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A DETERMINATION OF THE INTENSITY OF THE ATMOSPHERIC ENERGY CYCLE FROM THE MEAN VERTICAL DISTRIBUTION OF DIABATIC HEATING

by

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Abstract

Using the concept of gross static stability, the intensity of the atmospheric energy cycle is expressed as a function of the vertical distribution of diabatic heating. Numerical evaluation of the expression is made with the aid of semi-empirical data.

1. *Introduction*

Conversion of available potential energy into kinetic energy requires in the atmosphere an average upward heat transport across isobaric surfaces with the associated effect of cooling of the lower layers due to the flux divergence and heating of the upper layers due to the flux convergence. In the long run this stabilizing effect of the energy conversion must be counterbalanced by heating of the lower layers and cooling of the upper layers through diabatic processes. This fact has been realized by several authors (*e.g.* Lorenz, [2]; Palmén, [4]), but so far the relationship between the atmospheric energy cycle and the vertical distribution of diabatic heating has not been analyzed. The purpose of this note is to derive such a relationship and also to evaluate it quantitatively with the data available.

2. *The energy cycle and the vertical distribution of diabatic heating in the atmosphere*

One method of relating the vertical distribution of diabatic heating with the atmospheric energy cycle is to use the concept of gross static stability. With a slight modification of the definition given by LORENZ [2], the gross static stability is here defined as the deficit of the total potential energy (internal energy + potential energy) below the total potential energy which the atmosphere would have if it, through adiabatic redistribution of mass, would become isentropic. For the sake of simplicity, we neglect the variations of surface pressure and assume it to be everywhere 1000 mb. The expression for the gross static stability is then

$$G = \int_M c_p((1+\varkappa)^{-1}\Theta - T)\,dm = c_p(1+\varkappa)^{-1}\int_M (p_0 p^{\varkappa} - p^{1+\varkappa})\left(\frac{\partial\tilde{\Theta}}{\partial p}\right) dm\,, \quad (1)$$

where G denotes the gross static stability, m mass, T temperature; p pressure ($p_0 = 1000$ mb), c_p the specific heat of air at constant pressure, $\varkappa = R/c_p$ (R is the gas constant), $\Theta = T\left(\frac{p_0}{p}\right)^{\varkappa}$ and M the whole mass of the atmosphere. The circumflex ($\tilde{\ }$) denotes a horizontal area average around the whole earth along an isobaric surface.

An expression for the rate of change of the gross static stability can be obtained from the thermodynamic energy equation:

$$c_p \frac{dT}{dt} = \alpha\omega + Q\,, \quad (2)$$

where t is time, α the specific volume, $\omega = dp/dt$ and Q denotes the rate of diabatic heating per unit mass. Equation (2) may also be written as

$$c_p \frac{d\Theta}{dt} = \left(\frac{p_0}{p}\right)^{\varkappa} Q\,. \quad (3)$$

Multiplying (3) by $(1+\varkappa)^{-1}$, subtracting (2) from (3) and integrating over the mass M one obtains

$$\frac{dG}{dt} = -\int_M \alpha\omega\,dm + \int_M \left[(1+\varkappa)^{-1}\left(\frac{p_0}{p}\right)^{\varkappa} - 1\right]\tilde{Q}\,dm\,. \quad (4)$$

The first term on the right-hand side of (4) is the well-known integral representing the transformation of available potential energy into kinetic energy and thus measuring the intensity of the atmospheric energy cycle. The second term on the right-hand side represents the effect of vertical distribution of diabatic heating on the gross static stability. Because the conversion process tends to increase the gross static stability $\left(-\int_M \alpha\omega\, dm > 0\right)$, this second term must on the average be negative and of the same magnitude as the first term. Thus, a measure for the intensity of the atmospheric energy cycle, which we denote by W, is given by

$$W = -\int_M \left[(1+\varkappa)^{-1}\left(\frac{p_0}{p}\right)^{\varkappa} - 1\right] \tilde{\bar{Q}}\, dm\,, \tag{5}$$

where a bar is used to denote an average over a long period of time.

For quantitative evaluation of this term we first notice that the diabatic heating Q can be written as

$$Q = Q_R + Q_C + Q_T + \delta\,,$$

where Q_R, Q_C and Q_T denote, respectively, the heating due to radiation, condensation processes and the turbulent transfer of sensible heat and δ stands for the heating due to dissipation. Furthermore, for the total energy of the atmosphere not to change, we must have

$$\int_M (\tilde{\bar{Q}}_R + \tilde{\bar{Q}}_L + \tilde{\bar{Q}}_T)\, dm = 0\,. \tag{6}$$

With this requirement, the contribution to the expression (5) by the net diabatic heating $\int_M Q dm \left(= \int_M \delta\, dm\right)$ is found to be negligibly small due to the smallness of the vertical average of the weighting factor $(1+\varkappa)^{-1}\left(\frac{p_0}{p}\right)^{\varkappa} - 1$. On the other hand, at each level in the atmosphere δ is two or three orders of magnitude smaller than the »external» heating $(Q_R + Q_C + Q_T)$. Accordingly we can write to a good approximation

$$W = -\int_M \left[(1+\varkappa)^{-1}\left(\frac{p_0}{p}\right)^{\varkappa} - 1\right] (\tilde{\bar{Q}}_R + \tilde{\bar{Q}}_C + \tilde{\bar{Q}}_T)\, dm \tag{7}$$

Because the weighting factor $(1 + \varkappa)^{-1}\left(\frac{p_0}{p}\right)^{\varkappa} - 1$ is negative in high pressures and positive at lower pressures it follows from (6) and (7) that $(\tilde{\bar{Q}}_R + \tilde{\bar{Q}}_C + \tilde{\bar{Q}}_T)$ must be predominantly positive in low troposphere and negative higher up.

In Fig. 1 the vertical profile of Q_R for the annual mean conditions over the Northern Hemisphere is shown as obtained from the data given by London [1]. There are no data available on the vertical distribution of Q_C and Q_T. Qualitatively, the main part of the release of latent heat due to condensation takes place below 500 mb level. It is also likely that the turbulent transfer of sensible heat primarily heats the lower troposphere. These facts have been taken into account in constructing the hypothetical vertical profile of the combined effect of Q_C and Q_L in Fig. 1: $\tilde{\bar{Q}}_C + \tilde{\bar{Q}}_T$ is assumed to be constant from the surface up to 500 mb and from there on to decrease linearly to zero at 100 mb level in such a way that (6) is fulfilled. Accordingly, about thirty per cent of the total heating due to condensation and turbulence is assumed to occur above 500 mb level. The resulting curve for the net diabatic effect $\tilde{\bar{Q}}_R + \tilde{\bar{Q}}_C + \tilde{\bar{Q}}_T$ shows heating below 800 mb and cooling between 800 mb and 100 mb levels. Above 100 mb level thermal equilibrium has been assumed.

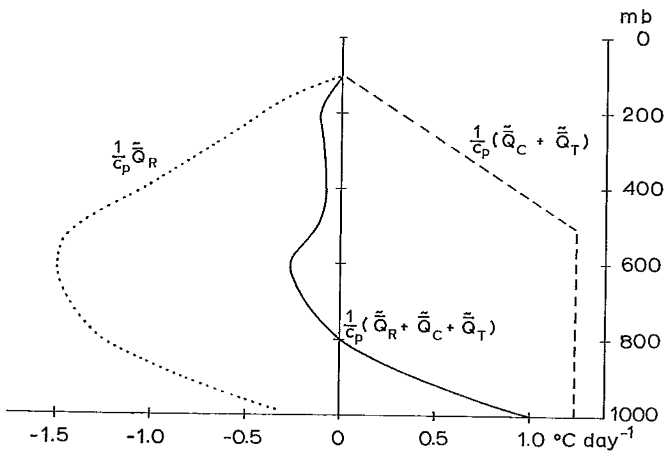

Fig. 1. The vertical distribution of the diabatic heating used in evaluation of W (see Eq. (7)). $1/c_p\, Q_R$ is taken from London [1].

With the $(Q_R + Q_C + Q_T)$-profile from Fig. 1 the value of the expression (7) for the Northern Hemisphere was evaluated to be 50×10^{10} kj sec^{-1} or 2 watts m^{-2}. This is the same order of magnitude as obtained in other independent ways for the annual mean intensity of the atmospheric energy cycle (for review, see *e.g.* OORT, [3]). Much emphasis cannot be put on the numerical value obtained due to the hypothetical nature of the $(Q_C + Q_L)$-profile adopted and because the expression (7) for W is rather sensitive to small variation of the heating profile. The main purpose of the numerical example is just to illustrate the new concept. The point to be emphasized is that whenever the vertical variation of diabatic heating is studied on a global or hemispheric scale, the result must be consistent with the energy transformations in the atmosphere.

REFERENCES

1. LONDON, J., 1957: »A study of the atmospheric heat balance.» *Final Report on Contract AF19 (122)-165 (AFCRC-TR-57-287)*, New York University, Department of Meteorology and Oceanography.
2. LORENZ, E. N., 1960: »Energy and numerical weather prediction.» *Tellus*, **12**, 364—373.
3. OORT, A. H., 1964: »On estimates of atmospheric energy cycle.» *Mon. Weath. Rev.*, **92**, 483—493.
4. PALMÉN, E., 1966: »On the mechanism of the vertical heat flux and generation of kinetic energy in the atmosphere.» *Tellus*, **18**, 1—8.