528.026.2
528.27
550.831

GRAVITY MEASUREMENTS ON THE ICE OF THE BOTHNIAN BAY

by

PEKKA LEHMUSKOSKI and JAAKKO MÄKINEN

Finnish Geodetic Institute
Helsinki, Finland

Abstract

The authors describe measurements made in March 1976 and February 1977. Worden Master gravimeters, Decca positioning and helicopter transport were used. The influence of background noise from ice motions on the gravity observations is discussed and a mean accuracy of 0.1 mGal is claimed despite an average noise level corresponding to 0.5 mGal on the gravimeter display. The standard error in the point anomalies is 0.2 mGal, which compares favourably with alternative measurement methods.

1. *Introduction*

During 1977 the Finnish Geodetic Institute completed its nation-wide gravity net where the spacing is five kilometres. The net now covers all of continental Finland and the archipelago. Including local and regional densifications it contains 21 100 stations. When this work was approaching completion, the next step was to start investigating methods to extend the net to the open sea, where only a 30 kilometre net measured with an underwater gravimeter currently exists (HONKASALO [8, 9]). The fact that during severe winters the Gulf of Finland and the Gulf of Bothnia are completely covered with ice several decimetres thick suggested that it might be possible to observe gravity with conventional gravimeters mounted on the ice. Such a method, if feasible, seemed to offer considerable geodetic and practical advantages over shipboard or bottom gravimetry.

In Finland the first gravity observations on ice were made in 1958 by VANNINEN [27] in a local survey around Helsinki. In 1966 KIVINIEMI [12, 13] made experi-

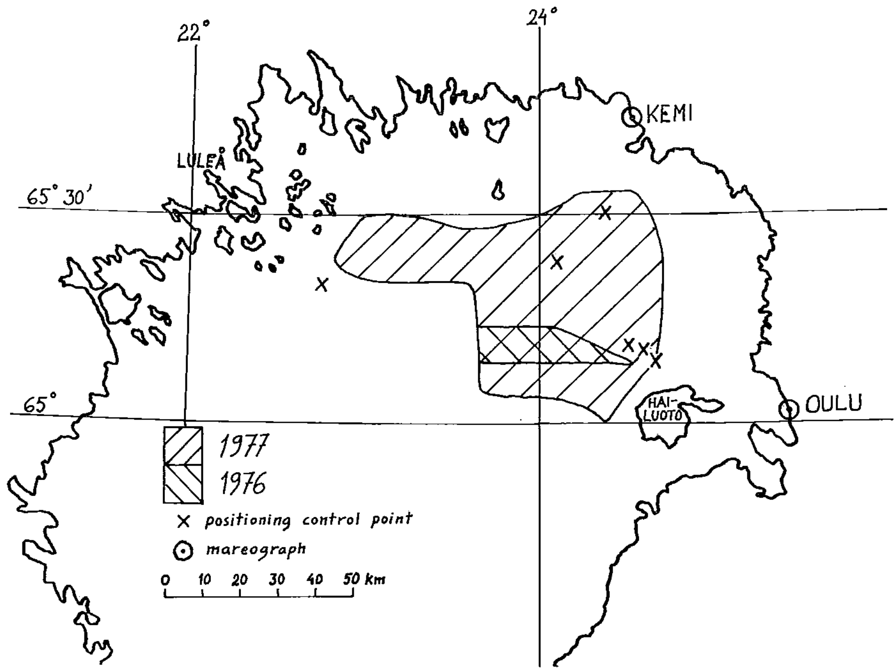


Figure 1. The areas surveyed in 1976 and 1977, the positioning control points, and the mareographs from which the sea level was determined.

mental observations when investigating ice movement. Then a series of test measurements was performed by Prof. Kiviniemi and P. Lehmuskoski on the ice of the Bothnian Bay from March 8th – 13th, 1976. Eleven new gravity stations were observed and the results were so encouraging that in February 1977 a larger-scale observation campaign was mounted. Between February 14th and 26th the authors, using Worden Master gravimeters, observed 160 new gravity stations. Sixty of these went into a local densification, and the net thus covers about 2500 km^2 (Figure 1). This paper describes the measurements of 1976 and 1977. An earlier version of it was published in Finnish (LEHMUSKOSKI and MÄKINEN [18]).

2. *Other gravimeter measurements on sea or lake ice*

The first description of gravity observations on ice, as far as we know, was that given by CRARY *et al.* [4]. Using a North American gravimeter they made observations at six sites in the Beaufort Sea. PLAUMANN [24] gives an account of observations on lake ice using a Worldwide instrument. Extensive measurements around the Sverdrup Islands in the Arctic Ocean are described by SOBCZAK [26], who used Worden gravimeters with some extra damping, and by WEBER [28], who used a LaCoste-Romberg recording gravimeter of the overdamped underwater type. The accuracy in their measurements was of the order of 0.1 – 0.2 mGal. CHOJNICKI [3] made experimental observations in the southern Baltic with an Askania Gs-11 tidal gravimeter.

In a closely related field, HUNKINS [10] and LESCHACK and HAUBRICH [20], among others, have used gravimeters in investigations on the movement of sea ice. A review of work in this direction is given by ROBIN [25].

3. *The gravity measurements*

3.1. The observation method

A gravimeter mounted on ice is influenced by the accelerations and tilting caused by its movements. The situation is thus similar to shipboard observations, in which context an analysis of the problems and their solutions can be found in LACOSTE [17]. On ice, however, the magnitude of the disturbances is often essentially smaller and they can be dealt with by simpler methods. We have been able to use ordinary land gravimeters. A form of averaging out the disturbances has been used, but the systematic corrections (LACOSTE [17]), which are mostly second order, have been neglected, as has the tilting. For a Worden gravimeter the direct effect of a 4.″1 tilting, for instance, observed by KIVINIEMI [13] under severe conditions, is

$$dg = g(\cos 4.''1 - 1) = -0.2\,\mu\text{Gal} \tag{1}$$

We have used two gravimeter models, namely the LaCoste-Romberg model G (no. 55), used only during the 1976 tests, and the Worden Master (nos. 227 and 934), no. 934 being used only in 1977. In these gravimeters the reading is ordinarily obtained by balancing a moving crosshair on a stationary line with the aid of a calibrated dial. The dial position then indicates the reading. On ice, the disturbances mentioned above cause a more or less irregular fluctuation of the crosshair. By watching the fluctuation long enough the observer can either visually, or by recording its progress, determine its centre. We have relied on visual averaging and subsequently adjusted the dial so that the centre coincides with the stationary line. Another alternative would be to determine a correction to the dial reading directly from the position of the centre, as KIVINIEMI [11] and PLAUMANN [24], who both recorded the crosshair positions, have done.

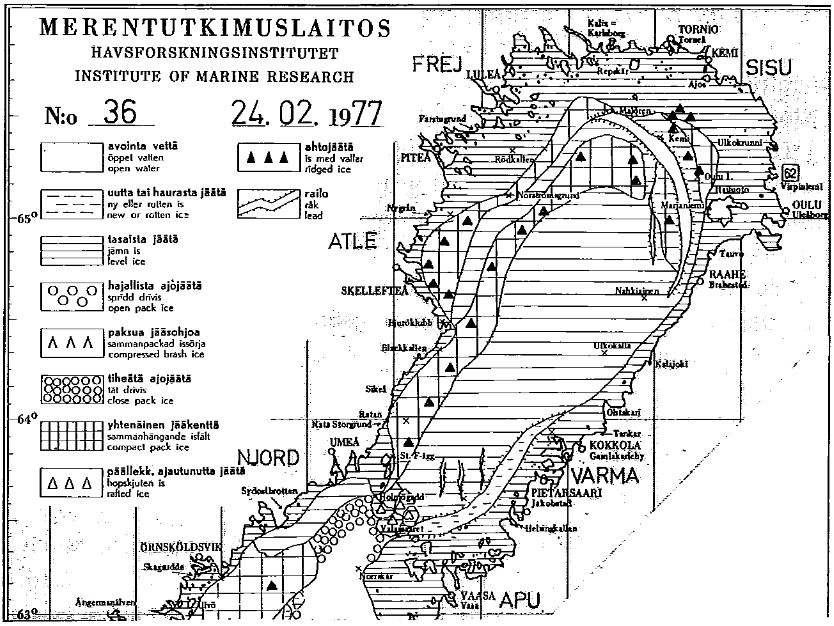


Figure 2. The ice situation on February 24th, 1977. The large lead across the observation area was formed by a strong eastern wind during the three preceding days (see Fig. 3).

In our measurements a gravity observation consists of two or three successive readings, each involving about two minutes of continuous crosshair watching.

Because of the visual averaging, the »centre» in our observations is a somewhat intuitive notion. It is not quite the time average, as in [24], nor is it the middle point of the average extreme positions, as in [13].

Of the two gravimeter models used, the Worden Master proved more suitable than the LaCoste-Romberg model G. The crosshair movements are more subdued and regular, and their potential range is considerably larger. Under good observation conditions the results of the two gravimeters agreed reasonably well (see chapter 3.4.3.).

The sensitivity of the G-55 was 8 eyepiece divisions for 10 counter units. In the Worden no. 227 the outer inside lines in the reticle corresponded to 0.7 mGals and in no. 934 to 0.6 mGals.

The amplitude of the crosshair fluctuation (hereafter also called noise level) was estimated at every station. If it was over 1.0 mGal, the gravity observation was not considered acceptable. During the 1976 field work acceptable observations could be made on four days out of five, and in 1977 on eight out of eleven. The r.m.s.

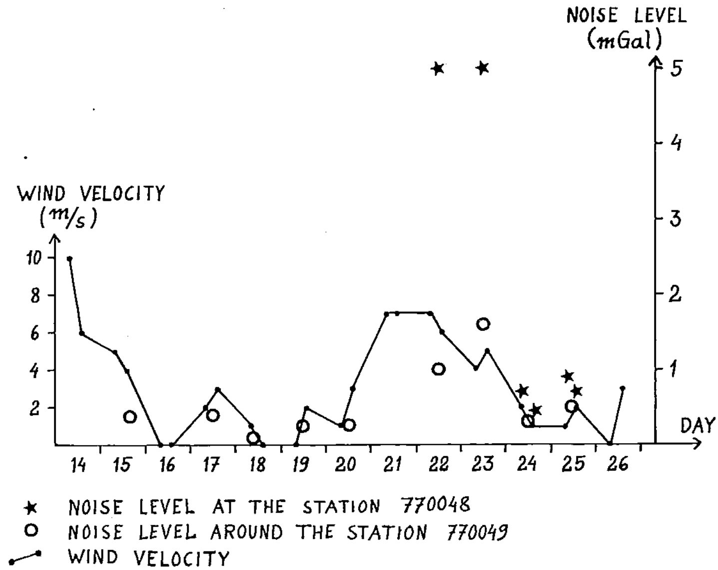


Figure 3. Wind velocity in Hailuoto and noise level at two sites in February 1977. For their positions see Figure 8. The 770049 is actually represented by the closest station of the day, if not further than 10 km.

noise level in these 1977 observations was 0.47 mGal. The largest amplitude observed by us was about 5 mGal, but on the best days in 1976 the disturbances were not worse than those caused by microseism in ordinary land measurements.

Both winters were about average as far as ice conditions were concerned. The situation towards the end of the 1977 field work is shown in Figure 2.

No systematic notes were made of other aspects of the crosshair fluctuation. However, the presence of a basic period of about twenty seconds, mentioned by KIVINIEMI [13], was noticed. A fast, rather irregular movement was often prominent on thin ice, especially near open water. Sometimes there seemed to be an underlying period of about one minute. It was, in general, possible to link sudden powerful shocks with the formation of breaks in the ice. Ice thickness was an obvious damping factor, as was distance from the open water. With the exception of wind velocity we were not able to detect any connection between weather conditions and noise level. Figure 3 shows the wind velocity and the noise level at two sites during the 1977 observations.

3.2. Transport

A technical problem appeared in connection with transport. A four-seater Agusta-Bell 206 Jet Ranger helicopter was used. It is powered by a turbine, and the turbine cannot be stopped and started, say, forty times a day. Thus the helicopter, if it is not sent away, disturbs the observations by transmitting vibrations into the ice. On the other hand, it cannot very well hover in the neighbourhood, because the pressure waves generated by the rotor have a similar effect. Therefore a plan was devised whereby the helicopter would shuttle between two observers working separately and move each one in turn by five kilometres (or the net spacing). This method worked excellently when a local densification with a 2.5 kilometre spacing was measured. However, in the 5 kilometre net, where large areas had to be covered, the increased flying time per station resulted in an impracticably short range. Now, the ice was actually a mosaic of small floes which had broken apart and frozen together again. It turned out that if the observer and the helicopter idling on ice were separated by a fault in the ice field, the vibration was generally damped to a tolerable level. Only in a few cases, on thin ice, was it necessary to send the helicopter away. This success must be attributed to the fact that the helicopter was equipped with air-filled rubber floats.

In the measurement of the five kilometre net we used an average of eleven minutes per station. This includes the total time from leaving the base in Hailuoto to returning: the flights to and from the measurement area, flying between the stations, and the observations themselves. In the 2.5 kilometre net (with the two observer method) the corresponding figure was five minutes.

3.3. Computation of the gravity values

The scales of all gravimeters were checked on the Vihti calibration line and the measurements were joined to the Finnish first order net (KIVINIEMI [11]). Drift was checked once or twice a day and removed linearly. The longest closure time was seven hours. The Worden no. 227 had to be run unthermostated because of a breakdown in the heating system, and showed temperature rises of up to 8°F between closures. Its greatest daily drift was 0.9 mGal. The average drift of no. 227 during transport was 49 μGal/h and that of no. 934 only 16 μGal/h. For comparison we might mention that in an extensive measurement programme with car transport the corresponding figure for no. 227 (thermostated) was 42 μGal/h (KIVINIEMI [11]).

3.4. Accuracy of gravity observations

3.4.1. General

The errors in our gravity observations can be roughly divided into three components, in this chapter referred to as errors of the first, second and third kinds, respectively:

(i) The error of the observer in estimating the mean crosshair position.

(ii) The instrument error, which can be thought to consist of the effect of the disturbances and a basic part present even in ordinary land measurements.

(iii) The deviation of the time average of vertical forces from gravity.

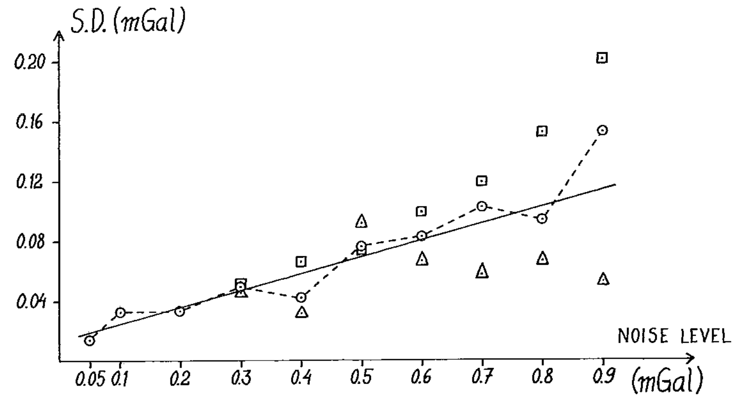
In order to estimate the errors we analysed

(a) the successive readings which make up the observations,

(b) simultaneous observations by two observers,

(c) repeated observations at two stations.

This material is presented in Figures 4, 5, 6 and 7. Only those observations where the noise level did not exceed 1.0 mGal have been used in this chapter.



		0.05	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
N	JM/W 934 (△)	1	20	17	27	13	2	24	3	10	5
	PL/W 227 (□)	3	14	19	31	4	11	19	6	3	6
	POOLED (⊙)	4	34	36	58	17	13	43	9	13	11

Figure 4. The standard deviation (s.d.) of the readings as a function of the noise level. The observers are identified by their initials and the instruments by their numbers. The number of degrees of freedom is denoted by N. The approximating straight line refers to the pooled s.d.

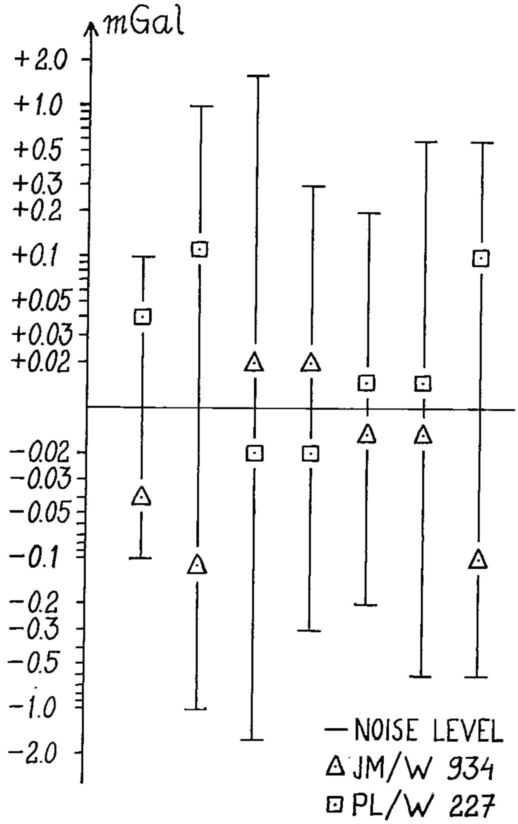


Figure 5. Simultaneous observations in February 1977. Observers/instruments are identified by their initials/numbers. All observations are at different stations. The station means lie on the horizontal axis. The observations are arranged chronologically (no time scale). The noise level is shown. Note the logarithmic vertical scale.

3.4.2. Reading accuracy and noise level

The first kind of error can be expected to depend primarily on the noise level, and its magnitude, with the exception of observer bias, should show in the dispersion of the readings (a). In Figure 4 we have plotted the standard deviation of the readings against noise level. While with the other observer (PL) the standard deviation of the readings grows fairly regularly with increasing noise, with the other (JM) the situation is more irregular, and there is a surprising drop at high levels. This might, for instance, be due to differing instrument characteristics, although it must be mentioned that in simultaneous observations the independent

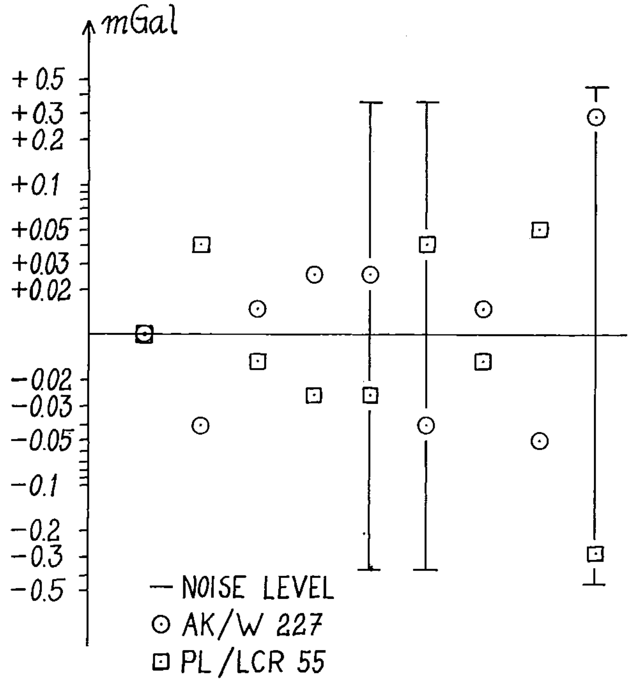


Figure 6. Simultaneous observations in March 1976. The explanations of Fig. 5 apply.

noise level estimates by the two authors never differed by more than 0.1 mGal. Another possibility is that the noise level does not describe the observing conditions sufficiently well; most of the high-noise observations by the two authors were made on different days.

In Figure 4 we have also plotted the pooled standard deviation, and drawn the corresponding least squares regression line, the slope of which is about 0.11.

When observations made under different noise conditions are compared, the expected differences in accuracy must somehow be allowed for. We tried the following simple model: According to an estimate by Kiviniemi [14], the basic instrumental error for the Worden no. 227 has been of the order of 0.05 mGal in the ordinary land measurements of the Finnish Geodetic Institute. If, in addition, only the reading dispersion is considered, the standard error of a gravity observation on ice is

$$E = \sqrt{0.05^2 + f(L)^2/n} \text{ mGal} \tag{2}$$

where $f(L)$ is the standard deviation of a reading for the observer/instrument com-

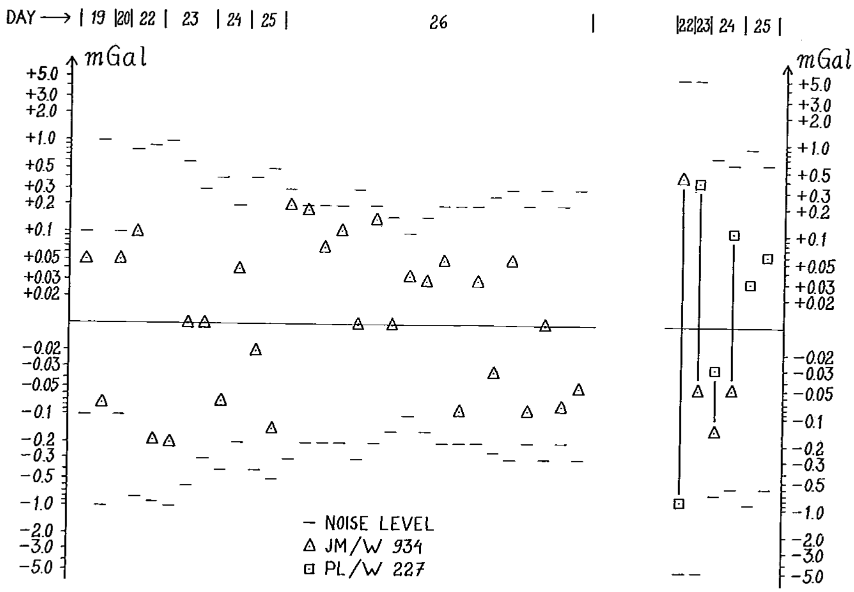


Figure 7. Observations at the test stations 770573 (left) and 770048 (right) in February 1977. The station means lie on the horizontal axis. Observation dates are shown (no time scale). Simultaneous observations are joined by a vertical line. The vertical scale is logarithmic.

bination in question at noise level L and n is the number of readings going into the observation. We have used the empirical values of Figure 4 for the functions f.

According to the model (2) the average accuracy of the 1977 observations would be

$$m_g' = 0.07 \text{ mGal} \tag{3}$$

calculated as the quadratic mean of all E values.

3.4.3. Does the result depend on the observer/instrument?

To get an idea of the magnitude of possible systematical differences between observers/instruments we then compared the simultaneous observations (b). It should be noted that many of the systematical errors caused by the disturbances are about the same for similar instruments under similar circumstances, and are not necessarily shown here. It is also impossible to separate observer/instrument effects since these combinations have been fixed.

Eight observation pairs in Figures 5 and 7 were used. The differences were

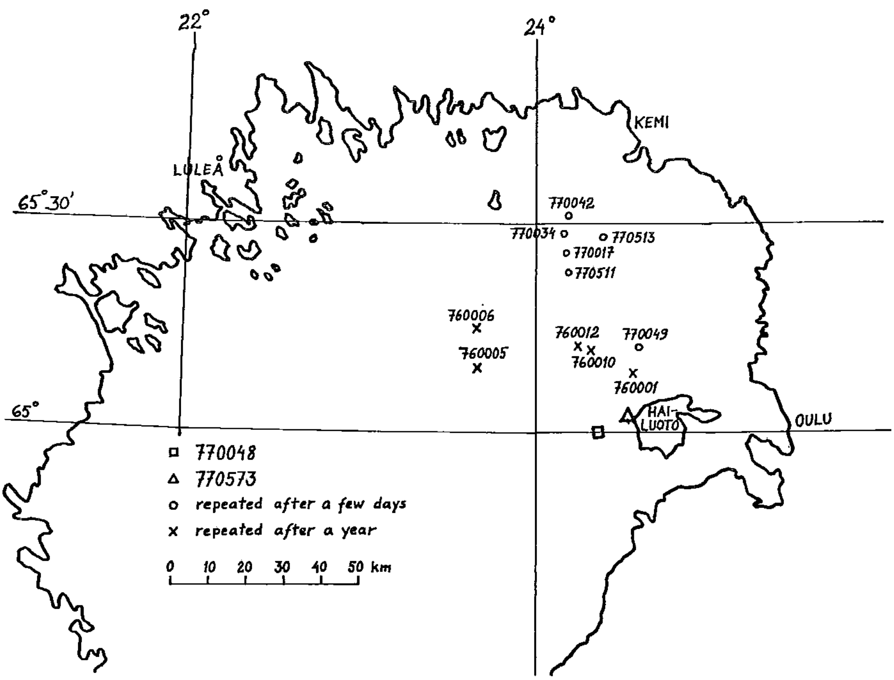


Figure 8. The positions of the test sites of Figures 3 and 7 and Tables 1 and 2.

compared with their expected standard errors computed from (2). Although in general formula (2) seemed to involve no serious underestimation of the error, there was a distinct systematic difference between the results of the two observers/ instruments, of about the same order as the error predicted by (2). At this stage it is impossible to tell whether this is caused by some factors peculiar to ice measurements, or simply by, say, parabolic drift due to temperature change. The fact that unexpectedly large differences were reached at high noise levels points to the former explanation.

The simultaneous observations made in 1976 are shown in Figure 6. Because of a battery failure and resulting highly irregular drift in the Worden no. 227 we have only computed their r.m.s. difference, which is 0.20 mGal. If the instruments are assumed equally accurate, this yields a standard error of 0.14 mGal for a single observation.

3.4.4. The average accuracy of the observations

Our estimate of the accuracy of the 1977 observations is based on the results (c) at the two test stations 770048 and 770573 (Figure 7). Their positions are shown in Figure 8. These observations are spread over several days and have been made under varying noise conditions. Their r.m.s. noise level (0.49 mGal) is about the same as that of all gravity stations. Thus we have assumed that their dispersion gives a picture of the true accuracy of the gravity observations. At first, weighting according to (2) was tested. From inspection of residuals it became obvious that formula (2) did not reflect the accuracy of the observations. For example, the standard error of the weight unit was

$$s_0 = 1.51 \qquad (36-2=34 \text{ degrees of freedom}) \tag{4}$$

constituting a significant deviation from unity. Thus, it seems that the reading dispersion does not sufficiently describe the error caused by the disturbances.

The accuracy of the observations (c) was then computed using unweighted observations, and the value

$$m_g = 0.10 \text{ mGal} \tag{5}$$

was obtained. We have assumed that this also represents the average accuracy of the 1977 observations. It should be noted that strictly speaking (5) also contains part of the reduction error from sea level determination (see chapter 4.1.).

The possible presence of certain errors of the third kind is discussed in chapter 5.3.

4. *Reduction of the measurements; supporting observations*

In order to compute free air anomalies at the stations we need their positions and elevations. For the Bouguer anomalies we also need the depths. In this section we describe how these data were obtained and how the errors affect the accuracy of the anomalies.

4.1. Vertical control

The variation in sea level with time is about two metres a year along the Finnish coast, although during our observations it was only thirty centimetres. In any case, both the actual elevation of the observation point and the gravitational attraction of the changing volume of water must be considered in order to make observations conducted at different times compatible.

We have approximated a water layer of thickness t by an infinite horizontal slab of density 1.00 g/cm^3; its gravitational attraction is then

$$dg = 0.0419\ t \quad \text{mGal} \tag{6}$$

if t is expressed in metres. The sea level has been reduced to zero elevation in the N60 system [15]. All gravity values have also been reduced to this same elevation, and consequently the complete formula reads

$$g = g' + 0.3086\,(h + h_0) - 0.0419\ h \quad \text{mGal} \tag{7}$$

where g and g' are the reduced and the observed gravity, respectively, h is the sea level in the N60 system at the station during the observation and h_0 is the elevation of the observation point above the free water surface. The ice mass requires no special consideration; if we make an idealisation similar to the one in the derivation of (6), the attraction of the ice equals that of the displaced water.

The reduction to the N60 zero elevation is consistent with the treatment of ordinary land measurements. Certain aspects must, however, be pointed out:

1) Because of tilting due to land uplift, the surface formed by points with zero elevation in the N60 system is no longer an equipotential surface of the gravitational potential. The greatest uplift difference along the Finnish coasts, between Raahe and Hamina is 6.3 mm/a, or an uplift difference of 11 cm during the time 1960.0 – 1977.0 (Kääriäinen [15, 16]).

2) The mean sea level of the Gulf of Bothnia and of the Gulf of Finland is, for geophysical reasons, not an equipotential surface, either. The largest difference, 61 mm, occurs between Kemi and Helsinki [15].

3) Because of land uplift the surface formed by points with zero elevation in the N60 system is rising away from mean sea level. The maximum occurs at Raahe, 8.3 mm/a [15, 16].

At this stage, the bias introduced by these discrepancies must be considered insignificant. If, however, at some later date the geoid surface determined from gravity observations on ice is compared with, say, results from satellite altimetry, the distortion must be considered.

The water level was interpolated from the mareographs at Oulu and Kemi (Figure 1). The error of this interpolation is of the order of 3 cm according to Mälkki [21], which causes an error of 0.01 mGal in the reduction, eq. (7).

The elevation of the observation points above the free water surface was determined indirectly from the ice thickness by assuming that one ninth of the total thickness lies above water. Since our stations are usually near minor ice ridges this assumption does not strictly hold. According to Palosuo [23] the deviation is maximally 3 cm, and one centimetre on average. In addition, the loading by snow (0.5 m in places) must be considered.

Ice thickness was measured by drilling at 18 sites and estimated elsewhere from these results. It varied from 0.3 to 0.7 metres. Thus we can assume that at the stations it was known with an accuracy of 0.2 m, which, according to the one ninth rule, corresponds to 2 cm in the elevation of the ice surface above water. Considering the error sources mentioned in the preceding paragraph, we have assumed (in order to be on the safe side) that the elevation of the observation point above the water surface was known with an accuracy of 5 cm. The effect of this error through eq. (7) is 0.02 mGal, and the total error made in the height reduction

$$m_h = \sqrt{0.01^2 + 0.02^2} = 0.02 \text{ mGal} \tag{8}$$

4.2. Horizontal control

The North Bothnian Decca Navigator chain was used for both navigation and positioning. The Decca coordinates for the five-kilometre grid were computed in advance and the landing was made within 0.1 lanes (about 300 m) of them, after which the final coordinates were read from the decometers.

In 1976 we had a Mk 19 receiver, and in 1977 a Mk 21. During the former field season the accuracy of the positioning was not controlled, but the receiver was checked later. During the 1977 observations we always read the decometers at the lighthouses in the measurement area when convenient, and during each flight at least one such check was made. The positions of the six lighthouses are shown in Figure 1.

From 22 readings at six sites we computed the following standard deviations for the Red and the Green decometer readings

$$s_R = \pm\, 0.012 \text{ lanes} \qquad s_G = \pm\, 0.014 \text{ lanes} \tag{9}$$

This corresponds to a 50-metre standard error of position. A previously measured site can thus be relocated with an accuracy of $50\sqrt{2} = 70$ m. This is slightly better than the manufacturer's performance estimate in [2].

The preceding discussion concerns only the internal accuracy of the Decca system. Its hyperbolic coordinates have yet to be converted into geographic ones. The data (transmitters' wavelengths and coordinates) needed for this were supplied by the Board of Navigation, and checked by computing the Decca coordinates of the six lighthouses mentioned above and comparing them with the observation means. The differences between the computed and observed coordinates, »fixed error corrections» in Decca parlance, were subsequently compared with those determined by FAGERHOLM and THUNBERG [5] in a large-scale survey covering the whole Gulf of Bothnia. We found that the difference between our set of corrections and theirs was nearly constant. Its mean (c) and standard deviation (s') were

$$c_R = 0.065 \text{ lanes} \qquad c_G = 0.091 \text{ lanes} \tag{10}$$

$$s'_R = \pm 0.020 \text{ lanes} \qquad s'_G = \pm 0.008 \text{ lanes} \tag{11}$$

for the Red and the Green components, respectively.

FAGERHOLM and THUNBERG's [5] set of corrections, amended by eq. (10), were then used for the whole measurement area.

We have assumed that the residual variation as given in (11) represents the errors of the two correction determinations, and that the determinations are equally accurate. Then the remaining positioning error from this source is estimated as

$$m'_R = \pm 0.014 \text{ lanes} \qquad m'_G = \pm 0.006 \text{ lanes} \tag{12}$$

In an earlier paper (LEHMUSKOSKI and MÄKINEN [18]) we quoted essentially larger values for the uncertainty of the fixed error corrections. This happened because we unfortunately misinterpreted the meaning of the error charts given in [5]. We now know that they express the computed minus the observed value, as is explicitly stated in [2], where they are reproduced.

Combining (12) with the accidental errors (9), we get the total positioning error in Decca coordinates.

$$m_R = \pm 0.018 \text{ lanes} \qquad m_G = \pm 0.015 \text{ lanes} \tag{13}$$

This corresponds to a 65-metre standard error of position with a 45-metre component in the north-south direction. The north-south error causes a 0.03 mGal error in the normal gravity and consequently in the anomalies. For practical purposes, the rest of the positioning error could well be ignored in a five or even in a two-and-a-half kilometre network. However, if we make assumptions about the horizontal gradient of the anomaly field, we can translate it into an error of the point anomalies. We have estimated that in the measurement area the r.m.s. horizontal gradient of the Bouguer (and the free air) anomaly field is 2 mGal/km. The 65-metre error of position then causes an average error of $2 \times 0.065/\sqrt{2} = 0.09$ mGal in the anomaly. Thus the total contribution of the positioning error is

$$m_P = \sqrt{0.09^2 + 0.03^2} = 0.10 \text{ mGal} \tag{14}$$

4.3. The determination of depths

The Bouguer reduction for gravity measurements on sea ice is calculated by replacing the water volume by a mass of normal density 2.67 g/cm³. We have made this reduction by simply adding an infinite horizontal slab of density 1.67 g/cm³. Thus the formula for the Bouguer reduction is

$$R_B = 0.0700\, d \text{ mGal} \tag{15}$$

where d is the sea depth in metres at the station. The correction for bottom topography has been neglected; a couple of spot checks yielded only insignificantly small values.

Depths we read from sounding charts prepared by the Board of Navigation on the 1:20 000 scale (for about 80 % of stations) and from Finnish and Swedish nautical charts on the 1:50 000 to 1:200 000 scales (for the rest). The greatest depth was 100 m, the smallest 3 m, and the average 28 m.

When the depths are obtained from the sounding charts, their error is apparently mostly due to positioning inaccuracies, those of the sounding expeditions as well as ours. Assuming equal accuracy, we estimated the error as ±2 m on the basis of a small (possibly unrepresentative) sample, which then causes an error in the Bouguer reduction

$$m_d = 0.14 \text{ mGal} \tag{16}$$

5. *The accuracy of the point anomalies*

5.1. Computed estimates

The error of a free air anomaly consists of the errors of the gravity observation, height determination and positioning. For the stations observed in February 1977 it is, on average

$$m_F = \sqrt{m_g^2 + m_h^2 + m_p^2} = \sqrt{0.10^2 + 0.02^2 + 0.10^2} = 0.14 \text{ mGal} \tag{17}$$

from (5), (8) and (14). For the Bouguer anomalies the error from depth determination (16) must be added:

$$m_B = \sqrt{m_g^2 + m_h^2 + m_p^2 + m_d^2} = \sqrt{0.10^2 + 0.02^2 + 0.10^2 + 0.14^2} = 0.20 \text{ mGal} \tag{18}$$

This figure is valid for the 80 % of stations where the depth has been determined from the sounding charts (chapter 4.4.).

5.2. Direct estimates

A direct check of the reproducibility of the Bouguer anomalies was made in February 1977 in the following way: Seven sites (Table 1 and Fig. 8) were reobserved after a few days. The stations were not marked and no attempt was made to get exactly the same decometer readings. The computed distance between the old and the new stations is up to 280 metres. For this reason Bouguer anomalies, not gravity values, were compared, and their differences were corrected using the horizontal gradient of the anomaly field, estimated from the five (or two-and-a-half) kilometre net. The standard error of a Bouguer anomaly, computed from the discrepancies between six »old» and »new» anomalies, was

$$m'_B = 0.15 \text{ mGal} \tag{19}$$

Table 1. Re-observations of February 1977 stations in the same month.
Key:
Column 1: Station no. (Fig. 8)
2: Old/new observation day
3: Old/new noise level (mGal)
4: Computed distance between the old and the new station (metres)
5: Old minus new Bouguer anomaly (mGal)
6: Correction for horizontal gradient
7: Corrected anomaly difference

1	2	3	4	5	6	7
770511	18/24	0.2/0.3	280	−0.12	+0.11	−0.01
770017	18/24	0.3/0.2	100	−0.49	0	−0.49
770513	18/24	0.8/0.1	80	+0.09	−0.19	−0.10
770034	19/24	0.3/0.3	50	+0.20	−0.07	+0.13
770042	19/24	0.2/0.2	200	−0.51	+0.52	+0.01
770049	22/24	0.9/0.3	260	+0.28	−0.37	−0.09
(770049	22/23	0.9/1.8	330	−0.48	−0.11	−0.59)

Table 2. Re-observations of March 1976 stations in February 1977. Key as in Table 1.

1	2	3	4	5	6	7
760001	8/17	0/0.3	20	−1.73	−0.02	−1.75
760005	8/25	0.3/0.7	130	+1.53	−0.25	+1.28
760006	8/25	0.3/0.5	140	+0.67	−0.09	+0.58
760010	11/19	0.6/0.3	0	−2.46	0	−2.46
760012	12/19	0.6/0.3	80	+1.92	+0.09	+2.01

which agrees reasonably well with (18). The r.m.s. noise level of the gravity observations was 0.42 mGal.

A similar check was made with five sites (Figure 8) from the preceding winter. It shows (Table 2) an essentially greater standard error, 1.2 mGal. Some possible explanations for this result are discussed in the following chapter.

5.3. Possible error sources

5.3.1. The Eötvös effect

If the ice is moving horizontally, the vertical component of the acceleration due to the Coriolis force, the Eötvös effect, is approximately (HEISKANEN and VENING-MEINESZ [7])

$$dg = 2\omega\nu\cos\varphi \tag{20}$$

Here dg is the error in the gravity observation, ω is the angular velocity of the earth's rotation, ν is the east-west component of the velocity of the ice and φ is the latitude. At $\varphi = 65°$ the formula (20) reduces to

$$dg = 6.2\ \nu\ \text{mGal} \tag{21}$$

if ν is expressed in m/s and if $\nu = 0.16$ m/s, the Eötvös effect is 1.0 mGal. According to PALOSUO [23], who, concurrently with the 1976 measurements, was investigating ice in the area, the maximum velocities at the time were 10 m/d, which corresponds to 0.7 μGal if we assume constant speed. For the 1977 observations, the velocities and drift directions were obtained from predictions computed at the Institute of Marine Research from the model constructed by LEPPÄRANTA [19]. These predictions are made for the ice-breakers working in the area, and give velocities with a precision of 0.1 knots, or about 5 cm/s. All the relevant values were zeroes, or 2.5 cm/s at most. This 2.5 cm/s corresponds to 0.16 mGal, and even if model errors are considered, it seems obvious that the Eötvös effect cannot explain the discrepancies mentioned in chapter 5.2.

The possibility of making a correction for the Eötvös effect on the basis of the model [19] or of its newer versions are currently under investigation. The error estimates given in this paper do not include the possible influence of the Eötvös effect.

5.3 2. Vertical accelerations with long periods

Our gravity observations are obtained by averaging over about four minutes, and if vertical accelerations of essentially longer periods are present, they are not sufficiently smoothed out in the process. Assuming that the acceleration obeys the harmonic law, we get the following relation between its amplitude and period and the amplitude of the corresponding movement

$$A_a = (2\pi/T)^2 A_z \tag{22}$$

where A_a and A_z are the amplitudes of the vertical acceleration and the movement respectively, and T is the period. Table 3 gives A_z as a function of T, when A_a equals 1.0 mGal.

Table 3. A_z as a function of T when A_a equals 1.0 mGal.

T (s)	120	180	240	300	360	480	600	900	1200	1800
A_z (mm)	3.6	8.2	14.6	22.8	32.8	58.4	91.2	205.2	364.8	820.7

According to MÄLKKI [21] the large amplitudes required at periods of over 600 s are highly improbable on the seas surrounding Finland. The occurrence of long coastal waves of sufficient amplitude also seems unlikely, mainly because of the damping effect of the ice cover. In any case, the shorter periods would probably be detected during gravity observations; if the period is ten minutes, the change during four minutes is at least 70 % of the amplitude.

5.3.3. Positioning errors

Since during the 1976 observations the Decca positioning was not checked, it might be suggested, for example, that abnormally large ionospheric disturbances could have caused positioning errors sufficiently large to explain the discrepancies. Such positioning errors, however, would have to be of the order of half a kilometre, which seems rather improbable in daytime observations distributed over several days.

6. *Map of Bouguer anomalies*

The adjoining Bouguer anomaly map (Figure 9) has been drawn using land measurements made in the archipelago and along the coasts in addition to the

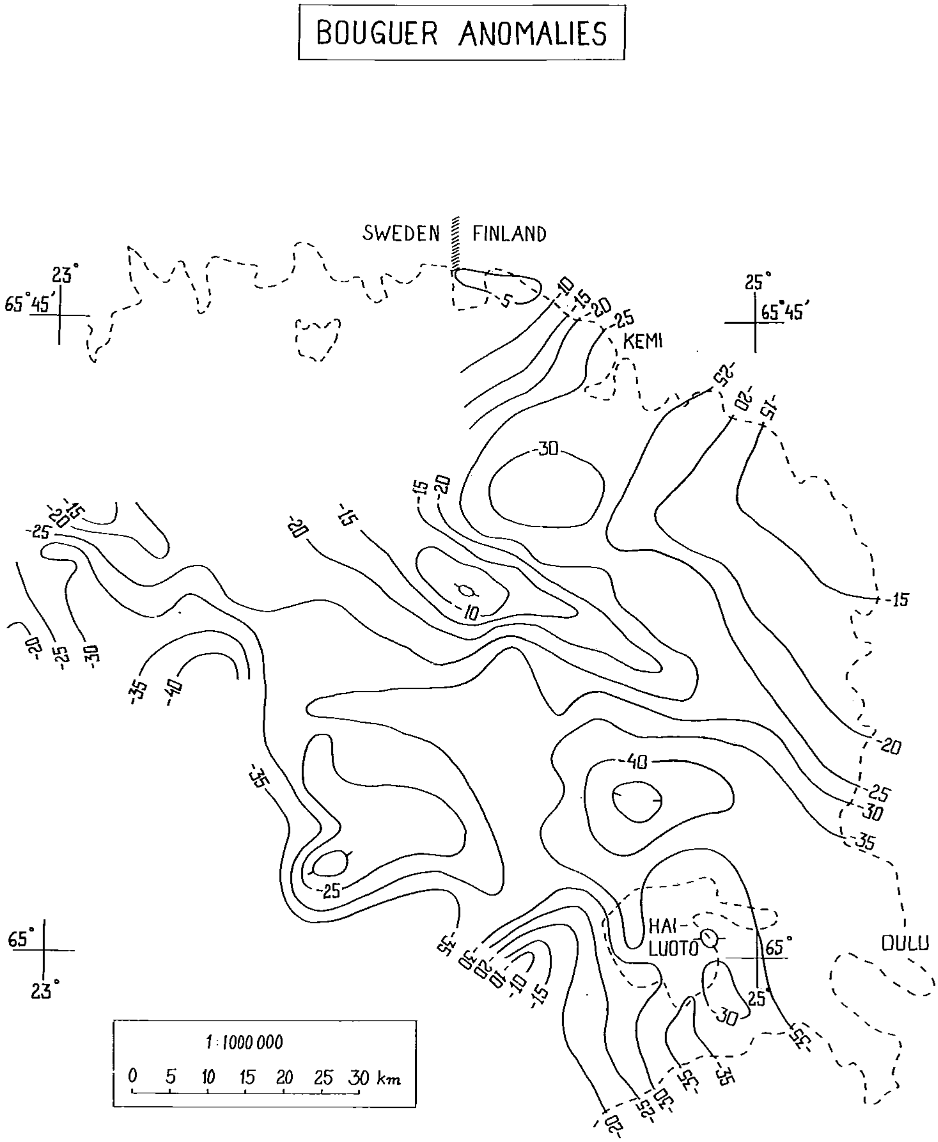


Figure 9. Map of Bouguer anomalies, scale 1:1 000 000.
International Gravity Formula (1930) – 14 mGal, IGSN 71. Densities 2.67 g/cm^3 (normal crust), 1.00 g/cm^3 (sea water). No terrain correction. Gauss-Krüger projection with 27° as the central meridian.

measurements discussed in this paper. Where the 1976 and 1977 results conflict, the latter have been preferred.

7. *Conclusions, future plans*

The measurement method presented here is feasible, at least under the conditions of the Bothnian Bay. The standard error of a free air anomaly is less than 0.2 mGal. This compares favourably with land measurements with barometric height determination. In these measurements the standard error of the height, about 1 m, already causes an error of 0.3 mGal in the anomaly.

The alternatives to ice measurements are, at present, bottom and shipboard gravimetry. In the former about the same accuracy in gravity observation is achieved (LACOSTE [17]). The observed value then refers to the sea floor, which for prospecting purposes is often advantageous, but for geodetic purposes is a drawback because of the uncertainty of the vertical gradient used to reduce the gravity to the geoid. In shipboard gravimetry, anomaly accuracies of the order of 2 mGal have been quoted under the conditions of the southern Baltic (ANDERSEN and ENGSAGER [1]).

Ice gravimetry has the practical advantages over both alternative methods of lighter logistic support (helicopters or possibly snowmobiles instead of ships) and greater speed. On the other hand, it is possible on the open sea only during a short part of the year. For instance, in the Bothnian Sea (excluding the coasts), ice remained immovable for only an average ten days each winter in the period 1931–1960 (PALOSUO [22]).

The rather high costs, 250 Fmk/station in February 1977, will probably go down when more experience is gained.

The conflicting results from successive winters need more investigation. A second remeasurement will be made as soon as possible. The extension of the net will be continued during the first sufficiently severe winter. In future measurements ice drift will be monitored.

8. *Acknowledgements*

The Board of Navigation shared the use of its helicopter during the 1976 observations, assisted in practical arrangements and made its sounding charts available to us. The staff of the Institute of Marine Research contributed advice about ice conditions and water level fluctuations, and mareograph results. The Worden Master no. 934 was loaned by the Geological Survey of Finland.

REFERENCES

1. ANDERSEN, O.B. & K. ENGSAGER, 1977: Surface-ship gravity measurements in Danish waters 1970–1975. *Mémoires de l'Institut Géodésique de Danemark,* 3:43.
2. ANONYMOUS, 1976: *The Decca Navigator. Operating instructions and marine data sheets,* 3, January 1976. London, The Decca Navigator Company Limited.
3. CHOJNICKI, T., 1966: Eksperymentalna metoda pomiarów grawimetrem Askania Gs-11 na zamarznietym morzu. Summary: An experimental method of measurement with the gravimeter Askania Gs-11 on a frozen sea. *Prace Instytutu Geodezji i Kartografii,* XIII, 127–141.
4. CRARY, A.P. COTELL, R.D. & J. OLIVER, 1952: Geophysical studies in the Beaufort Sea. *Trans. Amer. Geophys. Union,* 33, 211–216.
5. FAGERHOLM, P.O. & A. THUNBERG, 1964: Determination of fixed errors in navigational (and hydrographic) Decca chains. *Supplement to the International Hydrographic Review,* 5, 33–44.
6. HAMILTON, A.C., 1962: Gravity measurements in Canada, January 1, 1960 to December 31, 1962. *Publ. Dom. Obs.,* XXVIII:3.
7. HEISKANEN, W.A. & F.A. VENING MEINESZ, 1958: *The Earth and its gravity Field.* New York, McGraw-Hill, p. 158.
8. HONKASALO, T., 1960: Gravity survey of the Baltic and the Barents Sea. *Referat af det tredje nordiska geodaetsmøde i København 25.–30. maj 1959.* København, Geodaetisk Institut. 159–169.
9. –»– , 1962: Gravity survey of Finland in the years 1945–1960. *Veröff. Finn. Geod. Inst.,* 55.
10. HUNKINS, K., 1962: Waves on the Arctic Ocean. *J. Geophys. Res.,* 67, 2477–2489.
11. KIVINIEMI, A., 1964: The first order gravity net of Finland. *Veröff. Finn. Geod. Inst.,* 59.
12. –»– , 1966: Några undersökningar angående vågrörelsen hos havsisen. *Det femte nordiske geodetmötet i Stockholm 25–29 april 1966.* Stockholm, Nordiska Kommissionen för Geodesi, 81–85.
13. –»–, 1975: Measurements of wave motion in the ice surface. *Reports of the FGI,* 75:4.
14. –»–, 1977: *Oral communication.*
15. KÄÄRIÄINEN, E., 1966: The second levelling of Finland in 1935–1955. *Veröff. Finn. Geod. Inst.,* 61.
16. –»–, 1975: Land uplift in Finland on the basis of sea level recordings. *Reports of the FGI,* 75:5.
17. LACOSTE, L.B.J., 1967: Measurement of gravity at sea and in the air. *Rev. Geophys.,* 5, 477–526.
18. LEHMUSKOSKI, P. & J. MÄKINEN, 1977: Kokemuksia painovoimanmittauksista Perämeren jäällä. Abstract: Experiences in gravity measurements on the ice of the Bothnian Bay. *Maanmittaus,* 52, 79–84, 141.
19. LEPPÄRANTA, M., 1977: Pohjanlahden jääennustemalli. Abstract: Model for ice forecast in the Gulf of Bothnia. *Geofysiikan päivät Helsingissä 10.–11.3.1977.* Helsinki, Geofysiikan seura, 291–300.
20. LESCHACK, L.A. & R.A. HAUBRICH, 1964: Observations of waves on an ice-covered ocean. *J. Geophys. Res.,* 69, 3815–3821.
21. MÄLKKI, P., 1978: *Oral communication.*
22. PALOSUO, E., 1975: The formation and structure of ice ridges in the Baltic. *Winter Navigation Research Board,* 12.
23. –»–, 1978: *Oral communication.*

24. PLAUMANN, S., 1964: Schweremessungen auf dem zugefrorenen Steinhuder Meer. *Z. Geophys.*, 30, 97–104.
25. ROBIN, G. de Q., 1963: Ocean waves and pack ice. *Polar Record,* 11, 389–393.
26. SOBCZAK, L.W., 1963: Regional gravity survey of the Sverdrup Islands and vicinity. *Gravity map series of the Dominion Observatory,* 11.
27. VANNINEN, M., 1958: Helsingin Yliopiston tähtitornin ja Helsingin peruskolmiopisteen luotiviivanpoikkeamaeron määrittäminen gravimetrisesti. (A gravimetric determination of the difference in the deflection of the vertical between the Astronomical Observatory of the University of Helsinki and the first order triangulation point Helsinki). *Thesis for degree in Engineering,* Helsinki University of Technology, Department of Surveying.
28. WEBER, J.R., 1963: Gravity anomalies over the Polar Continental Shelf. *Dom. Obs. Contr.,* 5:17. (Ref. Hamilton, A.C. [6], p. 8).