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CONSTANTS OF LONG-PERIOD SEISMOGRAPHS AT NURMIJÄRVI (FINLAND)

by

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Abstract

The World-Wide Seismograph System (WWSS) calibration convention was supplemented with some additional measurements of parameters to determine the best fitting of the basic constants. The most probable constants of long-period seismographs were taken as follows: seismometer period T_s = 15 s and damping constant D_s = 0.9, galvanometer period T_g = 98.1 s and damping constant D_g = 1.0. The coupling coefficient is in the limits 0.001 – 0.5 for the standard maximum magnification range 375 – 6,000 at 15 seconds.

1. *Introduction*

In many cases of seismic data analysis it is necessary to know the accurate seismograph response, *i.e.* the absolute magnification and/or phase delay, used to correct seismograms. Both characteristics can be calculated (i) from the basic parameters of the seismometer, galvanometer and the attenuator network, or (ii) from the transient or steady-state response of the seismograph when the seismometer is excited with pulse or harmonic signals. The above-mentioned methods require a different time and instrumentation for measurements and different computing facilities for interpretation. Usually these basic calibration methods are combined to enable simple calibration, equipment and computation.

In the WWSS calibration convention the latter fashion was adopted [1–3]. Only three parameters of long-period seismograph, *i.e.* seismometer period T_s =

Table 1. Long-period seismograph constants

Reference Constant	Calibration convention [3]	USCGS [4]	USCGS [5c]
$T_s(s)$	15.0 ± 1 %	15.0	15.0
$T_g(s)$	100.0 ± 1 %	100.0	100.0
D_s		0.93	0.87
D_g	1.0	1.0	0.9
σ^2 for max. magn.			
375		0.003	0.0022
750		0.013	0.0094
1,500		0.047	0.035
3,000		0.204	0.130
6,000		0.805	0.46

15 s, galvanometer period T_g = 100 s and galvanometer damping constant D_g = 1.0, are checked prior to controlling the absolute magnification of the seismograph. The value of the next parameter, the damping constant of the seismometer D_s, was not published in Appendices A or B [1, 2] and is not measured. It is considered that the calibration constant K = 0.449 N/m used for to determine the absolute magnification at 15 seconds is independent of the attenuator setting. The phase response (Fig. 3 [2]) is also independent of the magnification. For both cases the influence of a different coupling between the seismometer and galvanometer is neglected. On the other hand the amplitude characteristics for higher maximum magnification do not have an identical shape (Fig. 2 [2]).

Not a great amount of constant measurements with different long-period instruments has been published. Table 1 shows the values from USCGS (U.S. Coast and Geodetic Survey) given in paper [4] as well as the data requested from USCGS [5c]. The damping constants of the seismometer are not the same. These values of the upper limit of the coupling coefficient are not negligible. If the coupling has a measurable influence on the course of the magnification curve, we can suppose there will be changes in the calibration constant.

To find the accuracy of the standard calibration procedure it should be compared with another independent calibration method. For this purpose the separate determination of the basic parameters of the seismograph was chosen. The amplitude and phase response can then be calculated under the assumption that the seismometer and galvanometer oscillating systems have linear behaviour.

The present paper describes the results of measurements of seismograph parameters. The work was done at Nurmijärvi (NUR) seismic station, where the Press-Ewing long-period set of instruments has been in regular operation since 1962. The decrease in the seismometer period from the initial 30 s to 15 s was made in 1965 and since then only the maximum magnification 1,500 at 15 seconds has been used during operation.

2. *Parameters of the long-period seismograph*

In addition to the standard procedure recommended for WWSSN (World-Wide Standard Seismograph Network), several supplementary measurements must be executed in order to determine the damping constant of the seismometer and the coupling coefficient of the seismometer-galvanometer system. The aim of the paper was to find the most reliable seismograph constants, taking into account the errors of different methods used for parameter determination.

2.1 Theoretical background

2.1.1 Attenuator and coupling

The attenuator of the magnification control box is composed of two T resistance terms (Fig. 1a): the first with resistors R_1, R_2 yields continuous control of attenuation, the second with resistors R_3, R_4 attenuation in 6 dB steps from 0 to 24 dB. For further discussion the real operation network is converted to a simple attenuator with resistances X, Y, Z (Fig. 1b). Their values are

$$X = R_1 + (R_1 + R_3) R_2/(R_1 + R_2 + R_3 + R_4) \tag{1a}$$

$$Y = R_3 + (R_1 + R_3) R_4/(R_1 + R_2 + R_3 + R_4) \tag{1b}$$

$$Z = R_2 R_4/(R_1 + R_2 + R_3 + R_4) \tag{1c}$$

The external resistance of galvanometer R_{ge} and seismometer R_{se} will then be

$$R_{ge} = R_{ga} + Y + (R_s + R_{sa} + X)\, Z/(R_s + R_{sa} + X + Y) \tag{2a}$$

$$R_{se} = R_{sa} + X + (R_g + R_{ga} + Y)\, Z/(R_g + R_{ga} + Y + Z) \tag{2b}$$

where R_{ga} and R_{sa} are additional resistances in series with the galvanometer coil resistance R_g and signal coil resistance R_s, respectively. (These resistances can be parts of the real T members to make them symmetrical.)

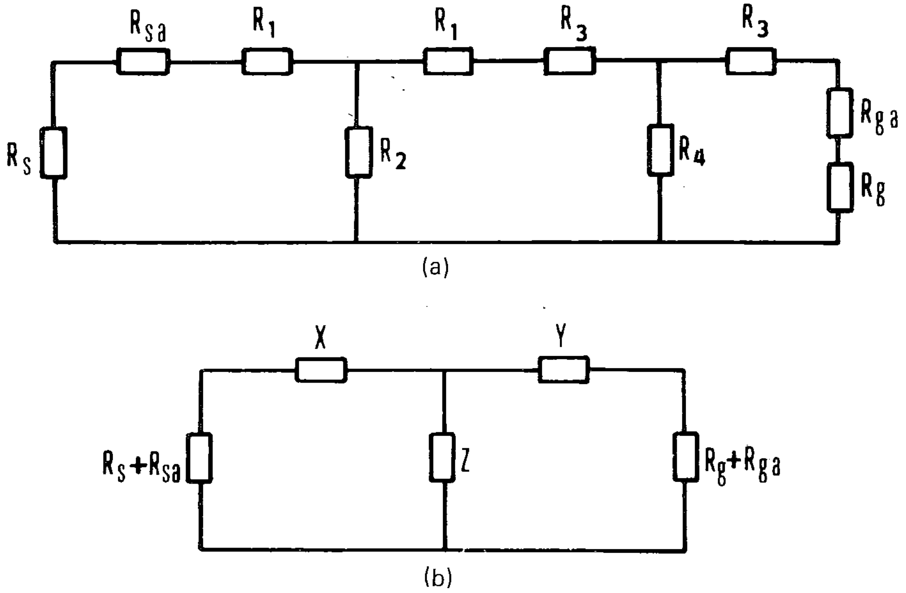

Fig. 1. Scheme of the attenuator network of the long-period seismograph (a) and its equivalent circuit (b). Explanation in the text.

Using the values of equivalent attenuator resistances the coupling coefficient can be calculated with the formula [6]

$$\sigma^2 = \frac{Z^2}{(R_s + R_{sa} + X + Z)(R_g + R_{ga} + Y + Z)} \; \frac{D_s - D_{s0}}{D_s} \; \frac{D_g - D_{g0}}{D_g} \tag{3}$$

Here D_{g0} and D_{s0} are the open circuit damping constants of the galvanometer and seismometer, respectively.

All the resistors in the circuit can be determined without difficulty. In our case the individual resistors were measured with a DC bridge to an accuracy of ±0.5 %. The whole input and output resistances were also measured to check the calculated values (2a, b). The unknown damping constants for coupling determination were derived during further galvanometer and seismometer tests.

2.1.2 Galvanometer test

Using the free period test the galvanometer is adjusted according to the direct measured period to the value $T_g' = 100$ s [3]. As the open-circuit damping constant

D_{g0} is considerable, the real free period T_g is smaller then T_g' and for ideal linear system it holds that

$$T_g = T_g'\sqrt{(1 - D_{g0}^2)} \tag{4}$$

The open circuit damping constant can be calculated from the recorded decay curve using the formula for linear oscillator

$$D_{g0} = 0.733\,\lambda/\sqrt{[1 + (0.733\,\lambda)^2]};\quad \lambda = \log_{10}(X_{i,\,i+1}/X_{i+1,\,i+2}) \tag{5a,b}$$

$X_{i,\,i+1}$ is the peak-to-peak amplitude between successive extremes i and $i+1$. For $X_{i,\,i+1}/X_{i+1,\,i+2} > 1.55$ ($D_{g0} > 0.138$) the measured period will be about 1 per cent greater than the real free period of the galvanometer. Great differences among particular determinations of D_{g0} will indicate non-linear behaviour of the galvanometer system.

The open circuit damping constant D_{g0} of individual galvanometers at NUR was in the range 0.18 – 0.21. The value of air damping of long-period galvanometers supplied by LOCKE [5b] has been measured as 0.194. For these damping constants the free period of the galvanometer is therefore about 2 per cent smaller than the direct measured value T_g'. The free period T_g = 98.1 s will be taken as best fitting the free period of the galvanometer for accurate adjustment of the galvanometer according to the WWSS calibration convention.

The critical damping of the galvanometer is checked using the damp test according to decay of the trace amplitude. Damping is regulated by means of magnetic shunt by the galvanometer so as to reach no further movement after 100 seconds. The necessary condition which must be fulfilled is that the external resistance inserted in the control box is the same as the operational external resistance of the network for the given magnification. Evaluation of the damp test requires some experience from the observer and more objective quantitative methods were therefore used.

The trace amplitude X at the time that $t = T_g$ from the start of the light spot to the return towards the zero line position is measured on the record. The ratio X to the initial deflection of galvanometer spot X_0 at the time $t = 0$ is 1.36 per cent for critical damping. The values of the decay ratio of some damping constants are in Table 2. The accuracy of damping derivation depends on the errors of free period determination and the time $t = T_g$ measurement. The ratio values for errors of 2 per cent in the damping constant are also given in Table 2.

Another suitable method for damping constant determination requires an additional record for critical resistance of galvanometer a_g measurement.

Then

$$D_g = D_{g0} + a_g/(R_g + R_{ge}) \tag{6}$$

Table 2. Amplitude ratio for galvanometer damping

D_g	X_{T_g}/X_0 (%)	X_{T_g}/X_0 (%) for D_g	
		−2%	+2%
0.90	−0.04	0.01	−0.07
0.92	0.21	0.28	0.16
0.94	0.48	0.57	0.40
0.96	0.76	0.88	0.66
0.98	1.06	1.19	0.94
1.00	1.36	1.52	1.22
1.10	3.01	3.26	2.78
1.20	4.82	5.16	4.51
1.30	6.72	7.13	6.33

where R_{ge} must be the operational external resistance of the galvanometer (2a). The critical resistance a_g is given by

$$a_g = (D_g' - D_{g0})\,(R_g + R_{ge}') \tag{7}$$

where D_g' is the damping constant for the galvanometer free-period test with external resistor $R_{ge}' \gg R_{ge}$, which is inserted in the control box, and D_g is calculated from the decay curve according to (5).

In order to get several swings of the galvanometer and a sufficient difference between D_g' and D_{g0}, the suitable external resistance of the long-period galvanometer is 25 kOhm and/or 50 kOhm.

Note: The determination of a_g and D_{g0} must be made without a change in the galvanometer period. The same applies to the seismometer (see further). If the operational period T_{op} differs from the period T_{cal} during calibration the open circuit damping constant and critical resistance might be multiplied by the ratio of periods T_{op}/T_{cal}.

2.1.3 Seismometer tests

The seismometer tests are not as simple as galvanometer calibration because the movement of the seismometer cannot be observed directly and it is checked and controlled through magnified movement of the long-period galvanometer connected to it. The basic condition for the application of this method is the following: the coupling coefficient during calibration must be sufficiently small to guarantee that the recorded curve corresponds to free oscillation of the seismometer with partial constants [7]. As for the free-period test of the seismometer a resistor (240 kOhm) is added in series with a seismometer coil and R_{sa}, this condition is fulfilled ($\sigma^2 < 0.01$, see further 2.2).

Open-circuit damping constant D_{s0} is much smaller than with the galvanometer: about 0.01 for Z component and 0.04 for horizontal components. Together with the electromagnetic damping of the circuit, the total damping constant during the test is $D_s < 0.05$. No correction of the seismometer period measured on the record is necessary for 1 per cent accuracy. The weak damped oscillations corresponding to the free movement of the seismometer modulate the nearly critical damped movement of the galvanometer. (The damping constant of the galvanometer is slightly smaller in this test than during operation due to the resistor 240 kOhm in the network.) According to a mathematical model of free movement of the system, the periods measured between extremes (not at crossing the zero axis) of the decay curve are sufficiently accurate after the third extreme. There is no distortion by the long term transient motion of the galvanometer.

The damping constant of the seismometer is given in general form by

$$D_s = D_{s0} + a_s/(R_s + R_{se}) \tag{8}$$

If we want to use the same record to determine the critical resistance a_s of the signal coil as in the preceding case for galvanometer critical resistance, this must be done more carefully. Two independent records of decay curve with different seismometer external resistances R_{se1}, R_{se2} must be made. If $R_{se1} < R_{se2}$, the corresponding damping constants will be $D_{s1} > D_{s2}$ and

$$a_s = (D_{s1} - D_{s2})(R_{s2} - R_{s1})/[(R_s + R_{se1})(R_s + R_{se2})] \tag{9}$$

and the air damping constant

$$D_{s0} = D_{s1} - a_s/(R_s + R_{se1}) = D_{s2} - a_s/(R_s + R_{se2}) \tag{10}$$

The same seismometer period is again necessary with both records.

To decrease the error in the evaluation of damping constants, a more usable form than (5b) is

$$\lambda = \frac{1}{\mathrm{n}} \log_{10} (X_{i,\,i+1}/X_{i+\mathrm{n},\,i+\mathrm{n}+1}) \tag{11}$$

where $\mathrm{n} > 1$. Then the error of logarithmic decrement is n times smaller for the same errors of amplitudes as in (5b).

The measured peak-to-peak amplitudes must not be affected by the transient of the galvanometer. For $D_s \leqslant 0.05$ we are allowed to start a measuring record from 11 – 13 extremes. The earlier part of the decay curve must not be used. In order to get more measurable amplitudes, the test with the recommended 6 dB attenuation should be used. This enables one to excite the seismometer in the linear range of amplitudes. According to amplitude attenuation the number n can be chosen from 5 to 10.

For the above experiments 25 and 50 kOhm resistors were inserted in original 240 kOhm resistor built in a control box for the vertical seismometer test and 50 and 100 kOhm for the horizontal seismometer tests. Their values must either be measured with minimum attainable errors not exceeding 1 per cent or else some calibrated resistance box should be preferred. For the calculation of external resistances the attenuator values during calibration are used. With the correct control box the external resistance of the attenuator does not change with the magnification adjustment.

The different resistances used for vertical and horizontal seismometers are due to the different open-circuit damping constants D_{s0} needed to hold $D_s \simeq 0.05$ at a maximum in both records and to attain a greater difference in damping constants in both tests. The record with the original 240 kOhm resistor can be used for checking.

All difficulties with transients of the galvanometer which limit determination of the open-circuit damping constant and the critical resistance of the seismometer coil can be removed using another short-period galvanometer critically damped. For $T_g \ll T_s$ the disturbing transient motion of the galvanometer is negligible after several extremes and both the period and damping ratio can be determined. For $T_g/T_s \leqslant 0.1$ the errors are smaller than 1 per cent starting with the second extreme. With the first zero-to-peak trace amplitude 50 mm on the record about 14 peak-to-peak amplitudes can then be read off to reach a 10 mm value (for $D_s = 0.05$) and more measurements for average damping constant determination are at one's disposal. The coupling coefficient for a maximum 1 per cent deviation of the damping constant is up to 0.3. The period distortion is only -0.3 per cent at its maximum (Table 2 in [7]).

2.2 Results of constants measurements

Measurements with the above-mentioned procedures were taken several times from 1970 onwards during regular calibration of seismographs. The results of a vertical seismograph composed of a seismometer with signal coil resistance R_s = 476 Ohm and galvanometer resistance R_g = 492 Ohm are given in detail. The attenuator network for 12 dB was: R_{sa} = 32.1 Ohm, R_1 = 39.6 Ohm, R_2 = 2287 Ohm, R_3 = 300.1 Ohm, R_4 = 268.7 Ohm, R_{ga} = 0. The equivalent attenuator has resistances X = 307.9 Ohm, Y = 331.6 Ohm, Z = 212.2 Ohm. The open-circuit damping constants were $D_{s0} = 0.006, D_{g0} = 0.19$, critical resistances $a_s = 872$ Ohm, $a_g = 756$ Ohm, free periods $T_s = 14.9$ s, $T_g = 98.0$ s and the damping constants $D_s = 0.89, D_g = 0.95$. The external resistances were $R_{se} = 509$ Ohm, $R_{ge} = 500$ Ohm. With step regulation of magnification the external resistances of the seismometer and galvanometer remain constant with deviations smaller than 1 per cent. Both damping constants are therefore without change for the magnification range.

The coupling coefficient is $\sigma^2 = 0.033$ for the maximum magnification of seismograph 1.410, derived by the standard calibration method. As the magnification is proportional to $\sqrt{\sigma^2}$, then under the assumption that there is no change in amplitude response caused by the coupling in the period $T = 15$ s, we get $\sigma^2 \approx 0.6$ for the maximum magnification 6,000 and $\sigma^2 \approx 0.002$ for the minimum magnification 375.

As mentioned in 2.1.3, the influence of the coupling on the free constants during calibration of the seismometer is negligible. The statement can now be proved. Additional resistance is recommended at minimum 25 kOhm, i.e. about 25 times greater than the total resistance of the circuit in operational conditions. If we overestimate the coupling coefficient for 6 dB at 0.5, the coupling coefficient during calibration will in any case be smaller than 0.02. For $D_s = 0.05$ the 1 per cent difference in constants is reached with a higher coupling coefficient $\sigma^2 = 0.1$ [7].

For horizontal seismographs we get similar results. The seismometer damping constants were again smaller than critical (0.85 – 0.93). For attenuator setting 12 dB the coupling coefficient is about 0.025. The maximum attainable coupling for 0 dB is about 0.4 and for minimum magnification we have $\sigma^2 \approx 0.001$. The limits for applying the seismometer calibration method are of course again fulfilled.

The electromagnetic seismograph has maximum theoretical coupling coefficient in case there is direct connection without shunt between the seismometer signal coil and the galvanometer coil. Then $Z = \infty$ and from (3) we get $\sigma^2_{\max} = (D_s - D_{s0})(D_g - D_{g0})/(D_s D_g)$ which yields the value $\sigma^2_{\max} \approx 0.8$ when $D_{g0} \approx 0.2$ and $D_{s0} \ll D_s$. According to Fig. 1 in Appendix B [2] the maximum magnification is $M = 6{,}000$ at the period of 15 seconds attained for 0 dB of step attenuator with residual 2 dB of vernier trim. The magnification ratio loss is therefore 0.8 and the corresponding coupling coefficient $\sigma^2_{\max}/0.8^2 \approx 0.5$. This is in agreement with the above rough estimates of coupling coefficients both for the vertical and horizontal seismograph.

3. *Conclusions*

Several measurements of the parameters of long-period WWSS seismographs made at NUR since 1970 provide some information about the constants of the seismograph set.

The seismometer free period can be adjusted without difficulty to the prescribed value 15 s ±1 %. The damping of the seismometer, which is not adjustable, is smaller than critical. Its value was found to be in the range 0.85 – 0.95. The galvanometer period is systematically about 2 % smaller than the adjusted value when air damping is neglected. The damping constants of individual galvanometers checked by different methods were between 0.95 and 1.05.

For the WWSS calibration convention we can accept the following seismograph constants: $T_s = 15.0$ s, $T_g = 98.1$ s, $D_g = 1.0$ and the average damping constant of seismometer $D_s = 0.9$. These values agree well with the other constant determination taking into account the accuracy of calibration methods. We denote the constants as »standard constants» of long-period seismographs.

The coupling coefficients for the same maximum magnification differ substantially. This is not important for smaller maximum seismograph magnification. In these cases the influence of coupling on seismograph response is negligible. For the highest seismograph magnification where the condition $\sigma^2 \ll 1$ is not fulfilled, this influence should be tested.

Checking all the seismograph constants is tedious. It takes a great deal of time in a seismic vault and for record evaluation, too. On the other hand, this procedure is the best way of determining the stability and deviations of individual constants from standard values and of checking errors in the application of standard seismograph response. The question of the accuracy of the magnification curve and phase response of long-period seismographs will be treated in another paper.

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