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On the Accuracy and Detection Efficiency of a Lightning Location System of Four Direction Finders

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Abstract

The southern half of Finland is covered by a lightning location system which is based on four magnetic direction finders. Flashes detected by all four allow an analysis of both the location accuracy and detection efficiency of the system. Supplementary information has been obtained by recording the pulse forms of lightning strokes detected by a flash counter, and by a comparison with the disturbance data in an electricity network. As a result, the average location accuracy is estimated to be better than or equal to 5 km in the main area of coverage, and the detection efficiency is about 70-80 %. Individual direction finders accept about 85 % of the cloud-to-ground flashes, and the probability of actually detecting an acceptable flash is about 55-80 %. Contrary to a flash counter, the acceptance of a direction finder is not sensitive to the strength or polarity of the pulse.

1. *Introduction*

In 1984, the Finnish Meteorological Institute (FMI) set up a lightning location system (manufactured by Lightning Location and Protection (LLP), Inc., USA) with two direction finders (DF) covering the southern half of Finland, the central unit (Position Analyzer) being at FMI in Helsinki. A third DF was added in autumn 1986 and a fourth one in August 1989; their sites were chosen to improve the location accuracy and detection efficiency in the southern half of Finland rather than to extend the coverage to Northern Finland (Fig. 1). Preliminary results, based on rather sparse data, of the performance of the system are analyzed in this report.

The LLP lightning location system is well-known and will not be described here (see *e.g.*, *Mach et al.*, 1986; *Tuomi*, 1990). Only some features relevant to the present analysis are pointed out. The direction finder, the detector of a lightning flash (actually the first stroke of a flash; it also counts the number of subsequent strokes), has certain criteria to accept only cloud-to-ground (CG) flashes occurring within its nominal detection range of about 400 km.

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While most CG flashes do have a chance to be accepted by a DF, it is found that some genuine CG flashes have pulse forms that do not satisfy the criteria and are absolutely rejected. This is suggested by a study where pulses were recorded (Section 4). For those flashes which meet the DF criteria, the probability to be actually detected (accepted) by a DF is well below certainty, typically 55-80 % within the nominal detection range, as can be seen by looking how the number of located flashes depends on the number of DF stations available (Section 3). Hence, the probability of locating an acceptable flash by at least two DFs (detection efficiency) is well below 100 %.

Additional information of the detection efficiency as well as of the location accuracy was obtained from the disturbance data of an electricity company (Section 5).

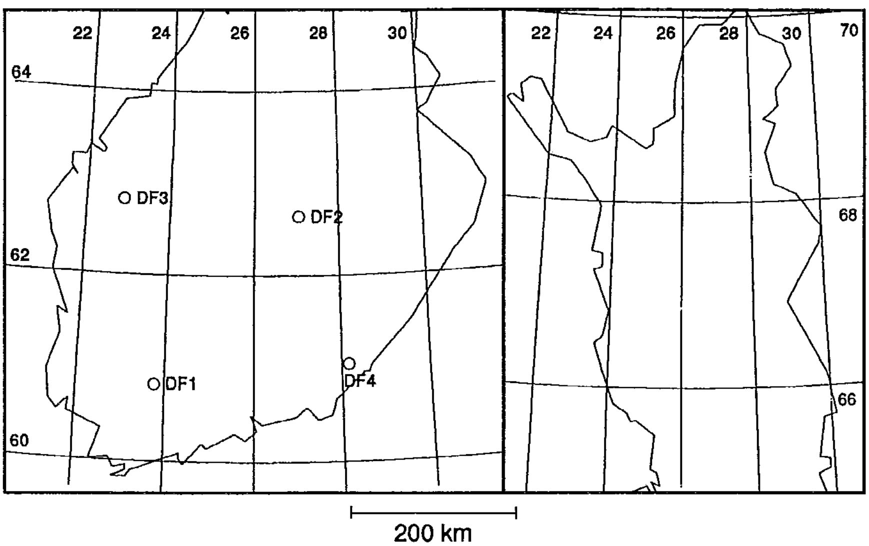

Fig. 1. The direction finder (DF) sites in the southern half of Finland (left).

Quite generally, DF stations suffer from so-called site errors, which are deviations from the true direction and which vary with the bearing angle. The average variation is typically more or less regular, often sinusoidal, and an amplitude of 10° is not uncommon. Systematic corrections for these errors can be loaded into the Position Analyzer.

All bearing errors are not systematic, however. While a direction finder can determine the bearing angle of an incoming lightning signal with an accuracy of 1° (according to the manufacturer), random variations of the order of ± 5° may occur in addition to the systematic site errors. The origin of the large random errors is unknown. Their effect can be minimized by using some optimization method, if at least three direction finders have observed the same flash. Errors are discussed in Section 2.

A theoretical study has been made of the effect of propagation on the lightning signal (*Janhunen*, 1990). It is found that a poor electrical conductivity of the ground (say 1 mS/m) does not damp significantly the signal at the frequencies and distances in question. For a period of 30 μs and a distance of 500 km the signal is attenuated only 13 % relative to an ideally conducting earth. Thus it seems that the detection range is restricted by other factors, e.g., by the curvature of the Earth and by ionospheric reflections. Time-domain computations show that topographic features may cause phase-shift distortions in the electric field when it is weak, but in practice the resulting angular errors remain below one degree because the direction is computed at the instant when the signal is strongest.

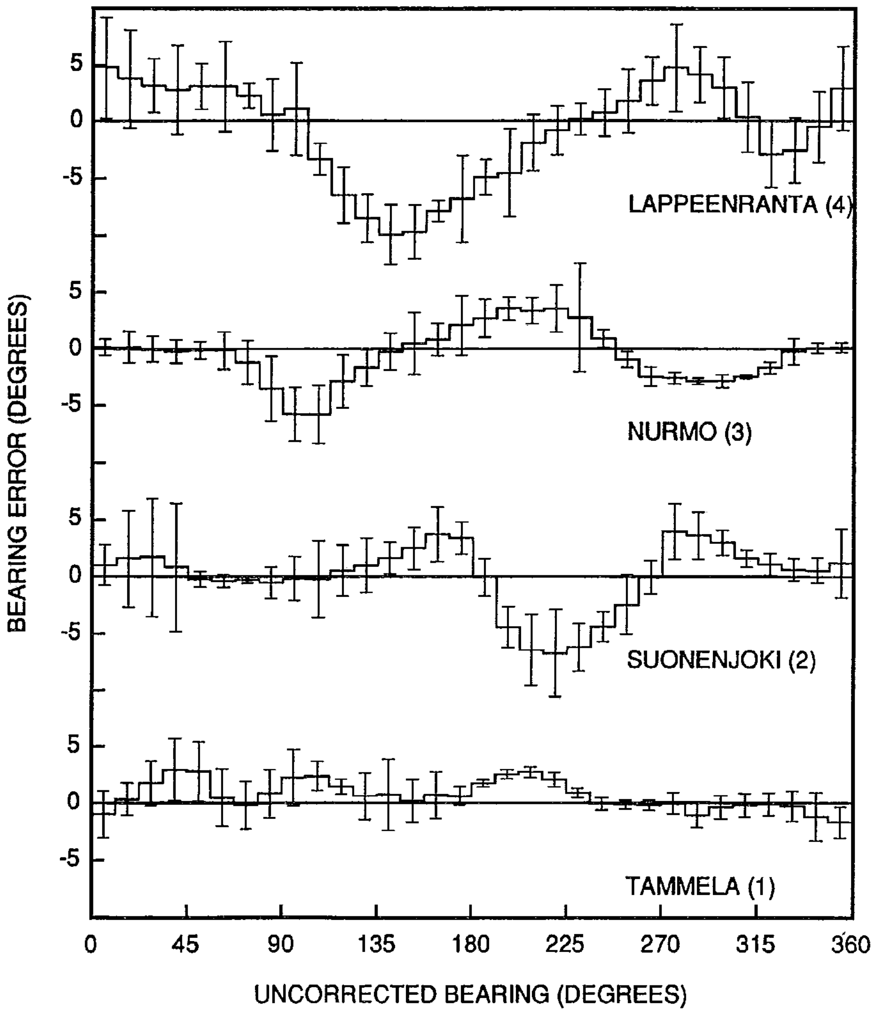

Fig. 2. Bearing errors in 11.25-degree sectors. The north is 0 and the east is 90 degrees. Computed from

Thus, theoretical explanations to the large random bearing errors and the less-than-perfect detection efficiency are difficult to find. The aim here is to study methods for determining these performance properties of the location system.

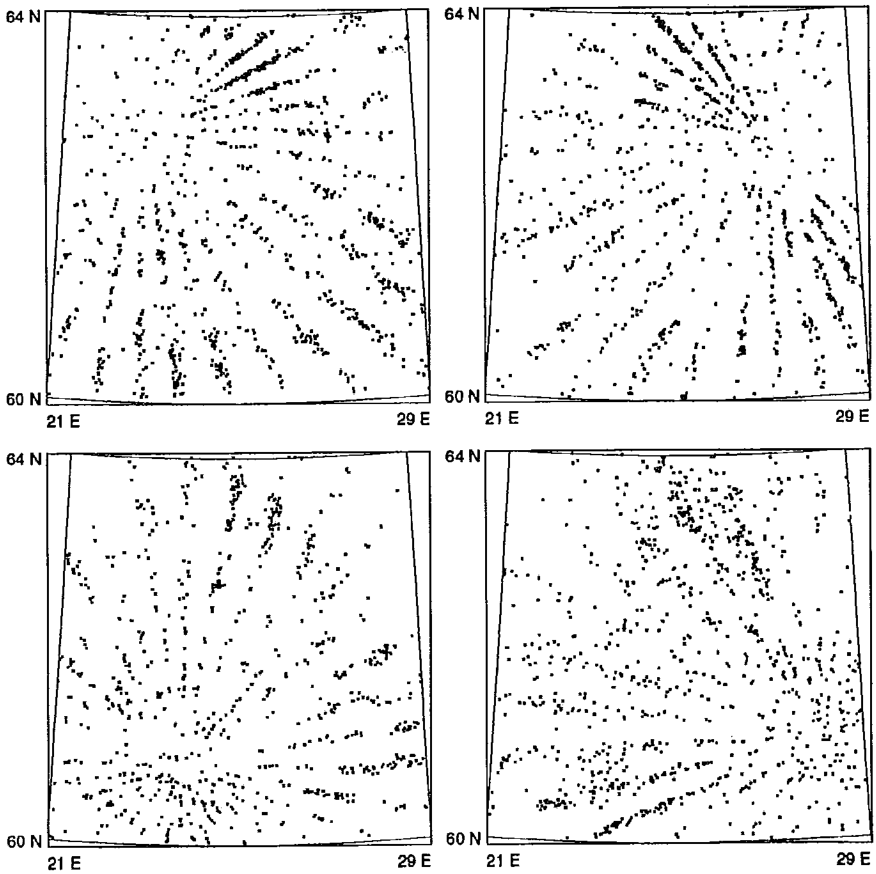

Fig. 3. Located flashes at 2-degree sectors as seen from each direction finder in directions 10, 20, ..., 360 degrees. Final locations have been found with an optimization procedure.

2. *Location errors*

When "ground-truth" data of lightning locations are not available, the only way to estimate the location accuracy is to look how the intersection points of a multiple-DF (more than 2) observation are scattered. The bearings of a 3-DF observation intersect at 3 points, and a 4-DF observation at 6 points.

Summer 1990 was the first whole season with four direction finders. The site errors of DF1-DF3 were determined earlier (*Tuomi*, 1988) with an iterative procedure described by *Mach et al.* (1986). In that method, one direction is corrected towards the intersection of two other directions. A new analysis, based on all four-DF data collected since August 1989, was made with the optimization method of *Orville* (1987). This procedure is a kind of least-squares fit and can also be used iteratively. In both methods, known errors can be introduced in the initial step. In fact, the disturbance data from an electricity company near DF1 (Section 5) revealed an error of about -8° in the SW sector of DF2. This error was underestimated both by the previous Mach's analysis and by the present Orville's method. After introducing the error in the initial data of the latter method, it did converge close to the initial value.

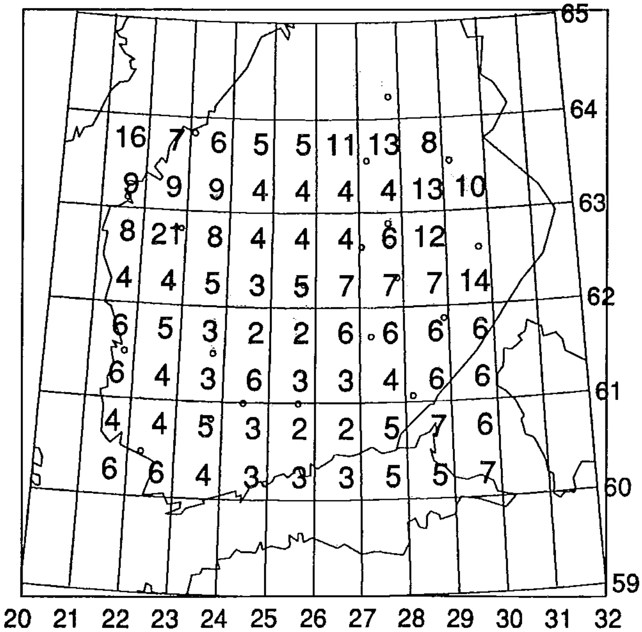

Fig. 4. Estimated errors (km) of 3-DF locations. Based on about 3000 4-DF observations in 1989-90. The error is the difference between an optimized 3-DF and 4-DF location and the result is the mean of the four distances for each flash.

The result is shown in Fig. 2. The errors in some sectors are as large as 10° and their largest standard deviations (due to the random errors) are typically 5°. Although the cause of the random errors is not known, their nature can be illustrated as shown in Fig. 3. Each frame shows the same area which is approximately the southern half of Finland (*cf.* Fig. 1), and each DF is in the focus of the sectors. The points are flashes observed by 3 and 4 direction finders and located by Orville's optimization procedure. Included are only directions (after correcting the systematic errors shown in Fig. 2) which fall within 2° sectors at 10° intervals as seen from the DF in question. Good and consistent accuracy can be expected when the sectors remain straight and narrow. The worst case is the northern and north-western sector of DF4 (lower right frame) but, fortunately, most of the flashes in that region are observed by at least two other DFs.

If a 4-DF observation is divided into four 3-DF observations and Orville's procedure is applied separately to all the 5 cases, the 4-DF point may be considered to represent the "true" location and its distances to the four 3-DF points give an accuracy estimate for a 3-DF observation. It turns out that the smallest of these distances is about one kilometre and is perhaps too optimistic. If the average of these distances is taken as the error measure, the map shown in Fig. 4 is obtained. One might say that within the area between the direction finders the mean accuracy is about 5 km or better and increases to larger than 10 km outside the area. (The large error (21 km) near DF3 is based on four flashes only and contains some occasional very bad bearings.)

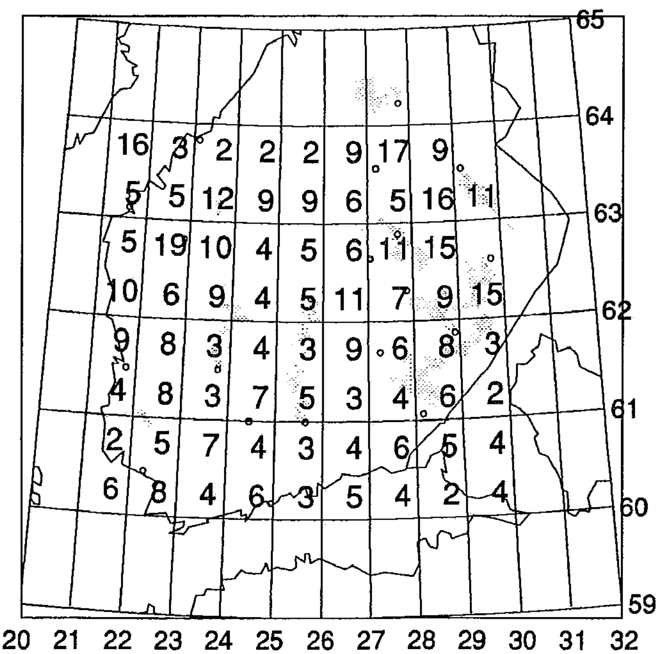

Fig. 5. Estimated errors (km) of 2-DF locations. Based on about 3000 4-DF observations in 1989-90. The error is the difference between the 4-DF optimized location and the best 2-DF triangulated location.

Almost one half of the flashes, even in the best detection area, are observed by two direction finders only in a 4-DF system. Hence, the 2-DF location accuracy remains important. If again the 4-DF optimized location represents the ground truth, a measure of the 2-DF triangulation accuracy is obtained by choosing the pair which intersects closest to perpendicular and calculating the distance between the 2-DF and 4-DF points. The result (Fig. 5) gives quite a similar impression as Fig. 4.

Passi and *Lopez* (1989) have developed a method by which the systematic site errors can be found without a bias caused by the random errors. On the other hand, one must assume a parametric form for the systematic-error curves, in this case two-cycle sinusoidals. The curves in Fig. 2 have large deviations from such a regularity, which implies that Passi and Lopez's method would cause large changes to the present site corrections, at least in some sectors. Their method will be applied to the present system after more data have been collected.

3. *Detection efficiency*

As the number of direction finders has increased, the number of located flashes has also grown. By finding how these changes are related, it is possible to estimate both the acceptances (detection probabilities) of individual direction finders and the detection efficiency of the whole system. Theoretical considerations of acceptance have been made *e.g.* by *Schütte et al.* (1988).

First, it is assumed that the acceptances remain fairly constant in the area in question. In summer 1990, a total of 8624 flashes were located in the area 60 - 64° N, 21 - 30° E. They were detected by different DF combinations as shown in Table 1.

Table 1. The numbers of flashes located by different combinations of direction finders.

DFs	Numbers	DFs	Numbers
1,2	384	1,2,3	406
1,3	585	1,2,4	300
1,4	593	1,3,4	640
2,3	580	2,3,4	1249
2,4	1269	1,2,3,4	2101
3,4	517		

If, for instance, DF1 and DF2 only had been present, they would have located 384 + 406 + 300 + 2101 = 3191 flashes, *i.e.*, less than one half. If the average acceptance of DF_i in the area is p_i and the true number of flashes is N, then for a 2-DF location system

$$p_1 p_2 N = 3191. \tag{1}$$

Note that here the acceptance is defined with respect to those flashes which, according to their pulse shape, have a chance to be accepted by a DF. (The signal strength has a minor role in this respect). In the next section it is concluded that about 15 % of the CG flashes do not meet the DF acceptance criteria.

Denoting $q_i = 1 - p_i$, the 2-DF observations of a 3-DF location system (1,2,3) would have resulted in

$$
\begin{aligned}
p_1 p_2 q_3 N &= 684 \\
p_1 p_3 q_2 N &= 1225 \\
p_2 p_3 q_1 N &= 1829
\end{aligned}
\tag{2}
$$

and the 3-DF observations in

$$p_1 p_2 p_3 N = 2507, \tag{3}$$

from which one can easily solve the acceptances $p_1 = 0.578$, $p_2 = 0.672$ and $p_3 = 0.786$. The exact solution is always found for a 3-DF system if the flashes detected by just one DF are not included (in 1990 they were not recorded); they would bring 3 equations more to the system and an exact solution would not exist in general. Correspondingly, in a 4-DF system there are five unknowns (N and four p) and 11 or 15 equations depending on whether the single-DF observations are known. An approximate solution can be found by minimizing the sum of squares

$$
\begin{aligned}
&(p_1 p_2 q_3 q_4 N - 384)^2 + (p_1 p_3 q_2 q_4 N - 585)^2 + (p_1 p_4 q_2 q_3 N - 593)^2 + \\
&(p_2 p_3 q_1 q_4 N - 580)^2 + (p_2 p_4 q_1 q_3 N - 1269)^2 + (p_3 p_4 q_1 q_2 N - 517)^2 + \\
&(p_1 p_2 p_3 q_4 N - 406)^2 + (p_1 p_2 p_4 q_3 N - 300)^2 + (p_1 p_3 p_4 q_2 N - 640)^2 + \\
&(p_2 p_3 p_4 q_1 N - 1249)^2 + (p_1 p_2 p_3 p_4 N - 2101)^2
\end{aligned}
\tag{4}
$$

which leads to $p_1 = 0.547$, $p_2 = 0.702$, $p_3 = 0.673$, $p_4 = 0.741$ and $N = 9387$, the minimum value being 1.014. The distance of this minimum from zero reflects the inaccuracy of the result, which is evidently caused by the unevenness of both the flash distribution and the acceptances in the area. When the probabilities of all the (p, q) combinations are added, the following estimates are obtained for the detection efficiencies of the different DF combinations: $p(1,2) = 0.38$, $p(1,2,3) = 0.71$ and $p(1,2,3,4) = 0.89$. If the observed numbers are divided by 9387, the estimates are 0.34, 0.67 and 0.92, respectively.

Still another estimate is obtained from the 3-DF system in 1988. A sample of relatively evenly distributed flashes gives $p_1 = 0.73$, $p_2 = 0.57$ and $p_3 = 0.82$. Then the 2-DF system (1,2) gives a detection efficiency of 0.42 and the 3-DF system 0.80. These numbers represent perhaps better the situation before 1990.

In the next section it is concluded that the detection efficiency of the 4-DF location system is 78 %. Comparison with the ideal detection efficiency 0.92 (or 0.89) obtained above leads to the conclusion that only about 85 % (88 %) of CG flashes are in principle acceptable by the direction finders.

The estimates of the detection efficiency given above refer to an average or representative situation in the main area of coverage. In the second approach, variations in the detection efficiency will be considered.

The differences in the results obtained by different methods or from different data, as well as the sparsity of the data so far, mean that trying to find the spatial variations of the acceptances of the individual direction finders may not be useful. In order to map the variations of the detection efficiency of the whole system without evaluating the DF acceptances, the following method is used.

Choosing arbitrary combinations of values for the probabilities (p_1,.., p_4) and a fixed value for N (see the different terms in eq. (4)), the sum of the six 2-DF observations N_2, the sum of the four 3-DF observations N_3, the number of the 4-DF observations N_4 and the total number N_{tot} are calculated for each combination. It turns out that the ratio $R = (N_3 + N_4)/N_2$ plotted as a function of N_{tot} forms a growing curve which is fairly smooth, that is, has only a weak dependence on the underlying variations in the values of p_i. A fit, for instance the function

$$R = 0.4\,(N_{tot}/(N - N_{tot}))^{0.75} \qquad (5)$$

can then be found where ratio of the detected flashes to all acceptable flashes, N_{tot}/N, is related to the directly available numbers N_2, N_3 and N_4. If the estimated fraction of

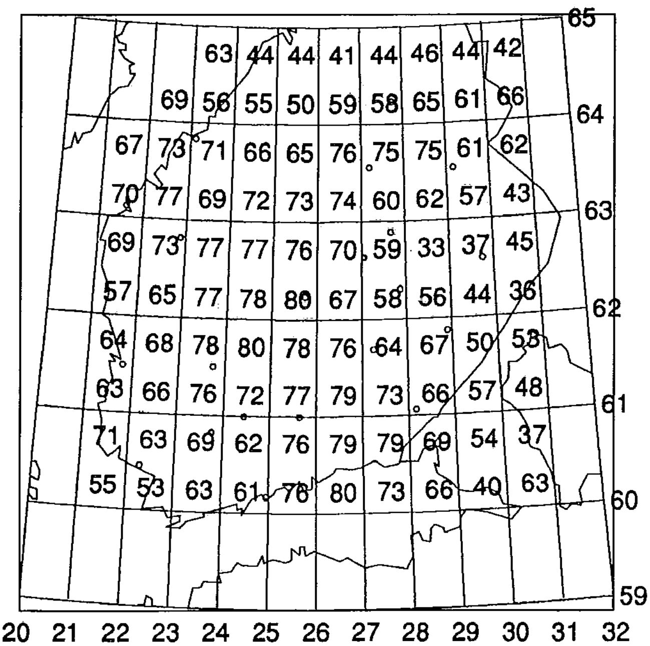

Fig. 6. The estimated detection efficiency (%) of the 4-DF location system. Based on the data of 1990 (about

acceptable flashes, 85 %, is included, the detection efficiency (%) of the location system, $P = 85\ N_{tot}/N$, is expressed with the ratio R as

$$P = 85 / (1 + (2.5\,R)^{-4/3}). \tag{6}$$

The deviations of the relation between R and N_{tot} from the smooth curve (5) may cause errors of a few per cent in P. The result for 1990 is shown in Fig. 6. Although the mean of the numbers clearly differs from the estimate given above, 78 %, this value is representative of many of the central squares. The low numbers in the eastern parts may be due to the sparsity of flashes (*cf.* Fig. 3).

4. *Comparison between located and counted flashes using pulse recording*

In July 1990, a lightning flash counter in Southern Finland, near DF1, was equipped with a recorder that takes 1024 one-microsecond samples of pulses triggered by the counter. The recording suffered from considerable noise, probably due to improper matching to the antenna, but a running average over 9 μs smoothed the pulses sufficiently for identification and classification. Various examples are given in Figs. 7-11, where the vertical scale is the electric field in arbitrary units; its relative range is larger for stronger pulses.

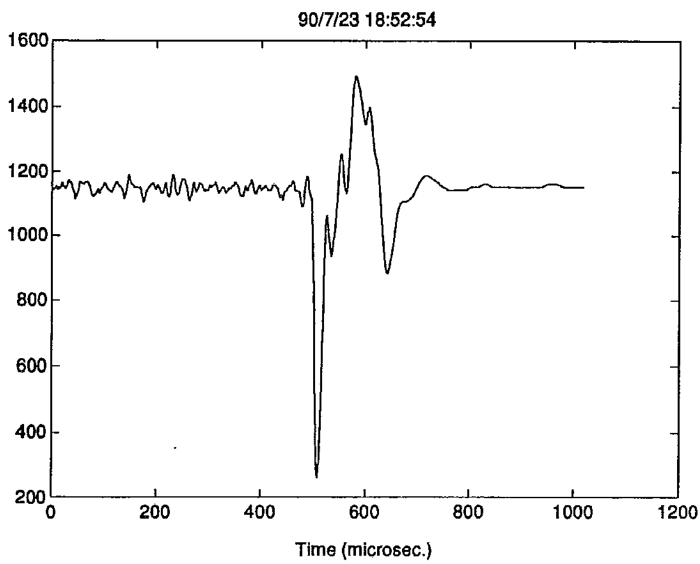

Fig. 7. Normal negative cloud-to-ground stroke.

Because of the quiet season, only 198 pulses were recorded; the location system detected 90 of them. The located flashes occurred within a radius of about 35 km from the counter, thus giving an estimate to its detection range. These flashes were stronger than average, only 22 being below 30 kA (when the signal strength given by a DF is normalized to a distance of 436 km, it is assumed to express the peak current of the return stroke in kA). On the other hand, within the same radius of 35 km there were 104 located flashes which were not detected by the counter. Most of these were negative and below 30 kA. Evidently, the counter has a high detection threshold (with respect to signal strength) compared to a direction finder, and, independently of strength, positive strokes tend to be ignored by the counter. 13 of the non-counted flashes occurred within 1 s after the previous count and therefore remained undetected by the counter.

There was one exceptional period (July 30) when the location system observed moderate lightning not very far from the region where the counter is situated, and the counter recorded 20 pulses during the same period, but there were no common observations. Few of these counted pulses resembled a CG flash, but they were associated with a certain type of electricity-network disturbances (see Section 5).

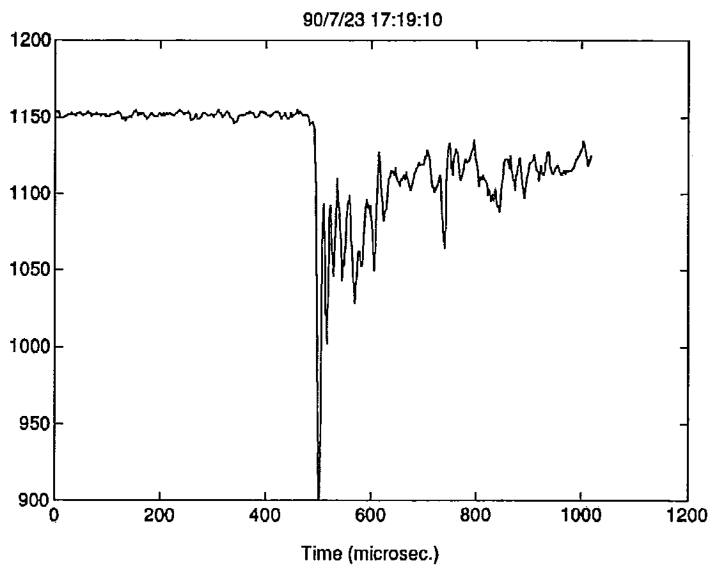

Fig. 8. Narrow negative pulse.

The recorded pulses were divided into five categories:

'S': normal negative strokes (Fig. 7), where the stepped leader (rapid weak variation) is followed by the negative return-stroke peak (base width 30-50 μs) and a positive overshoot.

'N': pulses resembling the S type (Fig. 8), with slightly narrower return-stroke peak (20-35 μs); the positive overshoot is missing and is often replaced by restless variation (noise).

'W': pulses resembling remotely the S type but being too wide (100 μs) and often having a slow rise; they may also be noisy (Fig. 9).

'C': bipolar pulses, where the highest peaks are often repeated at intervals of 100 μs; probably cloud flashes (Fig. 10).

'U': unidentified pulses, often weak and noisy (Fig. 11). Some of these may be stepped leaders which have triggered the counter before the return stroke.

The located 90 pulses were distributed in these categories as follows:

- 51 type S;
- 5 type N;
- 34 type U, of which probably
  - 24 stepped leaders,
  - 5 pulses contaminated by instrumental faults, and
  - 5 remaining unidentified.

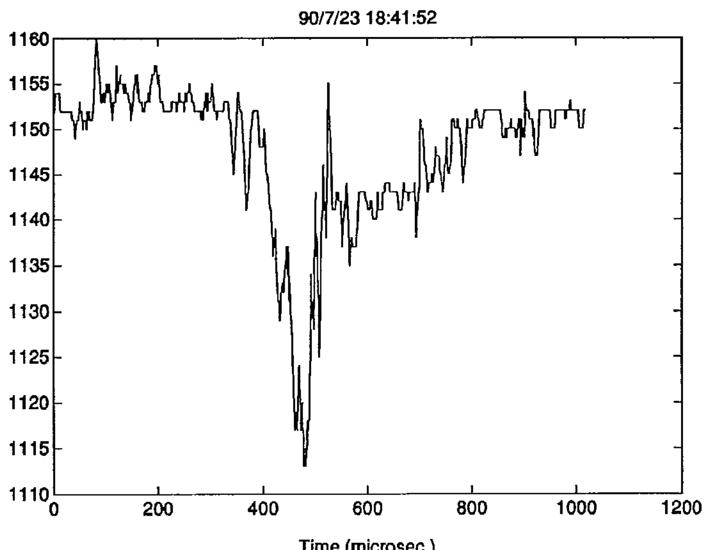

Fig. 9. Wide negative pulse.

In these U pulses, it is likely that the triggering was caused by other than the main peak of the pulse. In any case, it seems necessary to assume that all the located pulses were originated from true CG flashes; this is supported by the fact that they were at all accepted by two or more direction finders. Non-lightning pulses are usually so weak that at most one DF is close enough to detect them.

The non-located 108 pulses were distributed as follows:

11	type S;
15	type N;
17	type W;
40	type C;
25	type U.

The narrow pulses (N) resemble so closely negative return strokes that it is difficult to imagine sources other than CG flashes. But because there are few (5) narrow pulses among the located and more (15) among the non-located ones, the conclusion is suggested that part of the CG flashes — part of those of the narrow type — do not meet the criteria of the direction finders.

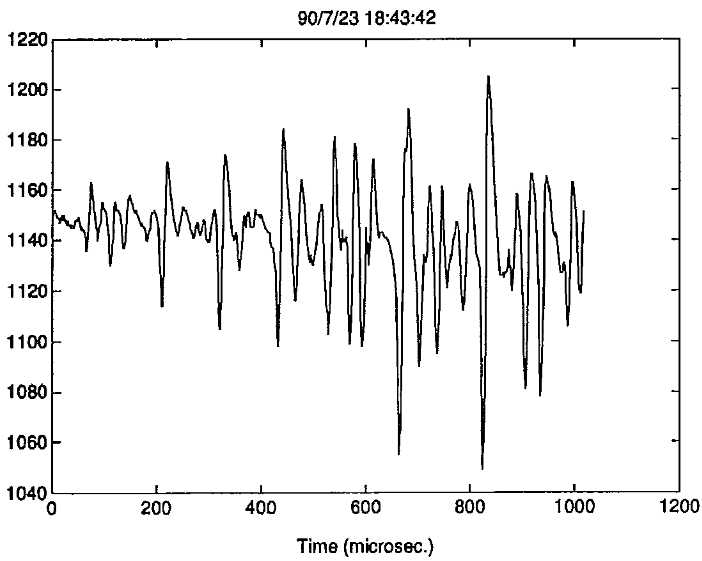

Fig. 10. Probably cloud flash.

Consequently, it is assumed that among the counted pulses, all the 90 located and the 26 non-located S ja N pulses are genuine CG flashes; hence the location system has detected 78 % of them. If this applies equally to the weaker located but non-counted flashes, one can imagine that these 104 flashes also represent 78 % of the number (133) that has occurred within the radius of the counter, and the true number of CG flashes in the area would be 249. The detection efficiency of 78 % is comparable with the result of about 70 % obtained by *Mach et al.* (1986) for a 4-DF system. The flash counter in turn has detected 80 % of the 249 flashes, that is, about an equal fraction, but many flashes have been replaced by other than lightning pulses, leading to a seemingly similar detection efficiency.

5. *Disturbances in a local electricity network*

The flash counter mentioned in Section 4 is situated in the area of a local electricity network (Hämeen Sähkö Oy), which has provided a list of disturbances (earth faults and automatic reclosings) from the pulse recording period. The number of probable lightning-caused disturbances was 116, of which 82, or 71%, were coincident with a located flash. This percentage is not very far from the detection efficiency of the location system obtained above.

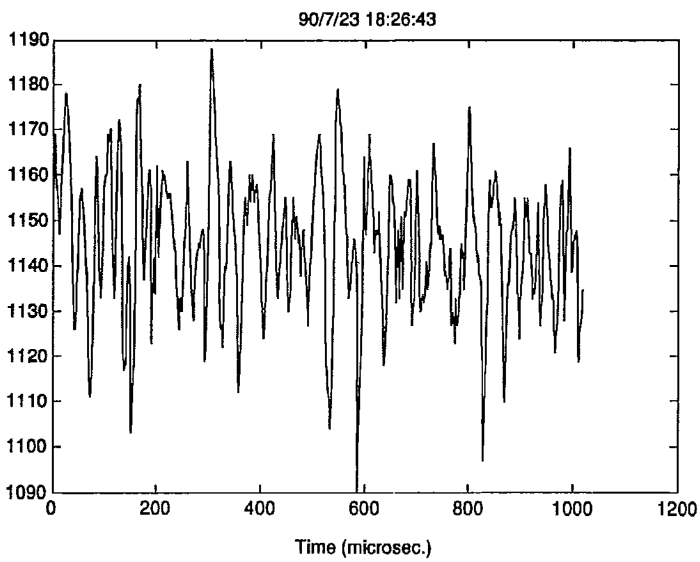

Fig. 11. Unidentified pulse.

The flash counter recorded 40 pulses coincident with the disturbances, and 36 of these were common with the located flashes. According to the distances of the flash counter from the disturbed power lines, the distance of the flash from the counter could be below 18 km in 31 cases, 18-30 km in 7 cases and over 30 km in 2 cases. This is not in contradiction with the estimated detection range of 35 km of the counter.

A systematic error of about -3°, in addition to an error of -5° already included in the location procedure, in the SW sector of DF2 was found when the flash locations were compared with the locations of the disturbed lines. The error has been taken into account in computing the locations for this report.

Except for the disturbances mentioned above, there were 96 cases of neutral-point voltage alarms (no interruptions caused) on July 30; none of these were located while 17 of the 20 pulses recorded by the counter in that period were coincident with these. 12 pulses were of type U, of which three may have also been cloud flashes. The remaining 5 pulses resembled narrow strokes. In spite of these N strokes, it seems likely that these events were not due to direct lightning, although their occurrence may be connected somehow to thunderstorms.

6. *Discussion and conclusions*

Lightning direction finders suffer from bearing errors, most of which are systematic and can be easily corrected once they have been found by some means. Optimization procedures, supported by knowledge of some of the largest errors, can be used to find the errors iteratively. After systematic corrections, the final locations for observations by more than 2 DFs can be adjusted by the same optimization method.

There remain random bearing errors of the order of 5° whose existence is difficult to explain, and so far, optimization seems to be the only method to minimize them. The mean location accuracy in the main area of coverage, which lies within 400 km from all direction finders, is about 5 km; for 4-DF observations, the number of which is 20-30 % of all observations, the error is generally smaller.

Direction finders have a certain set of criteria to discriminate other than genuine CG-lightning pulses. It seems that with the present criteria more than 10 % of CG strokes do not satisfy them. However, loosening the criteria is not necessarily justified because the possibility of accepting false lightning, already a drawback of flash counters, may increase considerably.

Apart from the absolute rejection of some of the CG strokes, a DF has a probablility of 55-80 % of actually accepting an acceptable stroke within the nominal range.

As a result of these factors, the overall detection efficiency of a four-DF location system (with DF separations of 200-300 km) is about 70-80 %.

A source of errors for the results of the pulse recording is the fact that not all located pulses were regular (type S). This means that some of the non-located pulses, in addition to those of the S and N types, may also have originated from CG lightning. As a result, the estimated detection efficiency would decrease closer to 70 %.

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