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ON THE Z-R RELATIONSHIP IN SNOWFALL

by

TIMO PUHAKKA

Department of Meteorology
University of Helsinki

Abstract

During 10 storms, which occurred at temperatures from $-7°C$ to $+1.5°C$, the amount of precipitation was measured at one point with a normal non-recording rain gauge. Simultaneous radar measurements showed that the variations in coefficient a in the Z-R relationship $Z = aR^2$ may be related to the variations in the surface temperature. This could be explained partly by the dependence of the agglomeration efficiency on the temperature and partly by the dependence of ice crystal shapes on the temperature. The variance in the fluctuations of the signal received was higher for rain and snow than for snow. The probable reason for that is that the reflectivity gradients in the target volume are greater due to the differences in the reflectivity of rain and snow.

1. *Introduction*

The possibilities of measuring rainfall by radar have been studied and recognized widely during the progress of radar meteorology. Much less work has been done on the measurement of snowfall by radar. This can be explained by two main factors. First, the exact theory concerning the scattering of electromagnetic waves is not as complicated in the case of nearly spherical water drops as it is in the case of ice crystals and snowflakes with a very complicated and variable structure. Secondly, no method exists today by which one could accurately estimate the actual intensity of snowfall on a given region for references to the radar estimates. Thus the development of radar methods also meets with practical difficulties; the results achieved by radar cannot be verified satisfactorily.

For the regions situated on northern latitudes where a reasonable amount of precipitation falls in the form of snow, it would be essential to have some means by which the horizontal distribution of snowfall could be estimated reliably and practically. Because radar is an instrument which reveals large regions at one single point, it can also offer a useful tool in the measurement of snowfall, provided the relationship between snowfall intensity and the parameters measurable by radar are known.

The measurement of snowfall (and rainfall) is based on the fact that the average power received by radar depends on the radar parameters, on the shape, size, number and dielectric properties of the scatterers in the target region and on the attenuation of electromagnetic waves between the target and the radar.

If the radar parameters, the dielectric constant of the scatterers and the attenuation are known, one can calculate the sc. radar reflectivity factor of scatterers Z based on the average power received by the radar. If the scatterers are small raindrops (small with respect to the wavelength used), then Z is simply the sum of the sixth powers of the diameters of the drops in a unit volume, but if the scatterers are snowflakes, ice crystals, or a combination of rain and snow, the radar reflectivity factor Z does not have such a simple interpretation as in the case of raindrops. If the dielectric constant for snow measurements is always assumed to be constant and equal to 0.197 (which value is valid for ice in all temperatures if the density is 1 g/cm^3) then Z can be approximated by the sum of sixth powers of the diameters of melted snowflakes (see *e.g.* Battan [1]).

However, for both rain and snow it is generally accepted that the relationship between Z and the intensity of precipitation expressed in units of water equivalent can be approximated by the formula

$$Z = aR^b \tag{1}$$

where a and b are empirical coefficients and R is the intensity of precipitation. The values of a and b are usually given so that if R is expressed in mm/h then unit of Z is mm^6/m^3.

In all experiments concerning the measurement of snowfall by radar it has been proved that both coefficients a and b in the Z-R relationship (1) are on the average higher than the corresponding values of a and b for rain. Coefficient b is quite generally accepted to be almost constant and equal to 2.0 while coefficient a varies from about 300 to 3000 according to the type of snow and sometimes meets with even higher values. (See *e.g.* GUNN and MARSHALL [5], IMAI *& al.* [8], OHTAKE and HENMI [15]).

In Finland two experiments concerning the radar snowfall measurements were carried out earlier. The first study (JATILA, PUHAKKA and TAMMELIN [10]) consisted of 7 storms. Radar measurements were taken by means of a single-range-bin analog integrator, which was connected to the output of the linear receiver of a 3.2 cm radar. Because of the small dynamic range of the linear receiver (order of 35 dB) the gain of the receiver was adjusted before each measurement so that the level of the average signal received was always near the midpoint of the dynamic range used. Measurements were taken at intervals of about 30 sec. Special attention was paid to the representativeness of the reference measurements. This was done using 5 various methods including some corrections for the effect of wind and the surroundings. No correction was made in order to take the possible attenuation between the radar and the target into consideration. Exponent b in the Z-R relation (1) was kept as a constant equal to 2 and only coefficient a was varied so that the daily amount of snow measured by the radar became equal to the amount measured by the reference gauge.

Based on the results achieved, it seems that the Z-R relation somehow depends on the surface temperature. The authors explained this dependence, at least partly, by the fact that different ice crystal shapes originate at various temperatures (FLEAGLE and BUSINGER [3]). The rough classification of Z-R relationships as a function of temperature suggested in the study was the following:

$Z = 1500\ R^{2.0}$	if the temperature is	from	-0 to -4°C
$Z = 1000\ R^{2.0}$	—»—		-4 to -8°C
$Z = 500\ R^{2.0}$	—»—		-8 to -12°C
$Z = 200\ R^{2.0}$	—»—	below	-12°C

These findings are, at least partly, in accordance with the results of IMAI [7], who produced the relation $Z = 540\ R^2$ for dry snow and the relation $Z = 2100\ R^2$ for wet snow. Another important fact verified in the first Finnish study on radar snowfall measurements was the strong effect of wind on the reference measurements.

Later Jatila [9] studied 9 snowstorms. He used the stepped gain method for radar measurements. The reference measurements were made at four points equipped with normal non-recording rain gauges. In this study no correlation between the surface temperature and the Z-R relationship was found. One possible reason for this may lie in the very rough method of measurements: the steps between various values of gain used were on the average 6 dB and no interpolation was applied between the steps. Measurements were taken at intervals of 5 min. The effect of the wind on the reference measurements was also not taken into account. According to Hitschfeld and Bordan [6], the power law relationships used in quantitative radar measurements may in many cases be sensitive even to small errors in the basic data. An indication of the instability of results caused by the inaccuracies in the measurements of Jatila may be the very high values for coefficient a achieved in some storms.

In summary Jatila suggests the use of the reference gauge technique in the measurements of snowfall by radar. However, keeping in mind the possible errors in the reference measurements itself, the results presented in his study do not give any clear indication of the superiority of the reference gauge technique.

2. *Measurements*

The present study consists of 10 storms measured during winter 1973. Radar measurements were carried out using the digital radar data logging system reported in more detail by Vuorela and Puhakka [18] together with the Selenia RMT-1L x-band radar. In this particular study the radar data logging procedure was arranged so that as a result of one measurement one can obtain on a paper tape 199 values which are proportional to the power backscattered from the radar target cell. These 199 values are echosignals corresponding to 199 successive pulses transmitted by the radar. The distance between the radar and the target was 21.4 km. The individual values were punched out to 7 bit resolution, which is the same as 1/128 of the full scale of the output of the digitizer.

This kind of measurement was made on the average at a frequency of one per 30 sec. The time needed for one measurement was composed of about 1 sec for digitalizing the 199 successive returns (the pulse repetition frequency of the radar used was 245 pulses per sec) and about 2 sec for the output onto 8-channel paper tape. The pulse length of the radar was 3 μsec, the beam was symmetrical with beamwidth 1.75° and the elevation angle of the antenna was 1°.

The entire system was calibrated by means of an x-band signal generator on the average once in two hours. Fig. 1 represents one typical calibration curve for the system. The need for frequent calibrations is clearly demonstrated by Fig. 2 where the outputs of the radar system, corresponding to some known values of input power, are drawn as a function of time.

The application of calibration curves, like the one in Fig. 1, to the real data was done as follows. Each calibration curve was approximated

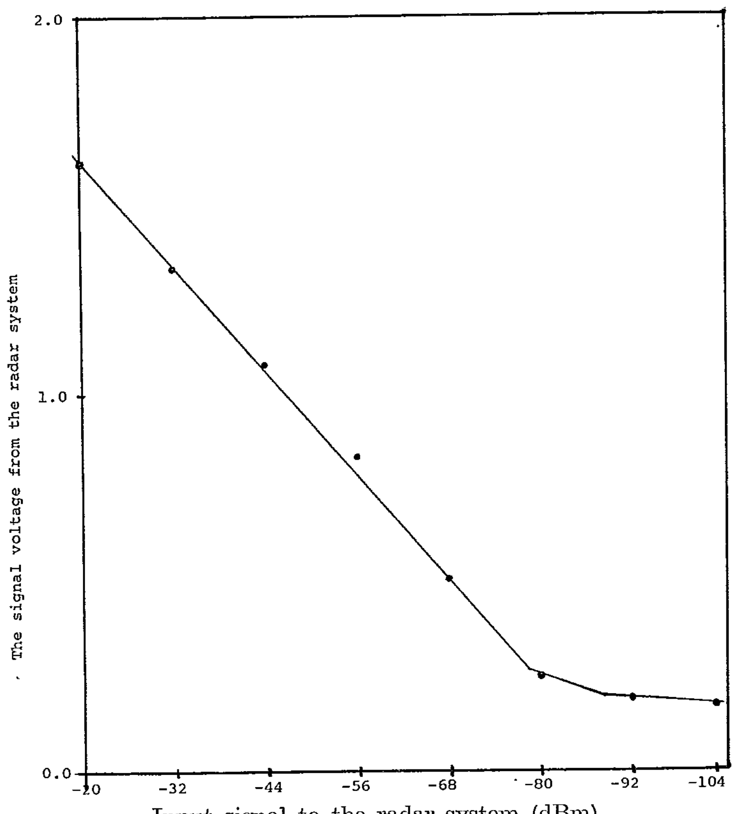

Fig. 1. Output voltage of the radar receiver as a function of the input power: an example of a typical calibration curve.

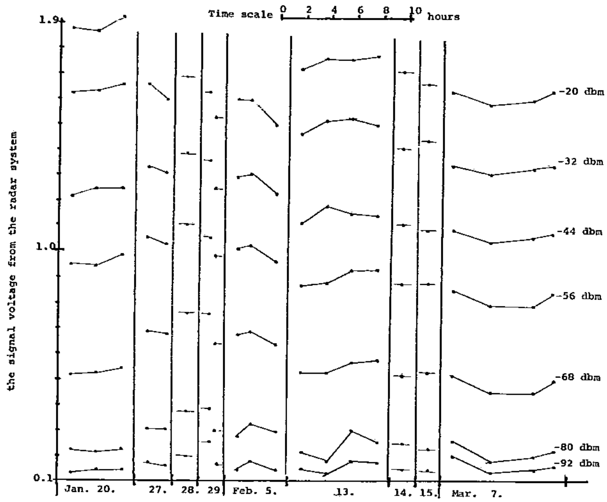

Fig. 2. Output voltages of the radar receiver corresponding to some known input powers as a function of time during the measurement periods in winter 1973. The sharp discontinuity on 29th Jan. as well as some other discontinuities between measurement periods are caused by adjustments of the receiver.

by a broken line, composed of three straight parts. The signal received at a known instant between two such calibrations was converted into a value of power using an intermediate calibration curve. The intermediate curve between two surrounding calibrations was worked out simply by interpolating linearily with respect to the time between the two real calibrations.

This kind of calibration procedure allows some amount of smoothing to the result. Slight smoothing is, however, desirable because of the possible random errors in the calibrations. Examination of the calibration curve in Fig. 1, which is a typical one, yields an estimate of about 3 dB for the possible maximum error in the calibrations. This estimate is valid for the values of power received higher than -80 dBm. In light of Fig. 2 even higher values for the error may sometimes occur. For the values of power received below -80 dBm the possible error increases rapidly, because the intervals used in signal quantizing (1/128 of 2 volts) are equal through the whole input scale of the digitizer.

The random error in this particular experiment was larger than it could be, because of a systematic error made in the calibrations. This error became clear and was corrected only when all data were under investigation. During the measurements all values of power received were assumed to be about 14 dB lower than they actually were. As a result of the error the resolution was reduced especially in the case of weak echoes.

Using the data obtained, the average power received and the standard deviation of individual radar returns expressed in terms of dBm and based on the 199 successive pulses were calculated. The values of the average power received were converted to values of the radar reflectivity factor Z with the aid of a radar equation, where the dielectric constant of scatterers has been taken as a constant equal to 0.197. The attenuation due to the precipitation between the radar and the target was assumed to be negligible. This assumption does not seem to be a very realistic one, especially in cases when rain and snow exist together, but because no information on the distribution of precipitation between the radar and the target was available, the possible attenuation could not be estimated.

The values of the radar reflectivity factor Z were converted to precipitation rates in terms of water equivalent in accordance with equation (1). The exponent b always had a constant value of 2.0 and coefficient a was adjusted so that the total amount of precipitation for each measurement period derived by radar became equal to the corresponding value measured by the reference gauge. The reference gauge was a normal non-recording rain gauge equipped with a Nipher shelter. For comparisons, the values of precipitation derived by radar were also calculated using the Z-R relationship $Z = 2000\ R^2$ given by GUNN and MARSHALL [5].

The radar measurements could not be made continuously. Calibrations, which were done at intervals of 2 hours, took about 2—5 min. For these intervals, and also for some other short interruptions caused by errors in the data logging system, the average power received was approximated by interpolating between the two nearest available values.

3. *Results*

3.1. General

Table 1 summarizes the main characteristics of the storms measured. Some storms, which lasted for a long time or during which the temperature or the type of precipitation was assumed to change appreciably, were studied in two parts. The total water equivalent of precipitation

Table 1. Some characteristics of the storms measured, together with the best values of coefficient a in the Z-R relationship, which made the radar estimate of the storm total of precipitation equal to the value measured by a reference gauge. The wind speed, the temperature and type of the precipitation are based on the observations made at the Helsinki Airport at a distance of 4 km from the reference gauge.

Date 1973	Duration hours	Total amount of precipitation mm water equiv.	The mean wind speed m/sec	The mean surface temperature °C	The best a	Type of precipitation
Jan. 20.	7.0	1.7	6	−6.7	720	snow
Jan. 27.	2.0	1.8	3	+0.4	1330	rain and snow
Jan. 29.	2.2	0.2	4	−3.8	730	snow
Feb. 5.	2.8	3.9	3	+1.3	2150	rain and snow
Feb. 5.	2.1	2.1	3	+0.2	1020	rain and snow
Feb. 13.	5.7	11.8	9 gusts 14	−2.5	2300	snow
Feb. 13.	2.8	3.9	9 gusts 14	−0.8	3100	snow
Feb. 15.	2.8	0.4	4	−0.2	1250	snow
Mar. 7.	5.0	3.3	4	−0.2	1420	snow
Mar. 7.	5.0	2.8	3	−0.5	620	snow

measured during all storms was 31.9 mm. In order to get some idea of the variability of the Z-R relationship from storm to storm, such values for coefficient a in relationship (1) were calculated, which made the radar estimate of the storm total of precipitation equal to the value measured by a reference rain gauge. The relationship between coefficient a and the surface temperature, type of precipitation and the standard deviation of signal fluctuations were investigated.

3.2. Z-R relationship and the surface temperature

In Fig. 3 the best values of coefficient a are plotted as a function of the mean surface temperature for each storm. A general feature of Fig. 3 is that for rather low values of the surface temperature coefficient a also has a low value. Near the temperature 0°C the values of a vary widely but the general tendency seems to be that a increases with increasing surface temperature.

A strong exception to the general distribution of the values of coefficient a can be observed on 13th February. The average temperature for both measurement periods during this day was well below zero degrees while coefficient a had very high values, 2300 and 3100. The possible

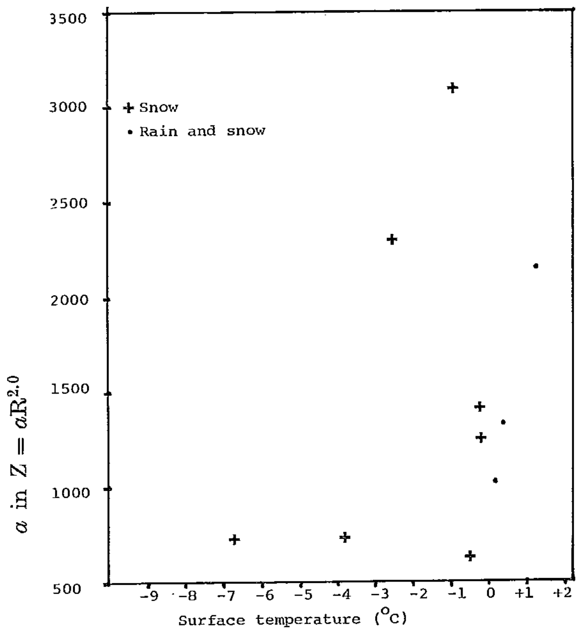

Fig. 3. The values of coefficient a (in the Z-R relationship $Z = aR^2$), which led the radar estimate of the total amount of precipitation equal to the value measured by a reference rain gauge, as a function of the mean temperature.

reason for this may lie in the inaccuracies in the reference measurements. Although the reference rain gauge was located on an almost ideal small open place within a rather low woods, the wind may cause serious errors in the measurements of snowfall with an ordinary rain gauge. This is clearly demonstrated by GOLUBEV [4]. According to his work a normal non-recording rain gauge equipped with a Nipher shelter is able to collect on the average 79 per cent of the actual amount of snow if the windspeed at the height of the rain gauge is on the order of 1.1—2.0 m/sec. If the windspeed is 2.1—3.0 m/sec the corresponding figure is 67 per cent and in the case of windspeed 4.1—5.0 m/sec only 49 per cent of the actual amount of snowfall would be measured.

Unfortunately no wind measurements were made just in the vicinity of the reference rain gauge of this study. However, the observations made at the Helsinki Airport show that the average windspeed during the measurement periods on 13th February was 9 m/sec and in gusts even

14 m/sec at a height of 10 m while on the other days the windspeed was much lower (Table 1).

Assuming that the average windspeed near the reference rain gauge on the day in question had been roughly 30—40 per cent of the value measured at the airport, the results of Golubev show that the reference rain gauge had very likely measured only about 60 per cent of the actual amount of snow. If this were true, the best values of coefficient a on 13th February would change to 830 and 1120 corresponding to the temperatures —2.5°C and —0.8°C respectively. These values are in quite good agreement with the values obtained on the other days.

In order to check the possible effect of wind, some simultaneous wind measurements were performed afterwards near the reference gauge and at the Airport. The results showed that the order of magnitude of the wind at the reference gauge may be roughly 25—35 per cent of the value measured at the Airport (which in this particular case was 6—7 m/sec). Of course the exact reduction depends on the direction, speed and gustiness of the wind as well as on the snow cover.

Another reason for the high values of a on 13th February can be the wind drift. Because the wind was strong, the snowflakes which actually has been measured with the radar, had been moved faraway before they fell to the ground. Nevertheless, in the authors opinion, the most probable reason for the exceptionality of the results on 13th February is the effect of wind on the collecting capabilities of snow of the reference rain gauge. If the wind drift were the main reason for the errors in the reference measurements, then the values for the »best a» sometimes had to be exceptionally low, too. This kind of effect cannot be found among the windy storms measured in this study.

In addition to 13th February, errors could also be found on 20th January in the reference measurements due to the moderate wind (6 m/sec). Assuming that the error of the rain gauge was about 10 per cent, the corresponding change in the »best a» would be from 720 to 600.

Another interesting case can be found on 7th March. The storm on this day was measured in two parts. For the first part the best value of a was 1420 while for the second period the corresponding value was 620. The slight changes in both the mean temperature and the wind-speed cannot explain the change in a. The most probable reason for the decrease in a may be the possible change in the type and in the vertical extent of snowfall as a front passed the check site. If we try to find the best possible average relationship between the surface temperature and

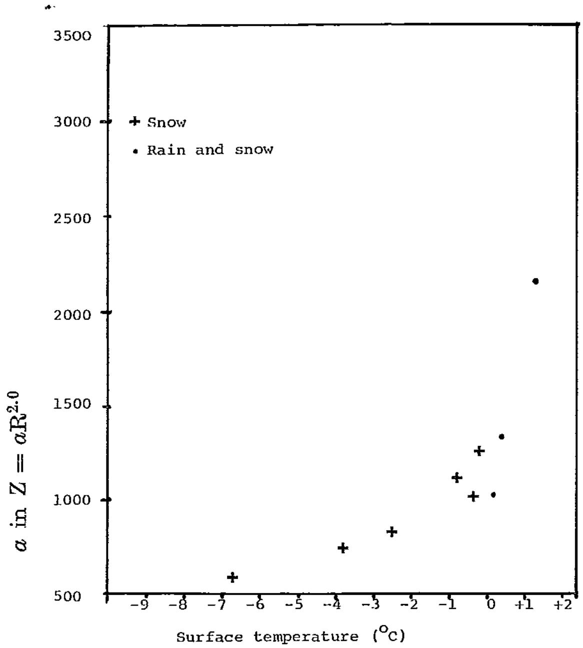

Fig. 4. The same as in fig. 3. but the reference measurements were corrected according to the assumed effect of the wind.

the *Z-R* relation, we are well justified in dealing with both measurement periods on 7th March as a whole because the temperature during both periods was almost the same. The combined value of a for the day is 1020 and the corresponding average surface temperature is -0.3°C.

Fig. 4 shows the same relationship between the surface temperature and coefficient a as in Fig. 3, assuming the corrections discussed above to be true. Compared with the results of JATILA, PUHAKKA and TAMMELIN [10] referred to earlier in this paper, the variability in coefficient a as a function of the surface temperature seems to be qualitatively similar below zero degrees. Absolute values of a however, are a little lower in the present study.

Unfortunately the original daily values of the »best a» achieved in the first Finnish study were not available. In order to get more data on the relationship between a and the temperature, the author also re-analysed the storms measured during winter 1971 in a manner similar to the storms of the present study. The reference measurements during 1971 were made using a normal non-recording rain gauge equipped with a Nipher shelter. The measured values of snow were corrected for the wind error according to the results of GOLUBEV [4] using wind measurements made just at the height of the reference gauge.

The results showed that the difference between the *Z-R* temperature relationships of the two studies may be mainly attributed to the errors made in the reference measurements during one windy day (23. March 1971) of the earlier study. On that day the windspeed was on the average 10 m/sec and in gusts even 21 m/sec, while the rain gauge measured only about 0.2 mm of melted snow. Because the accuracy in reading the amount is of the order of 0.1 mm there is no sense in trying to correct the amount against the wind error, which in this case must have been great. Another erroneous measurement period was on 7th March 1971. As already mentioned in the original study, during this day the blowing snow had very likely caused serious errors in the reference measurements. If these storms are left out of consideration the results agree very well with the new data as can be seen in Fig. 5.

The author also tried to reanalyse the data of JATILA [9] but, as pointed out earlier, the roughness of the measurement methods considerably restricted its use in this kind of analysis. However, during three of the four storms when coefficient a had exceptionally high values in

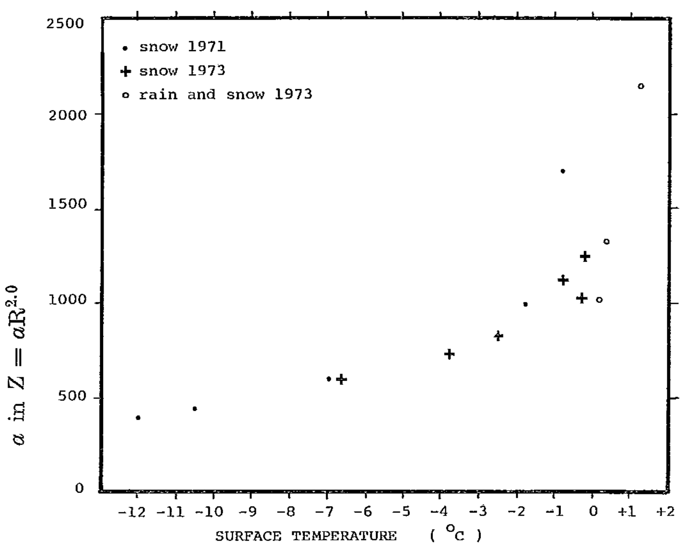

Fig. 5. The same as in fig. 4. but with the results of JATILA, PUHAKKA and TAMMELIN [10].

the study of JATILA, the windspeed was moderate to strong (7—11 m/sec), while on the other days the wind was weaker. Thus the high values of a may be explained partly by the effect of the wind on the reference measurements. On the fourth exceptional day the high value of a cannot be wholly explained by the wind error because the average windspeed was only about 4 m/sec. The reason for the error in this case may be that the general course of the reflectivity with time was relatively smooth during long time intervals; because the quantizing intervals used were as large as about 6 dB, large errors may arise in the case of rather steady echo or in the case of very rapidly changing echo. The remaining five storms agree qualitatively with the results of the present study, but because the rough measurement method has surely also affected the results of those days, the data of JATILA are not combined with the two other studies.

For the negative values of the temperature coefficient a increases with the temperature from 400 at -12°C to about 1700 near -0°C. Near the temperature 0°C there seems to be some kind of discontinuity and as the temperature rises above $+0$°C the value of a increases rapidly with temperature from 1000 at $+0$°C to about 2200 at $+1.5$°C.

A physical explanation for the relationship between coefficient a and the surface temperature below 0°C illustrated in Fig. 5 may be that the agglomeration efficiency of dendritic crystals is strongest at the temperatures just below 0°C but at temperatures much below -10°C no aggregation occurs (see *e.g.* MASON [14]). Because the terminal falling velocities of aggregated snowflakes of dendrites vary relatively little with the variations in the size of the snowflakes (LANGLEBEN [11]), the higher coefficients near the temperature 0°C may be associated with the relatively smaller number of small snowflakes in the target volume.

OHTAKE and HENMI [15] calculated *Z-R* relationships for various types of aggregated snowflakes. They found that snowflakes consisting of dendrites give a higher radar reflectivity than other crystal types provided the intensity of precipitation is the same. In the present study the average height of the target cell was about 0.5 km (from near the ground to about 1.0 km). Thus the temperature at that height was approximately 3—5°C lower than at the ground, but at the height where the ice crystals which has been measured with radar originate, the temperature was at least 5—10°C lower than at the ground.

According to FLEAGLE and BUSINGER [3] dendrites mainly originate at temperatures between -8°C and -12°C. At lower temperatures the crystal types are mainly plates and prisms. Thus, if the surface temperature is below $-5°C$ the relative number of dendritic snowflakes must be smaller in the target volume than if the surface temperature were higher.

The relationship between coefficient a and the temperature could also be explained using the results of MAGONO [12]. Examining a group of snowfalls, he found a relation between the air temperature at the observation point and the observed average size of snowflakes, as shown in Fig. 6. The average size of snowflakes increases with the surface temperature similarly to coefficient a until somewhere near the temperature $-1°C$ the sizes of snowflakes began to be reduced. The same effect could perhaps also be found in Fig. 4 and 5.

The temperature dependence of coefficient a at temperatures below 0°C according the results presented in Fig. 5 can be expressed roughly in the form:

$$a = (860 + 40\,T) \text{ if } -13°C < T < -4°C \text{ and}$$

$$a = (1500 + 200\,T) \text{ if } -4°C \leq T < 0°C.$$

Of course the expressions above are only approximations. In light of the present results not much can be said about the variations in the actual Z-R relation at some particular temperature, but it seems to be clear that the variability increases with temperature. It can also be assumed that the properties of the radar beam (beam width and elevation angle) as well as the target distance have an effect on the relationship between the Z-R relationship and the surface temperature.

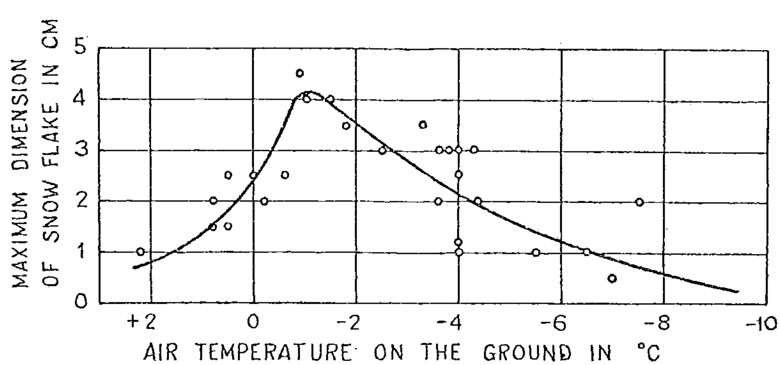

Fig. 6. Relation between the size of snowflakes and the air temperature at which the snowflakes were observed (MAGONO [12]).

As the surface temperature increases above 0°C the main reason for the increase in coefficient a with temperature may be the change in the dielectric constant of scatterers. In all calculations of this study the dielectric constant is assumed to be 0.197, a value valid for ice at all temperatures as the density is equal to 1 g/cm³. Actually, as the ice is melted, the value of the dielectric constant changes to 0.93, which is valid for water drops. The situation at temperatures just above 0°C is complicated because in one part of the radar target volume the scatterers are mainly raindrops while at the same time in some other part of the target volume scatterers are snowflakes.

3.3. Z-R relationship and the type of precipitation

The storms dealt with in this study have been classified into two main types of precipitation. The types are dry snow and wet snow or rain and snow. According to the results presented in Fig. 5 it seems obvious that for dry snow coefficient a is of the order of 400—1700 while for rain and snow a has on the average higher values varying between 1000 and 2500. This result is in good agreement with the results of Imai [7], who gave for rather dry snow the relation $Z = 540\ R^2$ and for wet snow $Z = 2100\ R^2$. Unfortunately the number of storms in the present study was not sufficient for some more detailed classification.

3.4. Signal fluctuation

In order to get some idea of the possible usefulness of signal fluctuation data as a predictor of the appropriate Z-R relationship, standard deviations of individual radar returns for each measurement were calculated based on the 199 successive pulses transmitted by the radar. Because the signal voltage of the logarithmic receiver of the radar is quantized using equal intervals, the final resolution depends on the signal level. For low values of the power received the resolution is much lower than for higher values of power received, as can be observed from the calibration curve of the system (Fig. 1). As a consequence of the variability of resolution with power level, for example, the standard deviations calculated from samples which are near —80 dBm are much greater than standard deviations for which the average power received is, say, —50 dBm.

In order to avoid these differences, which are not caused by the weather, the arithmetic mean of standard deviations of signal fluctuations of such measurements, for which the average power received was within an interval —69 to —71 dBm, was calculated for each storm. (In fact the standard deviations were calculated for intervals —65 to —67 dBm and —67 to —69 dBm too. The results were in all cases very similar.)

In Fig. 7 the best values of coefficient a for each storm are plotted as a function of the mean standard deviation of the backscattered power expressed in dBm. In general, no strong dependence between the value of a and the standard deviation of signal fluctuations can be observed. The standard deviation may be a little higher for low values of a than for higher values of a. Instead, in cases when rain and snow existed together, the standard deviations seem to be higher than for dry snow, regardless of the value of coefficient a. Unfortunately, the systematic

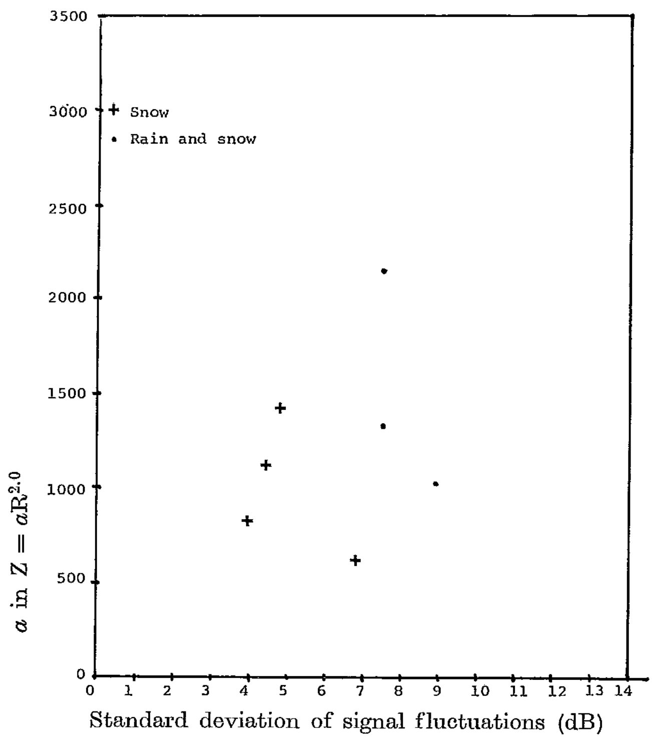

Fig. 7. The best values of coefficient a as a function of the mean standard deviation of signal fluctuations for each storm.

error made in the calibrations and discussed earlier in this paper restricted the accuracy of measurements especially when the signal was below −80 dBm. Hence the significance of the differences in the standard deviations is not absolutely sure.

MARSHALL and HITSCHFELD [13] derived the probability density distribution of independent signals received from a constant reflectivity target which is composed of randomly distributed scatterers. The standard deviation of the intensity levels of independent signals is in this case theoretically equal to 5.57 dB. Recently ROGERS [16] showed that if the reflectivity in the target cell is not constant, the results for the standard deviation of signal fluctuations as well as for the mean reflectivity itself can be very different compared with the theoretical values obtained in the case of constant reflectivity target.

This may be the most probable reason for the higher values of standard deviations achieved in rain and snow than in dry snow. If the temperatur is slightly above 0°C, it is very probable that even slight changes in temperature or wind may cause modifications in the concentrations of raindrops, wet snowflakes and rather dry snowflakes. Thus in turbulent windfield the precipitation may be composed of raindrops in some parts of the radar target cell while in some other part snowflakes predominate. Also if the temperature near the surface is above 0°C and in the upper part of the target volume the temperature is below 0°C, as it often is in rain and snow, the relative number of raindrops near the ground is greater than at the upper boundary of the radar target cell. Due to the differences between the reflectivity of rain and the reflectivity of snow, the reflectivity gradients are very likely greater in the case of rain and snow than in the case of pure rain or pure snow.

3.5. Results achieved with the relation $Z = 2000\ R^2$

In Fig. 8 the total amounts of precipitation for each storm measured by the reference raingauge are plotted on a log-log scale against the corresponding values derived by radar using the relation $Z = 2000\ R^2$ (GUNN and MARSHALL [5]). In most cases the values derived by radar are too small. The maximum underestimate is 43 per cent and the maximum overestimate is 26 per cent.

The slope of the regression line between the logarithms of amounts of precipitation derived by radar and measured by the gauge depends on exponent b in the Z-R relationship (1). Because the slope in Fig. 8

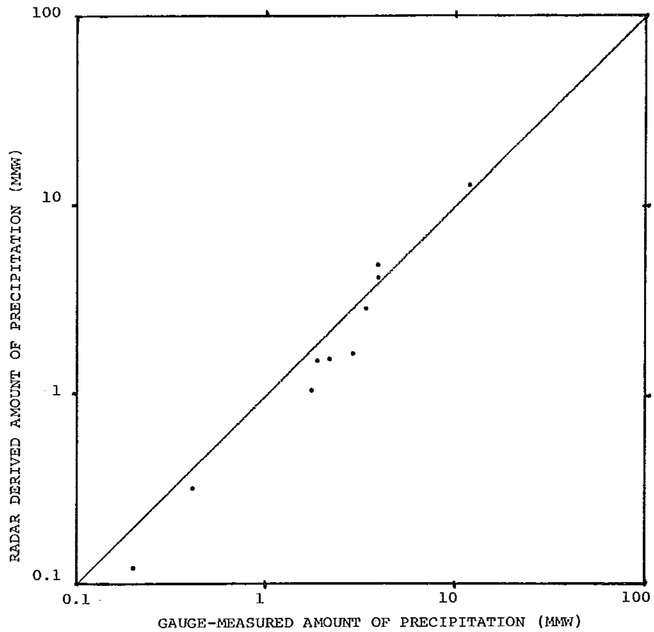

Fig. 8. Radar-derived amounts of precipitation as a function of corresponding gauge-measured values. The Z-R relation used is $Z = 2000\ R^2$. No correction for the wind error is made.

seems to be greater than 1.0 the results would be a little better if the value of b had been slightly higher than 2.0. The results of Carlson and Marshall [2] also indicated some evidence for a higher value than 2 for b. On the other hand, Jatila [9] came to the conclusion that the value of b had to be slightly lower than 2 in order to get the best results. However, if the assumed errors in the reference measurements caused by the wind were corrected in the present study, the results with the relationship $Z = 2000\ R^2$ would clearly suggest the value 2.0 for exponent b together with a lower value than 2000 for coefficient a (Fig. 9).

4. *Discussion and conclusions*

Although no very general conclusion can be presented based on the 15 storms measured at one point, it seems to be realistic to conclude that the temperature near the ground and especially in the target volume of radar plays an important role in the relationship between the radar

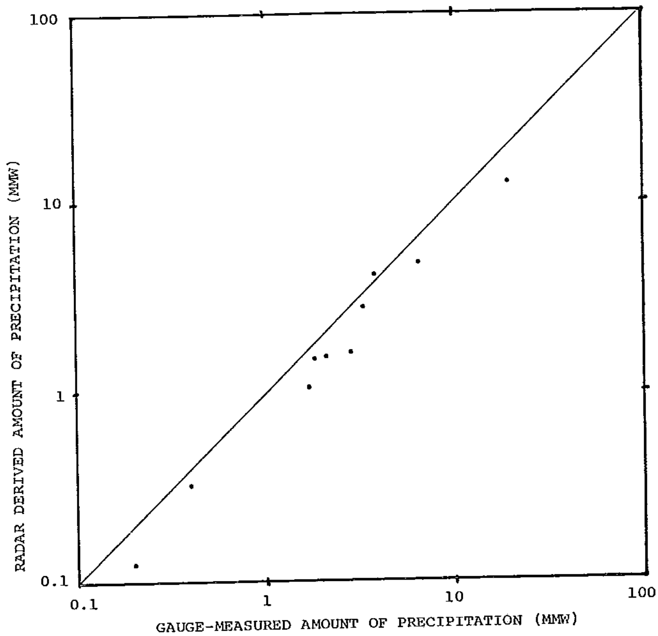

Fig. 9. The same as in fig. 8. but the gauge-measured values were corrected against the assumed effect of the wind.

reflectivity factor and the rate of precipitation if the scatterers are composed of snowflakes or of rain and snow.

The variation in coefficient a with surface temperature T may be approximated in the light of the result achieved by

$$a = 1500 + 200\ T \text{ if } -4°C \leq T < -0°C \text{ and}$$

$$a = 860 + 40\ T \text{ if } -13°C < T < -4°C$$

Near the temperature 0°C the variability of a may be considerable.

At temperatures above 0°C it is very difficult to give some exact relationship between a and the temperature. The values of a vary in this study between 1000 and 2200, increasing with temperature. Because the zero isotherm very probably is in the target volume, the situation is complicated. Not only the temperature but also the dimensions and the elevation of the target cell have an effect on the result. The relationship $Z = 2100\ R^{2.0}$ given by IMAI [7] for wet snow or the GUNN-MARSHALL relation $Z = 2000\ R^{2.0}$ may be a good compromise for rain and snow.

The standard deviation of signal fluctuations is related to the variability in the reflectivity in the target volume. Thus it should in principal be possible to derive from the signal fluctuation data some information about the type of precipitation. For instance in showery rains the possible reflectivity gradients are greater than in continuous precipitation, and hence the variance in signal fluctuations may be higher according to ROGERS [16]. Because of the small amount of data in the present study, not many synoptic types could be met. In any case, in rain and snow the variance in signal fluctuations seems to be higher than in dry snow.

This kind of knowledge may be valuable in regions where the temperature and the type of precipitation vary a great deal locally. A good example of this is the situation in winter on the shore of the Gulf of Finland. If the temperature on the shore is just above 0°C the precipitation near the sea is very often rain and snow or even pure rain but at a distance of say, 20 km north from the shore the precipitation is totally snow.

Unfortunately the normal methods used for the measurement of the average power received (multi-range-bin integrators) are not capable of this kind of analysis. The requirements for the data recording and processing equipment are also increased with the increasing speed of data flow.

The use of a reference gauge for the determination of the appropriate Z-R relationship was suggested by HITSCHFELD and BORDAN [6] and WILSON [17]. This technique is useful for rainfall measurements but if we are measuring snow or rain and snow by radar, the method is very questionable (with a normal rain gauge) due to the errors caused by the wind to the reference measurements. Although the scatter between the values derived by radar and values measured by a gauge apparently seems to decrease if the reference gauge technique is applied, the results achieved simply by using the relationship $Z = 2000\ R^2$ or some temperature-dependent relationship are very likely better in windy situation. Besides the wind error, the frequent existence of the zero isotherm near the surface in winter storms restricts the use of the reference gauge technique. In order to get a reliable radar estimate in such a situation, a dense network of reference gauges is required.

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