed; b) Lettau's (1969) about the roughness of a field of roughness elements with characteristic dimensions.

The resulting sea surface roughness is related to a wave spectrum consisting of a Kitaigorodskii (1983) inertial form and a Phillips (1958) saturation form.

The variation of roughness with wave age is discussed and related to the corresponding variation of the Phillips constant B.

Key words: Roughness, waves, wave spectrum, Charnock constant, wave age

1. Introduction

Kitaigorodskii (1973) derived an expression for the roughness of the sea surface. In his analysis the roughness of the wavelets were associated with the standard deviation of their height, following laboratory studies of the roughness of sand. The effects of the wave motion were included by considering the roughness in a coordinate system being translated with the phase speed, c, of the waves. For a wave spectrum of the form $F = Bk^{-4}$ (*Phillips*, 1958) where k is the wave number modulus and B is the Phillips constant, Kitaigrorodskii found that $z_0 \propto B^{1/2}u_*^2/g$. Here u_* is the friction velocity and g acceleration due to gravity. From various reported experiments it appears that B varies about a factor 2 in the range 0.005 to 0.01, while z_0g/u_*^2 is found to vary more than a decade for the same data set. Undoubtedly, part of this variation is associated with measuring difficulties. Still the discrepancy seems worth studying.

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Also many recent studies of the drag coefficient (e.g. *Geernaert* et al., 1987) indicate that z_0 of the sea depends on the wave age c_0/u_* , where c_0 is the phase speed of the dominating longer waves.

None of these phenomena are easily explainable in terms of the results derived by *Kitaigorodskii* (1973). In the present paper we therefore revisit his derivations to study the possibility of refining the results.

In section 2 we discuss the similarity between wavelets and land surface roughness elements, assuming that only wavelets with a steepness larger than a certain value (s ≃ 0.25>) will give rise to flow separation, making the wavelet a roughness element. The roughness length corresponding to a given distribution of roughness elements over the surface is further evaluated, using an empirical formula developed by *Lettau* (1969). In *Kitaigorodskii* (1973) this discussion was based on the results of *Nikuradse* (1932, cfr. *Kitaigorodskii* 1973) relating the roughness of a surface with roughness elements to the characteristic size of the roughness element. Lettaus formula has the advantage that it further incorporates the ratio of the roughness sizes and the distances between them.

In section 3 we rederive the *Kitaigorodskii* (1973) transformation of the roughness length from a coordinate system being translated with the phase speed, *c*, of the individual waves to a fixed coordinate system. The formulation in section 2 is applied, based on the spectral properties at the wave number scale *k* of each wave, i.e. assuming linear wave theory, then $c = \sqrt{g/h}$. The average roughness length is then obtained by integration over the full wave number range.

In section 4 we shortly discuss the influence of orbital motions of longer waves as well as the long wave influence on short wave breaking and thereby on their steepness distribution. This subject, which leads to quantitative models about the dependence of the roughness length on wave age, swell and swell direction, will be pursued in a forthcoming paper.

2. *The similarity between wavelets and land surface roughness elements*

Lettau (1969) formulated a simple empirical relation to represent his experimental results on the boundary layer, formed above a distribution of bushel baskets on the frozen surface of Lake Mendota,

$$Z_0 = \alpha_L\, h\, X/A\ , \qquad (1)$$

where *h* is the height of the roughness element, *X* its crosswind area, *A* the horizontal area available to each element, and α_L is a dimensionless coefficient of order unity. Eq. (1) was found to be valid for fairly isolated roughness elements in the range *X/A* = 0.01 to 0.4 (*Garratt*, 1992).

An application of this methodology for analysing an inhomogeneous distribution of roughness elements was given by *Kondo* and *Yamazawa* (1986), who examined roughness heights over some rural towns and cities. To represent the variation of the measured roughness length, Kondo and Yamazawa suggested the relation

$$z_0 = \alpha_K \sum \frac{AiHi}{A}; \alpha_K = 0,25 \qquad (2)$$

where H_i is the height of individual buildings or forest areas, A_i is the horizontal cross section area occupied by each roughness element, and *A* is the total area.

Equation (2) is similar to Lettau's formula (Eq. 1) because there is a uniform aspect ratio $\Sigma AiHi/\Sigma XiHi$, where X_i is the vertical crosswind area of individual roughness elements. Therefore, we may apply the result of *Kondo* and *Yamazawa* (1986) to randomly distributed wavelets, which also have a distribution of heights and lengths. We assume that the overall roughness length is obtained by a direct average of Eq. (1) over this distribution.

The principal feature of the flow across roughness elements is the formation of flow separation over the corner(s) of the roughness elements, with reattachment on the lee side and a subsequent redistribution of the momentum deficit within the atmospheric boundary layer. The complex nature of boundary layer flow makes it very speculative to estimate the force on each roughness element. Therefore, we build our approach on the empirical relation (Eq. 1). We emphasize, though, that the resulting profile wind speed near the height of the roughness elements excerts a force on the individual roughness element determined by a bluff body resistance coefficient of order unity[2]. To verify that this is indeed the case, insert Eq. (1) into the logaritmic profile at the roughness height *h*,

$$\frac{U(h)}{u_*} = \frac{1}{\kappa} \ln \frac{z}{z_0},$$

where $U(h)$ is the profile wind speed at height h, u_* the friction velocity and $\kappa = 0.4$ the von Karman constant.

Let l be the distance in the wind direction between the roughness elements; we then find the individual resistance coefficient

$$D = \frac{l}{h}\left(\frac{u_*}{U(h)}\right)^2 = \frac{l}{h}\left(\kappa / \ln \frac{l}{\alpha_L h}\right)^2.$$

With a characteristic value $l/h = 10$ we have $D = 0.3$, and with $l/h = 100$, $D = 0.8$. For waves we may estimate X/A as the roughness wavelet height to the wavelength ratio, h/L, times the fraction of wavelets, where flow separation occurs.

We assume that flow separation occurs where the steepness exceeds a critical value, s_0. The value of s_0 was evaluated by *Csanady* (1985) in his review on energy and momentum transfer to centrimetric waves. An experiment by *Kawai* (1982, *Csanadys* figure 7) indicates that flow separation occurs at a wave height to wavelength ratio $h/\lambda \approx 0.08$. This corresponds to a steepness $s_0 = \pi h/\lambda \approx 0.25$. As the air and water flow for these laboratory waves of wavelength $\approx$ 10 cm is characterized mainly by a nonrotational water orbital flow and a logarithmic wind profile, we expect the flow to be kinematically similar to the flow over the somewhat longer waves of a length of typically one meter, which we consider to be dominant roughness elements on a well-developed sea.

Consider a wavelet with wave number k and amplitude a. We take the following definition of the equivalent roughness element height of a wavelet:

$$h = 2a \text{ for } a > a_0 \text{ and } h = 0 \text{ for } a < a_0,$$

where a_0 is the amplitude corresponding to the critical wave steepness discussed above.

2 The common word in hydrodynamical textbooks is “drag coefficient”. However, we use the word “resistance” in order to avoid confusion with the boundary layer surface drag coefficient.

We may then rewrite Eq. (1) as:

$$z_c = \alpha_L hS / A = \frac{2}{\pi}\alpha_L a^2 k = \frac{2}{\pi}\alpha_L s^2 k^{-1} \quad \text{for } s > s_0 \qquad (3)$$

$$z_c = 0 \qquad \text{for } s < s_0$$

where $s=ak$ is the steepness of the wavelet and z_c is the roughness of the wavelet in the coordinate system moving with the wave. In that coordinate system, the overall roughness length is the average z_c of all wavelets.

We assume that the wave field is a random superposition of harmonic components in a narrow wave-number band, i.e. all of essentially one wave number scale $\bar{k}$.

The wave number scale $\bar{k}$ is defined from

$$\langle \eta^2 \rangle \bar{k} = \int_k^\infty k' F(k') k' dk' \qquad (4)$$

where $F(k)$ is the one-dimensional wave number spectrum and $\eta = \eta(x,t)$ is the surface displacement.

Note that if $F(k)$ is of the Phillips (*Phillips*, 1958) saturation form, Bk^{-4}, then $\bar{k} = 2k$ and $\langle \eta^2 \rangle = Bk^{-1}$.

To determine z_c for waves of a given angular frequency ω with a corresponding wave number $k = \omega^2 g$, we specify the amplitude of waves from

$$\langle \eta^2 \rangle = \int_k^\infty F(k') k' dk' = \int_\omega^\infty S(\omega) d\omega, \qquad (5)$$

where $S(\omega)$ is the omnidirectional frequency spectrum.

It is convenient to associate the steepness, s, of such waves with the local maximum slope between every two succisive local wave maxima and normalize the steepness with the overall slope parameter $\langle \eta^2 \rangle^{1/2} \bar{k}$

If the surface displacement $<\eta(x,t)$ has a narrow spectrum, then the square of the normalized steepness is expressed in the parameter

$$y = \frac{s^2}{(2\langle \eta^2 \rangle \bar{k}^2)}$$

The probability distribution of y is described by an exponential density distribution function

$$f(y) = e^{-y}, \qquad (6)$$

i.e. the probability of finding a wave steepness in an interval dy is $f(y)dy$ (see e.g. the review by *Srokosz* (1990) of Longuet-Higgins' exceedance theory). The observed wave spectra are, however, not sufficiently narrow for Eq. (6) to be valid. In fact the mean square wave slope determined from integration over the spectrum $\left(\int_k^{\infty} k^2 F(k) k dk\right)$ becomes indefinitely high for the case of a Phillips saturation spectrum (*Phillips*, 1958). But the underlying assumption that the wavy sea surface is a superposition of a spectrum of linear wave trains with random phases is probably violated by wave breaking events at high wave number scales. Therefore it is likely that Eq. (6) is still an adequate formulation.

In Appendix A we demonstrate the result of a numerical reconstruction of a one-dimensional sea surface from a spectrum of the Phillips form. From this reconstruction we determine the distribution of wave heights $2a$ and wavelengths λ defined as the maximum sea level difference and distance between successive upcrossings through the mean sea level. The distribution of the quantities $2a/\lambda$ is examined (the wave slope) and the frequency of waves exceeding various critical slopes is calculated as functions of $(2a)^2/\lambda$ (proportional to the wavelet roughness of Eq. 3). We find that the reconstructed mean wavelet roughness length compares well with the result (Eq. 10 below) which is based on the assumption of Eq. (6).

The wavelet roughness of Eq. (3) is expressed as a function of the normalized square steepness y:

$$z_c(y) = \frac{2}{\pi}\alpha_L \langle \eta^2 \rangle \bar{k}\, y \quad \text{for } s > s_0 \tag{7}$$

$$z_c = 0 \quad \text{for } s < s_0$$

The characteristic value of $\bar{z}_c$ for a given wave number $\bar{k}$ can now be found from Eq. (7) with the above considerations as an integral of the form:

$$\int_x^{\infty} z_c(y) f(y) dy$$

with

$$x = \frac{s_0^{\,2}}{2\langle \eta^2 \rangle \bar{k}^2}. \tag{8}$$

However, it was shown experimentally by *Rapp* and *Melville* (1990) that waves break (by spilling breaking) when their steepness s exceeds a value of about 0.3[3] >. This is close to our value of s_0 = 0.25, and may even coincide. Therefore, we approximate the integral by considering the individual roughness length of a wave at any $y>x$ approximately equal to $z_c(x)$. Thus we estimate the average roughness as

$$\bar{z}_c = z_c(x)\int_x^{\infty} f(y) dy = z_c(x) e^{-x}, \text{ where } x = \frac{s_0^{\,2}}{2\langle \eta^2 \rangle \bar{k}^2}. \tag{9}$$

3 We thank Professor Y. Papadimitrakis for pointing out to us that the steepness of Rapp and Melville corresponds to our definition.

Inserting Eq. (7) yields

$$z_c = \alpha_L \frac{2}{\pi}\langle \eta^2 \rangle \bar{k}\, x e^{-x} = \alpha_L \frac{s_0^{\,2}}{\pi} e^{-x}\, \bar{k}^{-1}. \qquad (10)$$

2.1 A theoretical upper bound for the roughness length of young waves

Under certain hypothetical conditions that may be established in a wave tunnel experiment or in case of a strong, steady wind blowing perpendicularly off a straight coastline, the wave field may be brought to a very high energy level with incipient breaking on almost all wave crests. In such a case x is somewhat smaller than one, so that the probability e^{-x} that a wave crest is a roughness element is close to unity. In such circumstances the wave phase speed is much smaller than the wind speed, and thus the effect of moving roughness elements described in section 3 can be ignored. Then from Eq. (10) the upper bound, which the roughness length cannot exceed, may be approximated as

$$z_0^{\,u} = \bar{z}_c(x=0) = \alpha_L \frac{s_0^{\,2}}{\pi} \bar{k}^{-1}. \qquad (11)$$

It is convenient to rewrite this expression in terms of the *inverse wave age* u_*/c_0, where c_0 is the phase speed of the spectral peak wave component. We define the dimensionless angular frequency scaled with the friction velocity ω_* and g as:

$$w_* = \omega u_*/g\,. \qquad (12)$$

Assuming linear gravity wave dynamics we have $\omega_* = u_*/c$, and the inverse wave age is equal to a dimensionless peak angular frequency $\omega_0{}_* = \omega_0 u_*/g = u_*/c_0$. This follows from the linear dispersion relation $c = \omega / k = \sqrt{g/k} = g/\omega$. For extremely steep waves the spectral peak wave number is approximately equal to $\bar{k}$. The dimensionless version of Eq. (11) then becomes

$$\hat{z}_0^{\,u} = \alpha_L \frac{s_0^{\,2}}{\pi} (u_*/c_0)^{-2}. \qquad (13)$$

In Eq. (13) $\hat{z}_0 = z_0 \dfrac{g}{u_*^{\,2}}$ is the dimensionless roughness length, sometimes called the Charnock's constant *(Charnock*, 1959).

There is a fundamental difference between such steep waves and Lettau's experiment, as steep waves produced in laboratory experiments tend to be long-crested and cross-wind oriented while Lettau (and others) made experiments with roughness elements that have horisontally isotropic geometries. Also for a well-developed oceanic wave field the roughness waves tend to have a broad directional distribution. This means that a specific value of Lettau's constant α_L cannot be inferred from one wave state to another.

However, the similarity may apply to an order of magnitude. Taking $\alpha_L = 1.0$, $s_0 = 0.25$ we find at the inverse wave age $u_*/c_0 = 1.0$ that $\hat{z}_0^{\,u} = 0.02$, which is typical for young laboratory waves. This value corresponds roughly to the bulk mean of published data from laboratory experiments as compiled e.g. by *Toba* et al. (1990).

As a wave field develops and the inverse wave age becomes smaller than of order unity, the effective roughness length of the spectral peak waves begins to drop off, and the significant contribution to the overall roughness length is transferred to high wave number components on the spectral tail. This will be quantified in the next chapter by application of *Kitaigorodskii*'s (1973) method. In order for the observed rate of energy increase near the spectral peak to be maintained, this shift to higher wave numbers of the roughness elements and associated momentum transfer demands a different mechanism to be responsible for wave energy input near the spectral peak. This is a qualitatively good reason that the spectral level at high frequencies, i.e the saturation level B of the Phillips range is observed to decrease with increasing wave age. The consequence is that the roughness length drops far below the upper limit of Eq. (13), even when the Kitaigorodskii effect is applied.

3. Extension of the Kitaigorodskii (1973) approach

The roughness length, z_0, of a flow over a flat surface is defined in terms of the logarithmic profile as the height in which the wind speed becomes zero,

$$u(z)/u_* = \frac{1}{\kappa}\ln\frac{z}{z_0} \tag{14}$$

Once the logarithmic profile is established in one inertial system, a logarithmic profile is found also in any other reference system moved with constant horizontal speed c. In the wavelet-following reference system the velocity profile is expressed in terms of the roughness length z_c defined in Eq. (3).

$$(u(z)-c)/u_* = 1/\kappa \ln(z/z_c). \tag{15}$$

Thus z_0 can be referred to z_c by a combination of the logarithmic profiles (14) and (15), which yields

$$z_0 = z_c \exp\left(-\kappa\frac{c}{u_*}\right). \tag{16}$$

Note that in this derivation we assume that all roughness generating wavelets travel along the wind direction. This assumption is appropriate, but an exact quantification is difficult, since the force on waves propagating in a high angle to the wind is strongly reduced. Neglecting this directional filter may cause the calculated roughness to be too high, or conversely be reflected in a value for the Lettau constant α_L somewhat lower than unity. However, it is our hope that the directional filter does not change considerably with the other parameters, i.e. with B.

We want to account for the variation of the phase speed with wave number k that appears in our expression (Eq. 10) for the moving-frame roughness length. Motivated by the experience that the effective roughness length is estimated as the sum over each roughness element in Lettau's expression, we will calculate the wave roughness as an integral over the wave number domain. The contribution from an infinitesimal wave number interval dk to the moving-frame roughness length $\bar{z}_c$ (10) is found from the differential of Eq. 4:

$$d(\langle \eta^2 \rangle \bar{k}) = kF(k)kdk.$$

Then from Eq. (10)

$$d\bar{z}_c = \alpha_L \frac{2}{\pi} xe^{-x} kf(k)kdk, \tag{17}$$

where x is found from Eq. (8) using the integral expressions (4) and (5) for the spectral tail above k.

The next step is to multiply Eq. (17) with Kitaigorodskii's filter z_0/z_C (Eq. 16) at each wave number. The filter is applied at the integral scale $\bar{\omega} = (g\bar{k})^{1/2}$. For simplification we rewrite the expressions in terms of the dimensionless angular frequency $\omega_* = \omega u_* / g = \left[u_* \sqrt{k/g}\right] = k_*^{1/2}$. The omnidirectional frequency spectrum $S(\omega)$ is defined such that $S(\omega)d\omega = f(k)\, kdk$, and the dimensionless frequency spectrum is $S_*(\omega_*) = S(\omega)g^3/u_*^5$. We then find for the differential roughness length in the fixed reference frame:

$$d\hat{z}_0 = \frac{g}{u_*^2} dz_0 = \alpha_L \frac{2}{\pi} xe^{-x} e^{-\kappa/\bar{\omega}_*} \omega_*^2 S_*(\omega_*) d\omega_*. \tag{18}$$

In figure 1 we present the contribution to $\hat{z}_0$ for different frequencies with the integral scale $\bar{\omega}_* = \bar{k}_*^{1/2}$, assuming a wave spectrum consisting of a *Phillips* (1958) saturation form $S(\omega) = 2Bg^2\omega^{-5}$ above some frequency ω_g and an inertial range of the form $S(\omega) = agu_*\omega^{-4}$ for $\omega < \omega_g$ as suggested by *Kitaigorodskii* (1983). The two ranges are matched at the frequency ω_g so that $\alpha\omega_g g u_* = 2Bg^2$

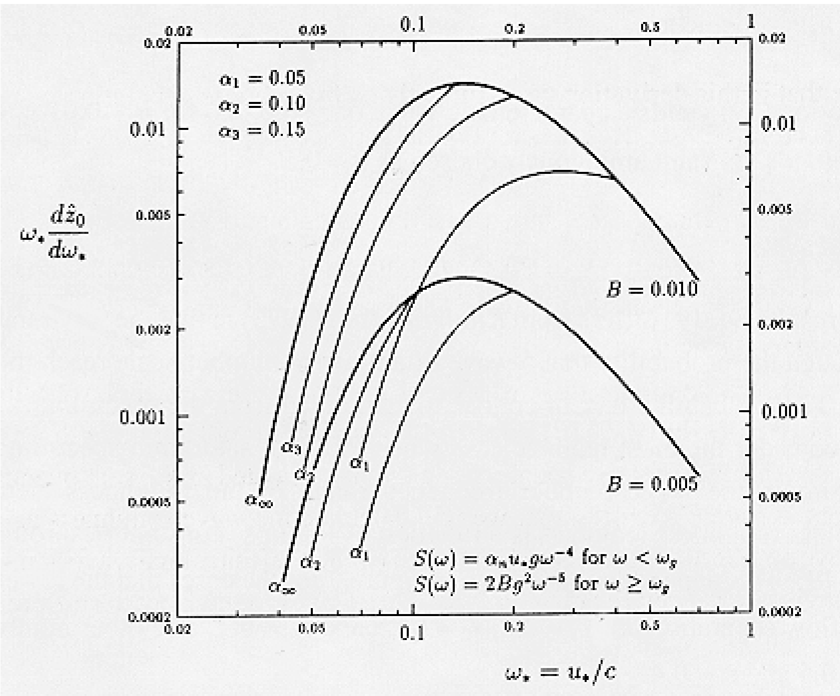

Fig.1. The contribution to the sea surface roughness from different parts of the wave spectrum, $S(\omega)$, derived from Eq.(16) where $\alpha_L = 0.5$, s0 = 0.25. The importance of the characteristic parameters of $S(\omega)$ is illustrated. A very high value of $\alpha(\alpha_\infty)$ means that the spectrum has the form of Phillips saturation on the whole range.

3.1 A special case: spectral tail of an all-over Phillips form

We consider the simplified case, where the spectrum is of the form $S(\omega) = 2Bg^2\omega^5$ over the whole spectral tail. This corresponds to the curves 'α_∞' in figure 1. Then x has the constant value

$$x = \frac{s_0^{\;2}}{4B},$$

and the term $xS(\omega)$ in Eq. (18) becomes simply $s_0^2 g^2 \omega^{-5}$, i.e. the saturation level B only remains in the e^{-x} term. Eq. (18) becomes

$$d\hat{z}_0 = \frac{g}{u_*^{\;2}} dz_0 = \alpha_L \frac{s_0^{\;2}}{\pi} e^{-x} e^{-\kappa/\varpi_*} \omega_*^{\;-3} d\omega_*.$$

Integration over all frequencies yields

$$\hat{z}_o = \frac{g}{u_*^{\;2}} z_0 = \alpha_L \frac{2s_0^{\;2}}{\kappa^2 \pi} e^{-x}, \tag{19}$$

Equation (19) yields a characteristic value of $\hat{z}_0 = 0.026$ for $B = 0.01$ $s_0 = 0.25$, $\kappa = 0.4$ and Lettau's constant ambigously chosen as $\alpha_L = 0.5$.

4. *Discussion*

Our result (Eq. 19) differs from *Kitaigorodskii* (1973) in that the wave statistics only enters through the probability that a wave of any wave number scale reach the threshold steepness. This probability has, of course, to be derived from the spectrum.

We consider the most realistic case when Phillips' saturation spectrum dominates the spectrum in the wave number frequency range of the roughness elements. The dimensionless roughness length $\hat{z}_0$ is a function of Phillips' constant B, through the term e^{-x} in Eq. (19). This dependence is shown in figure 2 for the cases that the lower steepness limits for flow separation are $s_0 = 0.2$, $s_0 = 0.25$ and $s_0 = 0.3$. The other numbers are: α_L chosen as 0.5 and $\kappa = 0.4$.

It is seen that $\hat{z}_0$ is increased by a factor of order 10 (depending on the value of s_0) when B is raised from 0.005 to 0.01.

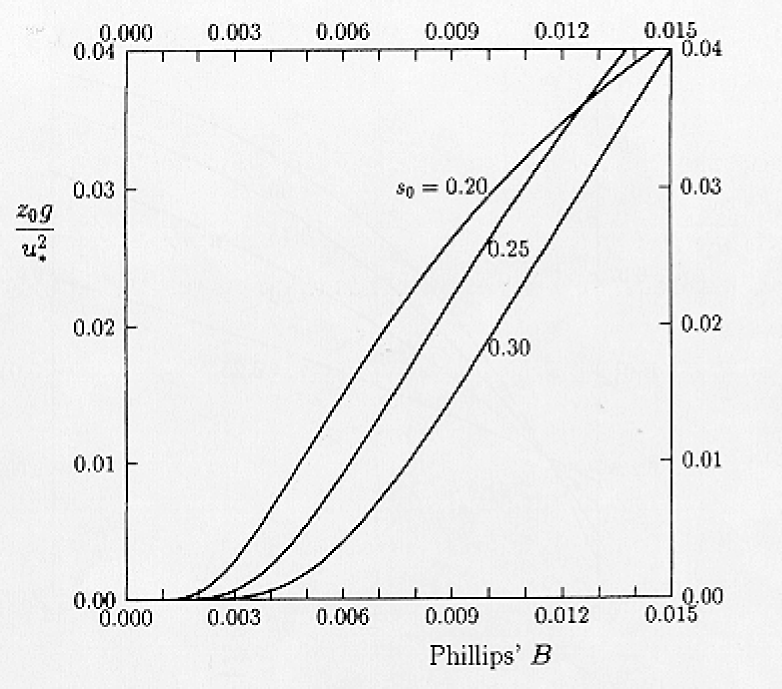

Fig. 2. The variation of z_0 with Phillips' constant B, for the case when the saturation range dominates the contribution to z_0 (case α_∞ cf. Fig 1). The variation is shown for different lower steepnesses of flow separation, s0 = 0.2, 0.25 and 0.3.

Thus we have obtained a formulation that has the potential that it can explain large variations of the roughness length as a consequence of a varying level of the wave spectral tail, within reasonable limits.

For example, various experiments seem to indicate that the level, B, of Phillips' saturation range depends on the wave age c_0/u_*. The dependence may be expressed as a power law, $B = B_0(c_0/u_*/40)^{-\gamma}$, where B has the reference value B_0 at very developed waves with $c_0/u_* = 40$. Figure 3 shows the dependence of $\hat{z}_0$ on the inverse wave age u_*/c_0 for various choices of the exponent γ, when $s_0 = 0.25$ and α_L=0.5, and $B_0 = 0.05$. We note that a modest value γ in the range $0.5 < \gamma < 1.0$ has the consequence that the dimensionless roughness length $\hat{z}_0$ is approximately proportional to the inverse wave age u_*/c_0 in the range $0.05 < u_*/c_0 < 0.1$.

Figure 4 shows a comparison between observations as compiled by *Donelan* et al. (1993) and our result for the cases $\gamma = 0.5$ and $\gamma = 1.0$. The comparison demonstrates that our model is realistic. Also shown is a modified version of the upper bound with Eq. (13) as its high-frequency asymptote. This upper bound is derived from Eq. (18) for a simplified spectrum of the form

$$S(\omega) = 2Bg^2\omega^{-5} \text{ for } \omega \geq \omega_0, \text{ and } S(\omega) = 0 \text{ for } \omega < \omega_0$$

which yields

$$\hat{z}_0{}^u = \alpha_L \frac{2 s_0{}^2}{\kappa^2 \pi}(1-(1+\kappa/\bar{\omega}_*)e^{-\kappa/\bar{\omega}_*}) \qquad (20)$$

In order to obtain Eq. (13) as a high-frequency asymptote, the result is plotted as a function of the (dimensionless) integral frequency scale, i.e. the abscissa is $u_*/c_0 = \bar{\omega}_*$.

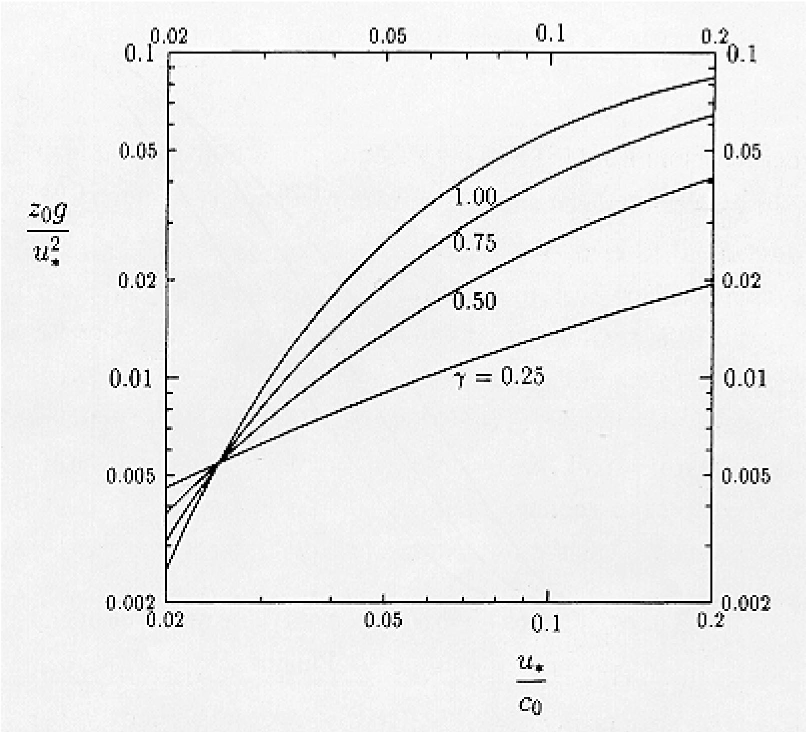

Fig. 3. The predicted variation of z_0 with inverse wave age for different values of the parameter γ in the wave age dependency of Phillips' constant $B \sim (c/u_*)^{-\gamma}$.

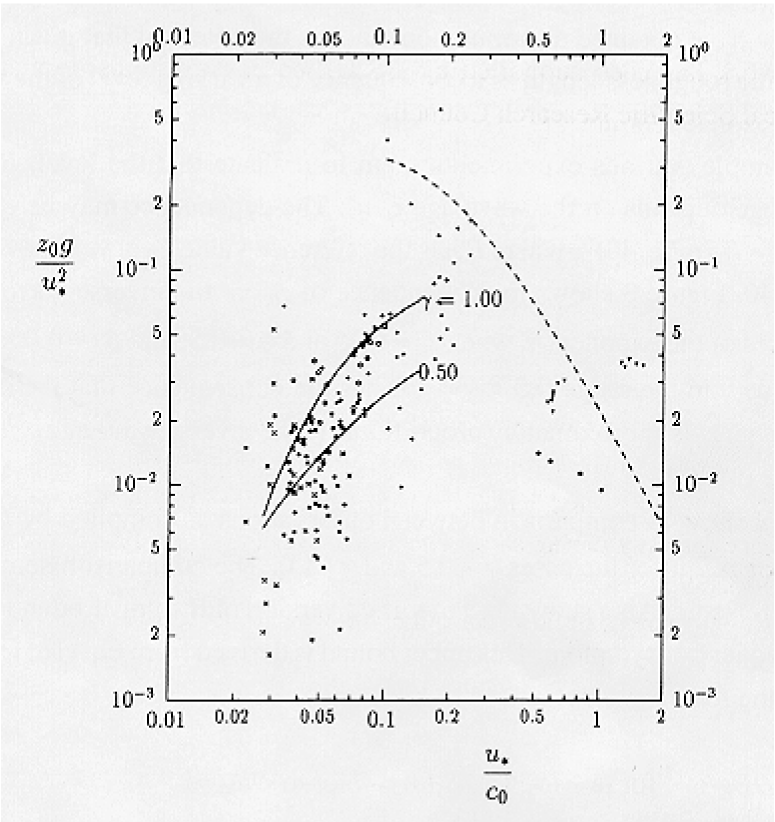

Fig. 4. A comparsion of our model with published field and laboratory data compiled by *Donelan* et al. (1993, from their figure 2). Full lines: Our model as in Fig. 3 with $\gamma = 0.5$ and $\gamma = 1.0$; broken line: Eq. (20).

The directional filter on the calculated roughness length is probably only a weak function of wave age. However, the spectral level is considerably affected by the presence of longer waves, as short waves are modulated by the longer waves in the wave spectrum via strong nonlinear interactions. It can be shown, following the results of *Phillips* (1981), that B is an exponential function of the long-wave instantaneous amplitude, providing the long-wave steepness is small. As a consequence of the exponential probability density function e^{-y}, wavelets have a considerable chance of reaching the level s_0 only on the higher crests of longer waves. Thus the overall probability of roughness wavelets will decrease with increasing wave age. Simultaneously the orbital water motions on the crests of the longer waves enter the Kitaigorodskii expression (Eq. 16) in addition to the wave phase speed and thereby further decrease the effective roughness. In a forthcoming paper we will demonstrate that this can be quantified in terms of an equivalent decrease of the Phillips' *<P12M>B* in the expression (Eq. 19).

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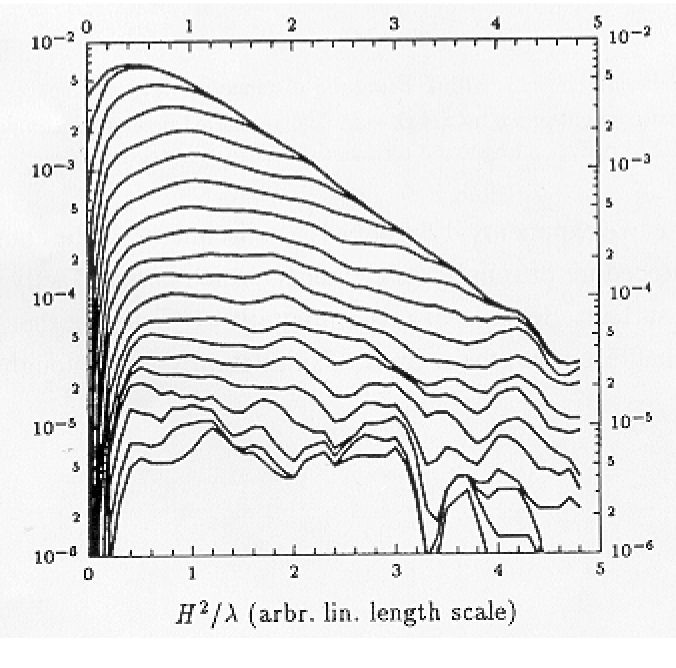
Appendix A: A numerical reproduction of surface wave roughness elements

In order to test the applicability of the simpilified probability density function of Eq. (6), we performed a numerical reconstruction of waves. A series of harmonic functions was defined with wave numbers k in the in the range [0.01,1.00] distributed at intervals $\Delta k = 0.02k$. The amplitudes define a variance spectrum of the form $S_k = 3k^{-4}$ for $k \le 1$, and $S_k = 0$ for $k > 1$, so that it integrates to the variance $\int_0^\infty S_k dk = 1$.

The phase of each harmonic was chosen at random for a series of calculations with different initialisations of the random number generator. The sum of the functions then represents the wavy surface. From this sum we determined the trough to crest level difference H between every two successive upcrossings of the zero level, and the distance λ between each upcrossing.

Then the wave steepness was defined as $s = \pi H/\lambda$. We chose series of the critical steepness s_0 normalized with the mean steepness determined from the spectrum to form the parameters $x = s_0^2 / (2\langle \eta^2 \rangle \bar{k}^2)$.



Density distribution function of H^2/λ when $\pi H/\lambda > s_0$ for $x = 0.05$ (top curve) and x incremented by 0.25 (subsequent curves).

Fig.1A. Results from a numeral experiment showing the number density distributions for H^2/λ, H being the maximum level difference between two zero crossings λ apart, for the steepness $\pi H/\lambda > s_0$, where s_0 is a lower limit steepness. The curves reflect different values of s_0.

In Fig. A1 we show the number density distribution over H^2/λ of the fraction of waves whose steepness $\pi H/\lambda > s_0$, plotted for various values of the parameter x. It is seen that there is a wide distribution of heights corresponding to each critical steepness. The density distribution level is seen to depend on x like exp(-1.3x) rather than the exp(-x) dependence we expect from the simple hypothesis of a narrow spectrum.

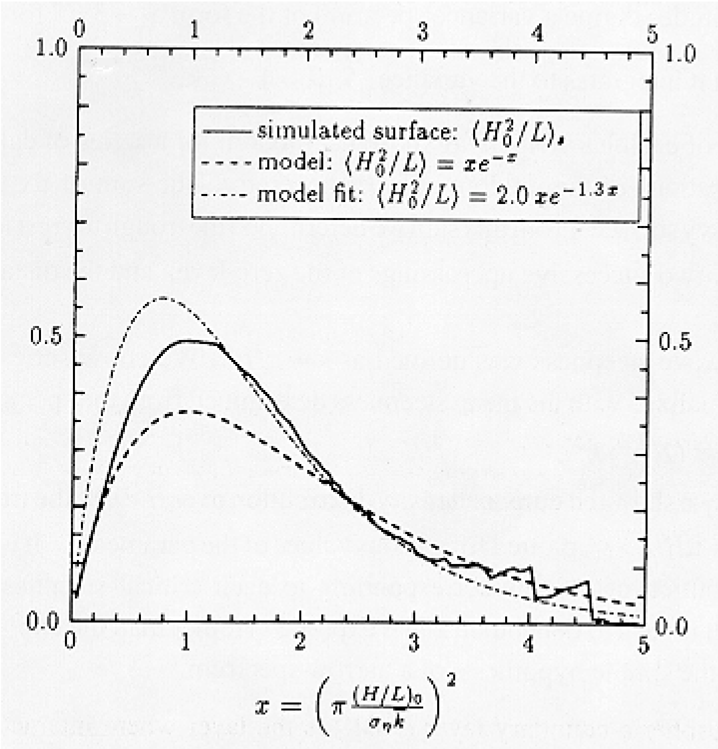


Fig. 2A. Calculated mean roughness height $\langle H_0^2/L \rangle$, with L being the distance between two succession roughness waves and H_0 a characteristic height given by $\pi H_0/\lambda = s_0$. The simulated results are compared with two different exponential variations corresponding to the formulation (Eq. 10) in section 2.

In Fig. A2 we show the calculated mean roughness height $\langle H_0^2/L \rangle$, where L is the distance between two successive rougness waves, and H_0 is the critical height defined by $\pi H_0/\lambda = s_0$. This result is compared with the model of the text, $\langle H_0^2/L \rangle = x \exp(-x)$ and a best fit similar expression $\langle H_0^2/L \rangle = 2.0\, x \exp(-1.3x)$.

The difference between the two exponents -1.3 x or -x is a consequence of the chosen "zero-upcrossing" selection procedure of roughness elements, which is not exactly the way the wind flow senses the surface. Because of this ambiguity we conclude that the exponent -x provides a model that resembles the numerical simulation "close enough".