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PRESSURE AND MOLECULAR SCALE TEMPERATURE IN THE UPPER ATMOSPHERE

by

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Several authors have extrapolated air pressures, densities, and temperatures for altitudes above 200 km, using certain approximations [1—8]. These calculations are based on measurements made below that level with the aid of rocket-carried instruments. Various physical methods allow an estimation of the temperature distribution in the upper atmosphere [2, pp. 550—558]. The estimated values vary widely.

Consistent and reliable average values for the air density at heights between 200 km and 1000 km have been deduced from the motions of artificial satellites [9—12]. The density and, accordingly, the corresponding pressure at 1000 km is so low that an integration of the hydrostatic formula through the satellite region will lead to pressure values which rapidly gain in reliability as the lower limit of integration becomes smaller. The pressure values can be checked against values obtained with rockets. According to Mitra [2, pp. 549—550], air of the very low density in question can be treated as a perfect gas and the hydrostatic formula is valid.

Hence, for the average pressure at height h we have the expression

$$p_h = \gamma\mu \int_h^\infty \frac{\varrho dh}{(r+h)^2} + p_\infty \tag{1}$$

where ϱ is the density, r the Earth's radius, γ the gravitational constant and μ the mass of the Earth. For the present purpose, instead of the upper limit ∞ we can take some value close to 1000 km, 1300 km say (*cf.* fig. 1).

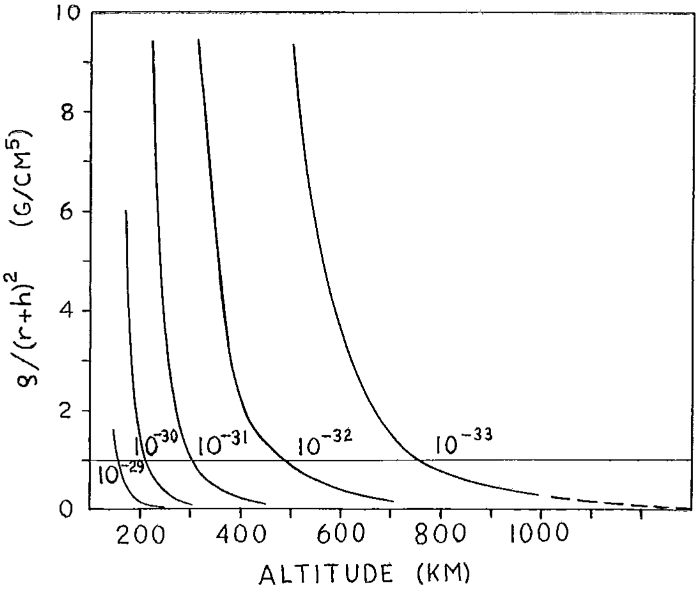

Fig. 1. Graph for the evaluation of the integral in formula (1).

The adopted values of $\log_{10}\varrho$ and the graphically evaluated values of $\log_{10}p$ are compiled in table 1. The integrand in formula (1) is illustrated in fig. 1. From this, a value was deduced for p_{1000} (p for the height 1000 km). Fig. 2 shows an excellent agreement with pressure data actually measured by rockets [13]. On the other hand, a divergence exists between

Table 1.

h (km)	$-\log_{10}\varrho$ (g/cm^3)	$10^{27}\cdot\int$ (g/cm^4)	$-\log_{10}p$ (dyne/cm^2)	T/M (°K/mol. weight)
1000	15.8	5	5.7	150
900	15.6	9	5.4	170
800	15.4	15	5.2	180
700	15.1	26	5.0	160
600	14.75	50	4.7	135
500	14.4	110	4.36	132
400	14.0	252	4.00	120
300	13.3	857	3.46	83
200	12.2	6140	2.61	46.9
150	11.1	36300	1.84	22.1

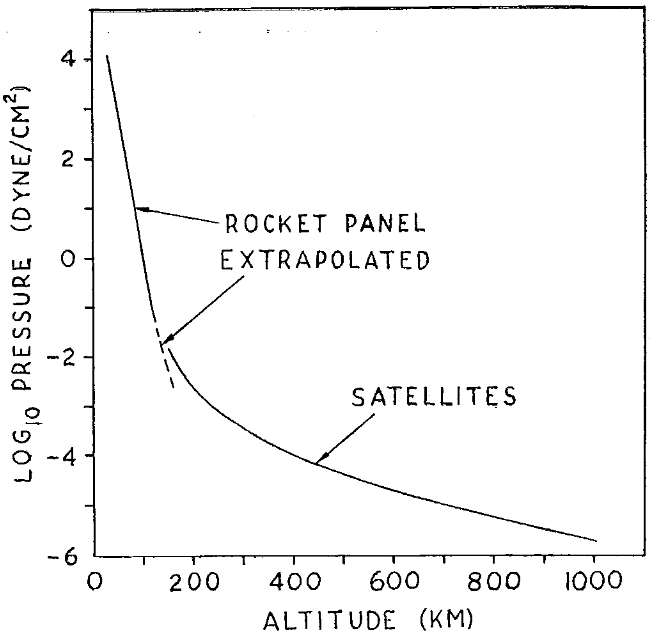

Fig. 2. The average pressure as a function of altitude, obtained with rockets and satellites.

the values obtained from satellite observations and those extrapolated from rocket observations [13].

The table also includes values of T/M computed according to the formula

$$\left(\frac{T}{M}\right)_h = \frac{p_h}{\varrho_h R} = \frac{\gamma\mu}{\varrho_h R}\int_h^\infty \frac{\varrho dh}{(r+h)^2}, ^* \tag{2}$$

where R is the gas constant. In order to calculate T, M was taken to be 16, which involves approximate assumptions concerning the prevalence

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[*] The exact formula for T/M is

$$\frac{T}{M} = e^{-\int\frac{\varrho'}{\varrho}dh}\left\{\text{const.} - \frac{\gamma\mu}{R}\int e^{\int\frac{\varrho'}{\varrho}dh}\cdot\frac{dh}{(r+h)^2}\right\}, \tag{3}$$

which gives

$$\left(\frac{T}{M}\right)_h = \frac{\varrho_\infty}{\varrho_h}\left\{\left(\frac{T}{M}\right)_\infty + \frac{\gamma\mu}{R\varrho_\infty}\int_h^\infty \frac{\varrho dh}{(r+h)^2}\right\}. \tag{4}$$

The first term is small in comparison with the second. Consequently formula (2) gives a good approximation.

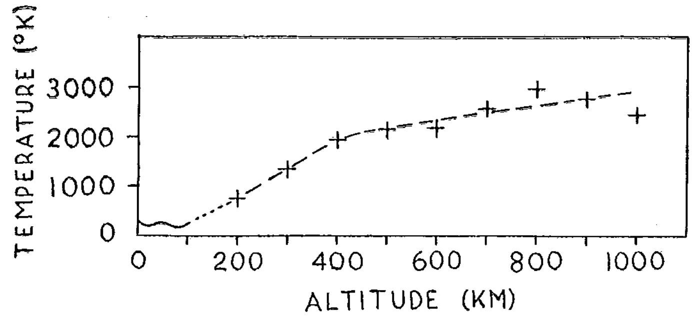

Fig. 3. Estimated average temperature as a function of altitude.

of O-atoms throughout the whole region in question. The temperatures obtained are illustrated in fig. 3, together with previously measured temperatures of the atmosphere at altitudes 0—100 km [13].

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