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Use of the Maximum Entropy Method to Predict Atmospheric CO_2 Content

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Abstract

The future of the Earth is threatened by the greenhouse effect, caused by the predicted rise in CO_2 content in the atmosphere. Past CO_2 observations are analysed and future values predicted by the maximum entropy method (MEM). The results suggest that the CO_2 content will increase only to 405 ppm by 2024 in contradiction to more dangerous values predicted earlier. Monthly variation within each year are utilized to predict the annual variations to increase from the present 8 ppm (peak to peak) to 16 ppm (peak to peak). No abrupt rise is to be expected in the atmospheric CO_2 content as long as the ecosystem retains its dynamic power.

1. *Introduction*

Recently several authors (*Clarke*, 1982) have been concerned about the CO_2 content in the atmosphere because the greenhouse effect threatens the environment. The present content of CO_2 is close to 350 ppm and the increase averages 1.3 ppm/year. The CO_2 content has been accurately monitored since 1957 in Hawaii (*Clarke*, 1982) indicating a stationary increase and there is less reliable evidence on the same growth rate from year 1850 onwards. The consumption of fossil fuels changes slowly. The opening of a new coal mine takes about 10 years and similarly a nuclear power plant becomes operational 10 years after the initial decision is made. Changes in the industrial energy demand and the land use practices are also stationary over decades.

The atmospheric CO_2-data are ideal for applying the autoregressive analysis, especially the maximum entropy method (MEM), to predict the future concentrations. Its feasibility follows from the multitude of trends superimposed in the data as e.g. the utilization of nuclear power, the oil price disturbances, the growth enhancement by using fertilizers. They all make up white noise in the CO_2 data and that makes it suitable for the autoregressive methods.

The use of fossil fuels intensified after the second world war and the annual release of CO_2 to atmosphere started to grow 4.5 %/year around 1950 and the trend continued until 1973. The growth rate has since dropped to 2.5 %/year (*Clarke*, 1982). The rather monotonous slow increase of the atmospheric CO_2 content does not reflect the strong increase in the consumption of the fossil fuels since 1950. The lack of this evidence can be understood with the dynamic action of the biosphere, each year the atmosphere exchanges some 30 % of its CO_2 content with the oceans and biosphere (*Clarke*, 1982). The monthly observations in Hawaii show annual fluctuations in the CO_2 content being 8 ppm higher during the winter in the northern hemisphere than during its summer. The curious fact in applying the MEM analysis is to look for a rapid starting trend buried in the CO_2 data warning for an environmental transient e.g. a collapse in the atmospheric exchange dynamics. That should be detectable years before it becomes evident with the statistical means.

2. *Method*

The CO_2 concentration recorded during the past 26 years can be used to extrapolate future values. A promising mathematical extrapolation program has been developed with the aid of the maximum entropy method (MEM) algorithm of Burg (*Russell*, 1978; *Andersen*, 1974). MEM uses N previous data to process the prediction with a digital filter the length of which M, can be selected. M must be smaller than N and the values of both N and M must be given in each analysis.

Let us consider a case in which a time series for the recorded data concerning a given process is known and the next value in the time series is needed. There are many possibilities for describing the process, and therefore some kind of mathematical model must be chosen (*Schlesinger and Mitchell*, 1986). In this case a linear model called as autoregressive model is applied, in which the process is represented by the following mathematical formula

$$future = x(t) = past + n(t) = \sum_{k=1}^{M} a(k)\, x(t-k) + n(t) \qquad (1)$$

where

$x(t)$ is the new value in the time series,
$n(t)$ is noise (the unknown during the transmission from the past to the future),
$a(k)$ are coefficients to be computed,

x(t–k) is the known time series and
M is the degree of the autoregressive model, or the length of so called predictor error filter.

The next problem is how to calculate the values of *a(k)*. This can be done in many ways but one of the most effective one is probably to use MEM, which maximizes the entropy of the information in a time series.

The entropy is defined by

$$H = \int_{-\infty}^{\infty} p(x) \log_2(p(x))\, dx \qquad (2)$$

where $p(x)$ is the probability associated with the value of x in a process.

Some restrictions must be made when maximizing the entropy. Let us assume that a process is almost or entirely stationary, so that the entropy will be maximized by considering also

$$\int_{-\infty}^{\infty} p(x)\, dx = 1 \quad \text{and} \quad \int_{-\infty}^{\infty} p(x)\, x^2 dx = \sigma^2 = \text{a constant}$$

In practice the entropy is not maximized directly by

$$\frac{dH}{dp(x)} = 0 \qquad (3)$$

but by various numerical methods, e.g. (*Andersen*, 1974; *Marple*, 1980).

In the present paper *a(k)*s are computed by the Burg MEM recursion algorithm which is as follows:

First the output power of the time series (S_1) is estimated after the digital filter order of one ($1, a_{11}$). The filter is operated in the time series both forward and backward, so

$$S_1 = \left(\sum_{t=1}^{N-1} ((x_t - a_{11}x_{t+1})^2 + (x_{t+1} - a_{11}x_t)^2)\right) / (N-1)/2 \qquad (4)$$

Minimizing S_1 as a function of a_{11} gives

$$a_{11} = \frac{2\sum_{t=1}^{N-1} x_t\, x_{t+1}}{\sum_{t=1}^{N-1} (x_t^2\, x_{t+1}^2)} \tag{5}$$

In the general case the length of the digital filter (m) is increased and the corresponding powers (S_m) are estimated that

$$S_m = [\sum_{t=1}^{N-m} ((x_t - \sum_{u=1}^{m} a_{mu}\, x_{t+u})^2 + (x_{t+m} - \sum_{u=1}^{m} a_{mu}\, x_{t+m-u})^2)] /(N\text{-}m)/2 \tag{6}$$

Smylie *et al.* (1973) have pointed out that

$$a_{mu} = a_{m\text{-}1\ u} - a_{mm}{\cdot}a_{m\text{-}1\ m\text{-}u} \tag{7}$$

where

m = 2,3,..., $N-1$
u = 1,2,..., $m-1$
$a_{mo} = -1$, $a_{mu} = 0$ for $u \geq m$

Using (6), (7) and

$$\frac{\partial S_m}{\partial a_{mm}} = 0 \tag{8}$$

the general a_{mm} can be solved and presented in the form

$$a_{mm} = \frac{2\sum_{u=1}^{N-m} BA_{mu}\, BB_{mu}}{\sum_{u=1}^{N-m} (BA_{mu}^2 + BB_{mu}^2)} \tag{9}$$

where

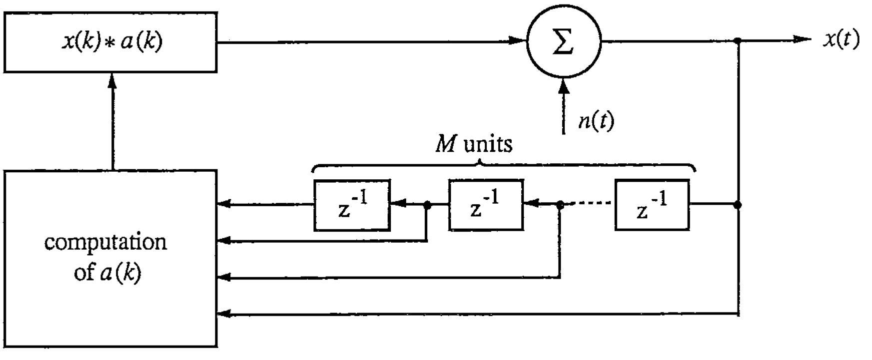
$$BA_{mu} = BA_{m\text{-}1\ u} - a_{m\text{-}1\ m\text{-}1}{\cdot}BB_{m\text{-}1\ u} \tag{10}$$

$$BB_{mu} = BB_{m\text{-}1\ u+1} - a_{m\text{-}1\ m\text{-}1}{\cdot}BA_{m\text{-}1\ u+1} \tag{11}$$

m = 2,3,..., M
u = 1,2,..., $N-m$
$BA_{1u} = x(t)$
$BB_{1u} = x(t+1)$

The final digital filter $(1,-a_{11},\ldots,-a_{MM})$ is normally presented in the form of $(1,-a_1,\ldots,-a_M)$ and it is called the prediction error filter.

In reality this kind of process can be described with a linear filtering of N recorded samples of $x(t-k)$. The system is as follows:



where

$*$ is the convolution,
z^{-1} is the time delay unit,
k = 1,...,M and
$n(t)$ is white noise

The by MEM calculated values of $a(k)$ are the best e.g. in the sense that the estimated mean square error of an M-order prediction is minimized and the loss of entropy in a linear predictive filter is maximized.

In practice when the new value in the time series was computed the oldest value was removed off at the beginning of the next cycle.

3. *Results*

The survey was based on the monthly and annual values of the CO_2 content during the past 26 years in Hawaii (*Clarke*, 1982). The reliability of the method was at first tested by the data prior to 1975 by calculating the values of CO_2 content for years

1975-1984, and by comparing the measured values with the calculated ones. The results are listed in Table 1, where the measured average values from 1975 to 1984 are compared with the predicted values. The predictions were based on either 15 or 10 previous years and the length of the filter, M, is marked. The predictions differed in average less than one ppm from the measured values. A similar test was carried out with the monthly values in the years 1981, 1982 and 1983, see Table 2. The predictions were based on the 48 previous monthly values with the length of the filter 30, and on the 24 monthly values with M equal to 15. The differences were listed and they exceeded one ppm only during the third predicted year in the calculation based an two previous years. The tests demonstrated the reliability of the extrapolations and assured that the selected ratio between N and M was correct.

Table 1. Extrapolation band of the observations during the period 1960-1974 (N = 15) and 1966-1975 (N = 10). N is the number of previous values. M is the filter length.

Year	Measured	Predicted $N = 15$ $M = 5$	$M = 10$	Predicted $N = 10$ $M = 9$
1975	330.7	331.6	331.8	
1976	331.7	333.1	333.2	331.9
1977	332.8	334.5	334.8	333.5
1978	334.6	335.8	336.7	334.7
1979	336.1	337.2	337.9	336.0
1980	337.6	338.5	339.5	337.0
1981	339.0	339.8	341.2	338.0
1982	340.0	341.1	343.0	339.0
1983	341.4	342.4	344.2	341.0
1984	343.7	343.7	345.7	342.6

Figure 1 presents the predicted annual CO_2 content for 40 years since 1984. The calculated values were based on 10 previous years, with $M = 5$, and 26 previous years with M equal to 10 or 25. No sign of rapid changes were found, the trends were mainly linear. A fluctuation with a period of 6 years was detected when the N and M numbers were increased. The accuracy of the prediction probably weakens towards the end of the period of 40 years.

Table 2. Extrapolation band of the observations during the period 1977-1980 (N = 48) and 1979-1980 (N = 24). The letters J,F,M,A *etc* are the first letters of each month. N is the number of previous values. M is the filter length. Δ is the absolute difference between the observation and the prediction.

Year Months	Measured	Predicted $N = 48$ $M = 30$	Δ	Predicted $N = 24$ $M = 15$	Δ
1981					
J	340.0	340.5	0.5	339.5	0.5
F	341.1	341.5	0.4	340.4	0.7
M	341.8	341.6	0.2	340.4	1.4
A	342.5	341.2	1.3	339.6	2.9
M	341.4	340.0	1.4	338.1	3.3
J	339.8	339.5	0.3	337.6	2.2
J	338.9	338.2	0.7	336.7	2.2
A	337.5	337.3	0.2	335.8	1.7
S	336.1	337.4	1.3	336.3	0.2
O	336.1	337.7	1.6	337.2	1.1
N	337.5	338.2	0.7	338.4	0.9
D	338.9	339.5	0.6	338.9	0.0
1982					
J	339.6	341.4	1.8	339.8	0.2
F	341.8	342.6	0.8	340.4	1.4
M	342.5	343.1	0.6	339.5	3.0
A	343.2	343.0	0.2	338.5	4.7
M	342.5	341.6	0.9	337.5	5.0
J	341.4	339.8	1.6	337.4	4.0
J	339.3	338.1	1.2	336.7	2.6
A	337.5	337.5	0.0	336.4	1.1
S	337.5	338.6	1.1	337.4	0.1
O	338.9	339.3	0.4	338.0	0.9
N	340.2	340.8	0.6	338.7	1.5
D	341.1	342.6	1.5	339.1	2.0
1983					
J	342.3	343.3	1.0	339.7	2.6
F	343.2	343.4	0.2	339.7	3.5
M	344.8	343.1	1.7	338.7	6.1
A	345.7	342.8	2.9	338.6	7.1
M	345.2	341.6	3.6	338.1	7.1
J	343.9	340.6	3.3	337.6	6.3
J	342.1	339.9	2.2	337.0	5.1
A	339.6	340.0	0.4	337.1	2.5
S	339.6	340.7	1.1	337.6	2.0
O	341.1	340.8	0.3	337.4	3.7
N	342.5	342.3	0.2	338.1	4.4
D	343.6	343.2	0.4	339.0	4.6

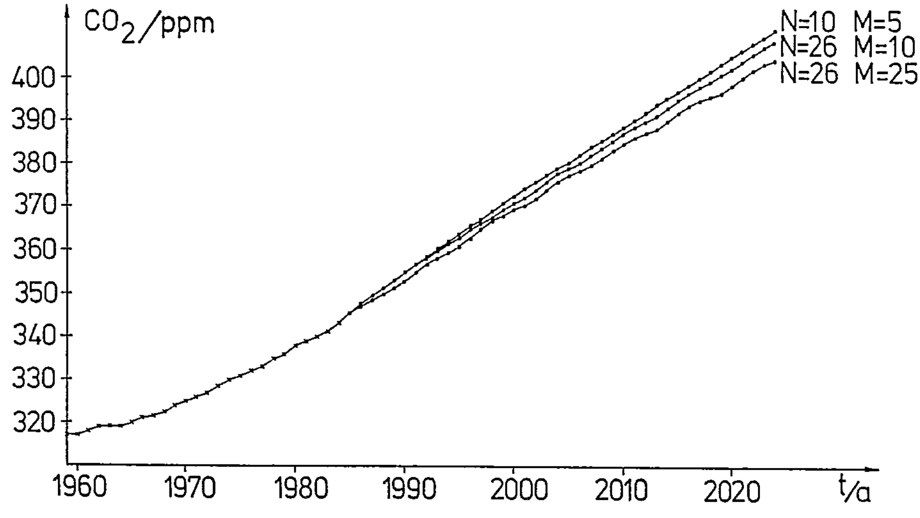


Figure 1. MEM predictions for the annual CO_2 concentration (dots) and the observed results (crosses). The number of initial values (N) is 10 (1975-1984) or 20 (1965-1984). The degree of the autoregressive model (M) is 5, 10 or 25.

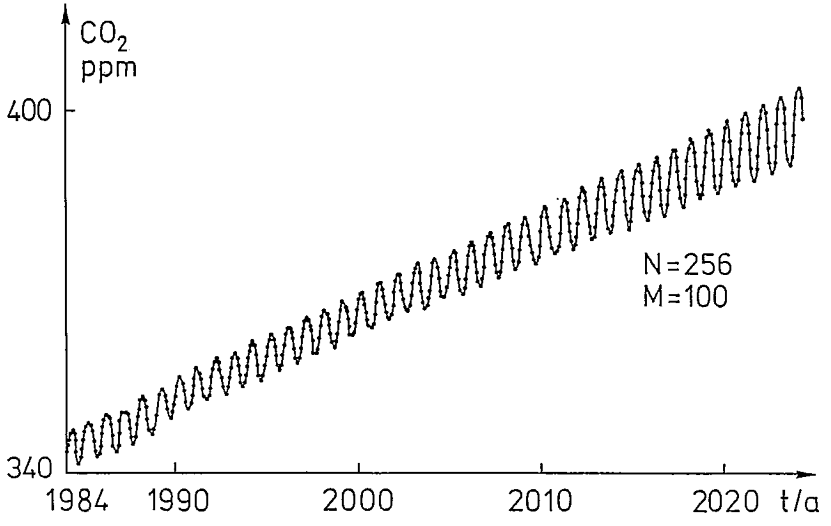


Figure 2. MEM predictions for the monthly CO_2 concentration in the atmosphere. The number of initial values is 256 (from about 1963-1984 and the degree of the autoregressive model is 100).

Figure 2 shows the predicted annual fluctuations for 40 years. The prediction is based on 256 monthly values with $M = 100$. The annual peak to peak variation of the CO_2 - content seems to gradually increase from the present value of 8 ppm. The prediction was extended to year 2024 in order to display the present trends and not to believe that the prediction accuracy remains high towards the last decade.

The trends can also be studied with the power density spectrum $S(f)$ in Figure 3, which is computed by MEM based on the last 256 monthly CO_2 values. The periods longer than the one year are probably connected to the skewness of the annual variation. The spectrum also reveals longer periods, 6 years and 43 years.

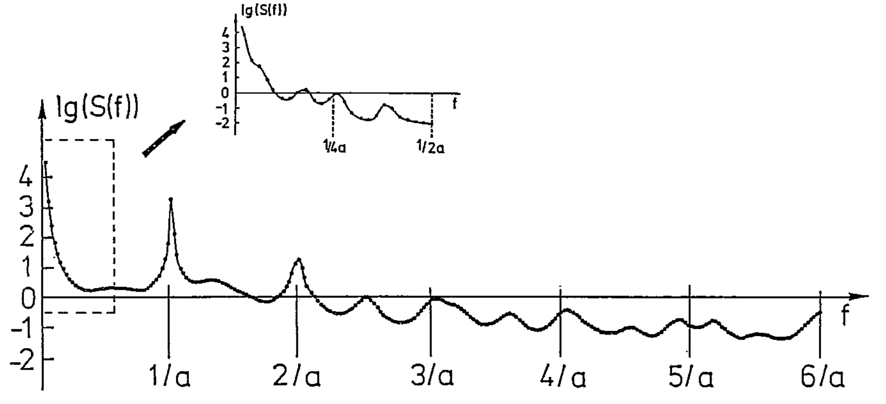


Figure 3. Logarithm of the power density spectrum of the time series for CO_2 concentration in the atmosphere. The length of the time series is 256 values. The sampling interval is one month. The filter length of 40 is used for the higher frequencies and the inset shows the lower frequencies which is based on 25 annual averages with the filter length of 20.

4. *Discussion*

This study was started because recently articles in newspapers and in other popular magazines claim that an exponentially growing increase in the CO_2 content is to be expected within the next few years. The statistical analysis should reveal the beginning of this trend, but nothing turned out. Usually the MEM analysis does not emphasize the linearity of the data, but reveals the exponential trends in them. The many superimposed factors affecting the CO_2 release into the atmosphere form a white noise type statistical distribution and are suitable for the MEM treatment. The anthropogenic release of CO_2, the technological improvements in industrial energy demand and the changes in the land use practices all continue to fulfil the white noise pattern in the future as long as it can be estimated in the literature (*Clarke*, 1982). In the past, the most abrupt single change in the CO_2 input was the discontinuation in the growth rate of the CO_2 release into the atmosphere at 1973. Its effect can not be detected in the CO_2 data in Hawaii during the next decade. This demonstrates the complexity of the atmospheric CO_2 model on which we do not present any other facts than the peaks in the $S(f)$-spectrum. The new trend discovered in this research was the gradual growth in the annual peak to peak variation which indicates a doubling of the annual amplitude after more than one hundred years. It would be feasible to recalculate in future the short range prediction every year in order to reveal a warning signal of a rapid change in the CO_2 content as soon as it appears.

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