551.521.1
551.508.11

Solar Radiation Intensity at the Ascending Radiosonde

By

VILHO VÄISÄLÄ

In our study concerning the radiation error of the radiosonde [1] we assumed that the radiation intensity in the stratosphere is constant because of the small air density. This assumption, of course, is not exact, especially when the Sun is near the horizon or below it. The purpose of this study is to form a basis for further examining the radiation error taking into consideration the depletion of the radiation intensity particularly at small height angles of the Sun. For this purpose we compute the air mass, which the beam has penetrated before reaching the radiosonde, and represent it as isopleths using the Sun's height angle as abscissa and the air pressure as ordinate. Since in radiosonde practice the Sun's height is not observed but calculated, it is advisable to do the plotting against the true height angle of the Sun, consequently without refraction.

Constitution of the Atmosphere

For the purpose of determining the radiation error in temperature soundings it is sufficient to consider a simplified atmosphere. We assume a polytropic troposphere from 1,000 to 200 mb and an isothermal stratosphere from 200 mb upwards. Let

r = distance from earth centre,
p, T, ϱ = pressure, temperature, density of air,
γ = lapse rate of temperature,

(1) $H = \frac{RT}{g}$ = height of the homogeneous atmosphere,

$R = 287$

(2) $\varkappa = \frac{g_0}{\gamma R} = \frac{T_0}{\gamma H_0}$

(3) $\Delta r = r - r_0, \quad d(\Delta r) = dr$

(4) $x = \frac{\Delta r}{H_0}.$

The index 0 refers to the values at pressure p_0. Then in a polytropic atmosphere:

(5) $$T = T_0 - \gamma \Delta r$$

(6) $$p = R\varrho T, \quad p_0 = R\varrho_0 T_0$$

(7) $$\frac{T}{T_0} = 1 - \frac{x}{\varkappa}$$

(8) $$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{\varkappa}$$

(9) $$\frac{\varrho}{\varrho_0} = \left(\frac{T}{T_0}\right)^{\varkappa - 1}.$$

We consider the gravity as constant $g = g_0 = 9.8$ m sek^{-2}. Index 2 refers to the values at 200 mb and index 1 to values at 1,000 mb. We assume that

(10) $$r_2 = 6{,}380{,}000 \text{ m}, \quad g_2 = 9.8 \text{ m sec}^{-2}.$$

It is very convenient to determine T_2 so that H_2 becomes $= 6{,}380$ m. Then

(11) $$T_2 = 217^\circ.8 \text{ and } \frac{r_2}{H_2} = 1{,}000.$$

Supposing

$$\varkappa = 6$$

we get from eq. (8, 2) for 1,000 mb

(12) $$T_1 = 284^\circ.8, \quad \gamma = 5.69 \text{ C}^\circ \text{ km}^{-1}.$$

Table 1 contains some values of the troposphere computed with these constants from eq. (5—9). It also shows $\Delta n = n - 1$ in which $n =$ the index of refraction of air, as well as

$$\varepsilon = \frac{H}{r} \quad \text{and} \quad k = \frac{\Delta n}{\varepsilon}; \tag{13}$$

Table 1. The Polytropic Troposphere

p mb	Height Δr m	T °K	ϱ kg m^{-3}	H m	$10^3 \Delta n$	$10^3 \varepsilon$	k
200	11,779	217.8	0.320	6,380	0.0724	1.000	0.0724
250	10,320	226.1	0.384	6,620	0.0868	1.040	0.0835
300	9,106	233.0	0.448	6,820	0.1013	1.071	0.0946
400	7,089	244.5	0.569	7,160	0.1287	1.124	0.1143
500	5,467	253.7	0.687	7,430	0.1553	1.167	0.133
700	2,888	268.4	0.907	7,860	0.2050	1.234	0.166
1,000	0	284.8	1.222	8,350	0.2763	1.309	0.211

As mentioned, we assume that the stratosphere is isothermal with the temperature $T = T_2 = 217.8°$ K.

Table 2. The Isothermal Stratosphere

p mb	Δr m	ϱ kg m^{-3}	
10	30,890	0.016	$T = 217.8$
20	26,470	0.032	$H = 6380$
50	20,620	0.080	$\varepsilon = 0.001$
100	16,200	0.160	$g = 9.8$
150	13,610	0.240	
200	11,779	0.320	

As to the moisture content of the atmosphere, we assume a constant relative humidity of 50 % in the troposphere and dry air in the stratosphere.

Refraction, Air Mass and Height Angle of the Sun Beam

The air mass, penetrated by the Sun beam, is the determining factor for the depletion of the radiation intensity. When computing this mass, the refraction, particularly at small and negative height angles, must be taken into consideration. The influence of the beam path curvature is relatively small. E.g. at 200 mb the mass of the horizontal beam relative to the vertical one is 40.8 for the curved beam, and 39.6 for the straight one.

The practical formulas for the computation of the refraction and air mass of the beam are derived from the fundamental formulas [2]:

$$\Delta h = \int \cot h \, \frac{dn}{n} \tag{14}$$

$$M = \int \varrho ds = \int \frac{\varrho dr}{\sin h} \tag{15}$$

$$nr \cos h = n_0 \, r_0 \cos h_0 \tag{16}$$

where

h = height angle of the beam curve = apparent Sun height,
n = the index of refraction of air,
ds = length differential of the beam curve,
Δh = refraction,
M = air mass.

When the integrals between two points of the beam curve are computed, we get the respective values of the refraction and mass. The index of refraction is

$$n = 1 + \alpha \frac{\varrho}{\varrho_1}. \tag{17}$$

Proceeding from the value of Bessel

$$\alpha_B = 0.00029243 \text{ (0° C, 760 mm Hg)}$$

we get

$$\alpha = 0.0002886 \text{ (0° C, 1,000 mb)}. \tag{18}$$

According to (17) we calculate

$$(19)\qquad \nu = \frac{\alpha}{\varrho_1} = \frac{n - 1}{\varrho} = \frac{\Delta n}{\varrho} = 0.0002261,$$

hence

$$(20)\qquad \Delta n = \nu\varrho.$$

The height angle h can be computed from (16). Without taking into consideration relatively small quantities, we get

$$(21)\qquad \begin{aligned} F(x) = \sin^2 h &= \sin^2 h_0 + 2\,\varepsilon_0\, x - 2\,\nu\,(\varrho_0 - \varrho) \\ &= \sin^2 h_0 + 2\,\varepsilon_0\, x - 2\,\Delta n_0\,(1 - y), \end{aligned}$$

where

$$(22)\qquad y = \frac{\varrho}{\varrho_0}$$

Then equations (14—16) give us the values

$$(23)\qquad \Delta h = \nu\,\varrho_0 \cos H_0 \int_x^0 \frac{\frac{dy}{dx}}{\sqrt{F(x)}}\,dx,$$

$$(24)\qquad M = \varrho_0\, h_0 \int_0^x \frac{y}{\sqrt{F(x)}}\,dx$$

between the level p_0 where $x = 0$, and the height $\Delta r = x\,H_0$ above the level p_0. The formulas (23, 24) are valid for all atmospheres. Special formulas for our atmosphere may be derived from them. Considering the equations (5—9) of the p o l y t o p i c t r o p o s p h e r e and writing

$$(25)\qquad k = \frac{\Delta n_0}{\varepsilon_0}$$

$$(26)\qquad E_6(x) = \frac{1 - (1 - \frac{x}{6})^5}{x},$$

$$(26^*)\qquad u_6(x) = \left[\frac{\sin^2 h_0}{2\,\varepsilon_0\,x} + (1 - k\,E_6)\right]^{-\frac{1}{2}}$$

$$(27)\qquad v_1(x) = \int_0^x x^{-\frac{1}{2}}(1 - \tfrac{x}{6})^5 dx$$

$$(27^*)\qquad v_2(x) = \int_0^x x^{-\frac{1}{2}}(1 - \tfrac{x}{6})^4\, dx$$

we get from (21—24):

$$(28)\qquad \Delta\, h = \tfrac{5}{6}\,\nu\,\varrho_0 \cos h_0 \sqrt{\frac{1}{2\,\varepsilon_0}} \int_0^x u_6\, dv_2,$$

$$(29)\qquad \frac{M}{\varrho_0 H_0} = \sqrt{\frac{1}{2\,\varepsilon_0}} \int_0^x u_6\, dv_1.$$

If the beam is horizontal at the pressure p_0, then $h_0 = 0$ and

$$(26^*\ \text{a})\qquad u_6(x) = (1 - k\,E_6)^{-\frac{1}{2}}.$$

In the isothermal stratosphere

$$(30)\qquad y = e^{-x}, \text{ with } x = \frac{\Delta r}{H_0}$$

H_0 being constant. In this case we get from (21—24)

$$(31)\qquad \frac{\Delta h}{\nu\,\varrho_0 \cos h_0} = \frac{M}{\varrho_0\, H_0} = \sqrt{\frac{1}{2\,\varepsilon_0}} \int_0^\infty u\, dv$$

with

$$(32)\qquad E(x) = \frac{1 - e^{-x}}{x},$$

$$(32^*)\qquad u(x) = \left[\frac{\sin^2 h_0}{2\,\varepsilon_0\,x} + (1 - k\,E)\right]^{-\frac{1}{2}},$$

$$(33)\qquad v(x) = \int_0^x x^{-\frac{1}{2}}\, e^{-x}\, dx = \sqrt{\pi}\,\Phi(\sqrt{x}),$$

where Φ is the Gauss error function. The formulas (31) contain the Laplace's extinction law for an isothermal atmosphere: $H_0 \, \Delta h = \nu \, M \cos h_0$.

The integrals in (28, 29, 31) are carried out mechanically. Especially for horizontal beams, u_6 and u vary slowly only.

The mass M is expressed in absolute units. We shall, however, take the vertical mass above 1,000 mb level as unit mass. This unit mass is

$$M_1 = \varrho_1 \, H_1 = 10{,}200 \text{ kg m}^{-2}. \tag{34}$$

The mass relative to M_1 becomes

$$m = \frac{M}{M_1} = \frac{g_1}{g_0} \frac{p_0}{p_1} \frac{M}{\varrho_0 H_0} \tag{35}$$

because of $p = g \, \varrho \, H$. Assuming $g_1 = g_0$, it follows

$$m = \frac{p_0}{1000} \frac{M}{\varrho_0 H_0}. \tag{36}$$

Table 3 B. Troposphere

mb	S (1,000)				S (700)				S (500)			
	$\odot h$	Δh	m	W	$\odot h$	Δh	m	W	$\odot h$	Δh	m	W
				cm				cm				cm
250	2°.98	0°.08	3.76	0.07	2°.54	0°.08	4.17	0.08	2°.04	0°.09	4.83	0.11
300	2.78	0.09	4.68	0.23	2.29	0.10	5.33	0.28	1.72	0.12	6.26	0.35
400	2.40	0.12	6.89	1.02	1.81	0.14	7.97	1.24	1.04	0.18	10.0	1.70
500	2.04	0.16	9.36	2.88	1.34	0.19	11.2	3.64				
700	1.33	0.25	15.4	12.4								
1000	−0.56	0.56	38.5	83.4								
700	−2.45	0.87	61.6	154	−0.42	0.42	27.1	26.6				
500	−3.16	0.96	67.6	164	−2.17	0.64	43.0	49.6	−0.32	0.32	19.6	7.68
400	−3.51	0.99	70.1	166	−2.64	0.69	46.2	52.0	−1.69	0.47	29.2	13.7
300	−3.90	1.03	72.3	167	−3.12	0.73	48.8	52.9	−2.37	0.53	33.0	15.0
250	−4.10	1.04	73.2	167	−3.37	0.75	50.0	53.1	−2.68	0.55	34.4	15.3
200	−4.32	1.06	74.0	167	−3.65	0.78	51.0	53.2	−3.00	0.57	35.7	15.4
	S (400)				S (300)				S (250)			
250	1.64	0.11	5.34	0.13	0.94	0.13	6.69	0.18				
300	1.24	0.14	7.14	0.43								
400	−0.27	0.27	15.9	3.32								
300	−1.79	0.41	24.6	6.21	−0.22	0.22	12.1	1.02				
250	−2.19	0.44	26.4	6.51	−1.38	0.31	17.4	1.86	−0.19	0.19	10.1	0.42
200	−2.57	0.46	27.9	6.64	−1.94	0.35	19.6	2.04	−1.46	0.27	15.1	0.84

Table 3. Sun's True Height Angle (⊙h), Refraction (Δh), Relative Air Mass (m) and Water Mass (W) on some Solar Beams (S) at Different Pressures (mb). — Table 3 A. Stratosphere

mb	S (1000)			S (700)			S (500)			S (400)			S (300)			S (250)			Water mass
	⊙h	Δh	m	⊙h	Δh	m	⊙h	Δh	m	⊙h	Δh	m	⊙h	Δh	m	⊙h	Δh	m	W cm
10	5°.45	0.00	0.10	5°.21	0.00	0.10	4°.99	0.00	0.10	4°.86	0.00	0.10	4°.66	0.00	0.11	4°.52	0.00	0.11	
20	5.02	0.00	0.20	4.77	0.01	0.21	4.51	0.01	0.22	4.35	0.01	0.23	4.12	0.01	0.24	4.00	0.01	0.24	
50	4.38	0.01	0.56	4.09	0.01	0.60	3.82	0.01	0.63	3.62	0.01	0.66	3.35	0.01	0.70	3.18	0.02	0.73	None
100	3.84	0.03	1.25	3.51	0.03	1.34	3.17	0.03	1.46	2.94	0.03	1.54	2.59	0.03	1.68	2.36	0.04	1.73	
150	3.46	0.04	2.04	3.13	0.04	2.20	2.72	0.05	2.43	2.45	0.05	2.61	2.01	0.06	2.92	1.71	0.07	3.18	
200	3.20	0.06	2.88	2.81	0.06	3.16	2.36	0.07	3.56	2.03	0.08	3.80	1.50	0.09	4.50	1.08	0.11	5.10	
10	—6.57	1.12	76.8	—6.05	0.84	54.1	—5.63	0.64	39.2	—5.40	0.54	31.6	—5.10	0.44	24.0	—4.90	0.38	20.1	The same
20	—6.14	1.12	76.7	—5.61	0.83	54.0	—5.15	0.63	39.1	—4.89	0.53	31.5	—4.56	0.43	23.9	—4.38	0.37	20.0	as at
50	—5.50	1.11	76.4	—4.93	0.83	53.6	—4.46	0.63	38.6	—4.16	0.53	31.1	—3.79	0.43	23.4	—3.56	0.36	19.5	200 mb
100	—4.96	1.09	75.7	—4.35	0.81	52.8	—3.81	0.61	37.8	—3.48	0.51	30.2	—3.03	0.41	22.4	—2.74	0.34	18.5	in
150	—4.58	1.08	74.9	—3.97	0.80	52.0	—3.36	0.59	36.8	—2.99	0.49	29.1	—2.45	0.38	21.2	—2.09	0.31	17.0	Tab. 3 B
200	—4.32	1.06	74.0	—3.65	0.78	51.0	—3.00	0.57	35.7	—2.57	0.46	27.9	—1.94	0.35	19.6	—1.46	0.27	15.1	

mb	S (200)			S (150)			S (100)			S (50)			S (20)			S (10)			
10	4.37	0.00	0.11	4.17	0.00.	0.12	3.81	0.00	0.13	3.23	0.00	0.14	2.13	0.00	0.19				None,
20	3.82	0.01	0.25	3.59	0.01	0.27	3.18	0.01	0.29	2.43	0.01	0.35							because
50	2.93	0.02	0.77	2.62	0.02	0.83	2.08	0.02	0.97										the
100	2.03	0.04	1.95	1.55	0.05	2.25													Strato-
150	1.25	0.08	3.66																sphere
10	—4.71	0.34	16.2	—4.42	0.25	12.1	—3.98	0.17	7.93	—3.31	0.08	3.86	—2.16	0.03	1.41	—0.01	0.01	0.40	is
20	—4.16	0.33	16.1	—3.84	0.24	11.9	—3.35	0.16	7.77	—2.51	0.07	3.65	—0.02	0.02	0.80				supposed
50	—3.27	0.32	15.6	—2.87	0.23	11.3	—2.25	0.15	7.09	—0.04	0.04	2.00							dry.
010	—2.37	0.30	14.4	—1.80	0.20	9.93	—0.08	0.08	4.03										
150	—1.59	0.26	12.7	—0.13	0.13	6.09													
200	—0.17	0.17	8.16																

Tables 3 contain data for some solar beams $S(\bar{p})$ which pass through the atmosphere and are horizontal at a given pressure $\bar{p}$. E.g. the beam $S(1000)$, after penetrating the troposphere, reaches the 10 mb level the true height angle of the Sun being — 6°.57, refraction 1°.12, apparent height — 5°.45 and air mass = 76.8.

Table 4. $\frac{M}{\varrho_0 H_0}$ *at Different Pressure Levels (mb) and Apparent Height Angles (h) in the Isothermal Stratosphere*

mb \ h	0°	1°	2°	3°	5°	10°	15°	20°	30°	60°	90°
10	39.7	26.8	19.6	15.3							
20	39.8	26.9	19.7	15.3							
50	40.0	27.1	19.7	15.3	10.3	5.59	3.81	2.89	2.00	1.15	1
100	40.3	27.2	19.8	15.3							
150	40.6	27.4	19.8	15.3							
200	40.8	27.5	19.9	15,3							

Table 5. Relative Mass (m), Refraction (Δh) and True Height Angle (⊙h) at Different Apparent Heights (h) in the Stratosphere which is supposed dry (W = 0).

mb	h	0°	1°	2°	3°	5°	10°	15°	20°	30°	60°	90°
10	*m*	0.40	0.27	0.20	0.15	0.10	0.06	0.04	0.03	0.02	0.01	0.01
	Δh	0°.01	0.00	0	0	0	0	0	0	0	0	0
	⊙h	—0.01	1	2	3	5	10	15	20	30	60	90
20	*m*	0.80	0.54	0.39	0.31	0.21	0.11	0.08	0.06	0.04	0.02	0.02
	Δh	0.02	0.01	0.01	0.01	0	0	0	0	0	0	0
	⊙h	—0.02	0.99	1.99	2.99	5	10	15	20	30	60	90
50	*m*	2.00	1.36	0.98	0.76	0.52	0.28	0.19	0.14	0.10	0.06	0.05
	Δh	0.04	0.03	0.02	0.02	0.01	0	0	0	0	0	0
	⊙h	—0.04	0.97	1.98	2.98	4.99	10	15	20	30	60	90
100	*m*	4.03	2.72	1.98	1.53	1.03	0.56	0.38	0.29	0.20	0.12	0.10
	Δh	0.08	0.06	0.04	0.03	0.02	0.01	0.01	0	0	0	0
	⊙h	—0.08	0.94	1.96	2.97	4.98	9.99	14.99	20	30	60	90
150	*m*	6.09	4.11	2.97	2.30	1.54	0.84	0.57	0.43	0.30	0.17	0.15
	Δh	0.13	0.08	0.06	0.05	0.03	0.02	0.01	0.01	0	0	0
	⊙h	—0.13	0.92	1.94	2.95	4.97	9.98	14.99	19.99	30	60	90
200	*m*	8.16	5.50	3.98	3.06	2.06	1.12	0.76	0.58	0.40	0.23	0.20
	Δh	0.17	0.11	0.08	0.06	0.04	0.02	0.02	0.01	0.01	0	0
	⊙h	—0.17	0.89	1.92	2.94	4.96	9.98	14.98	19.99	29.99	60	90

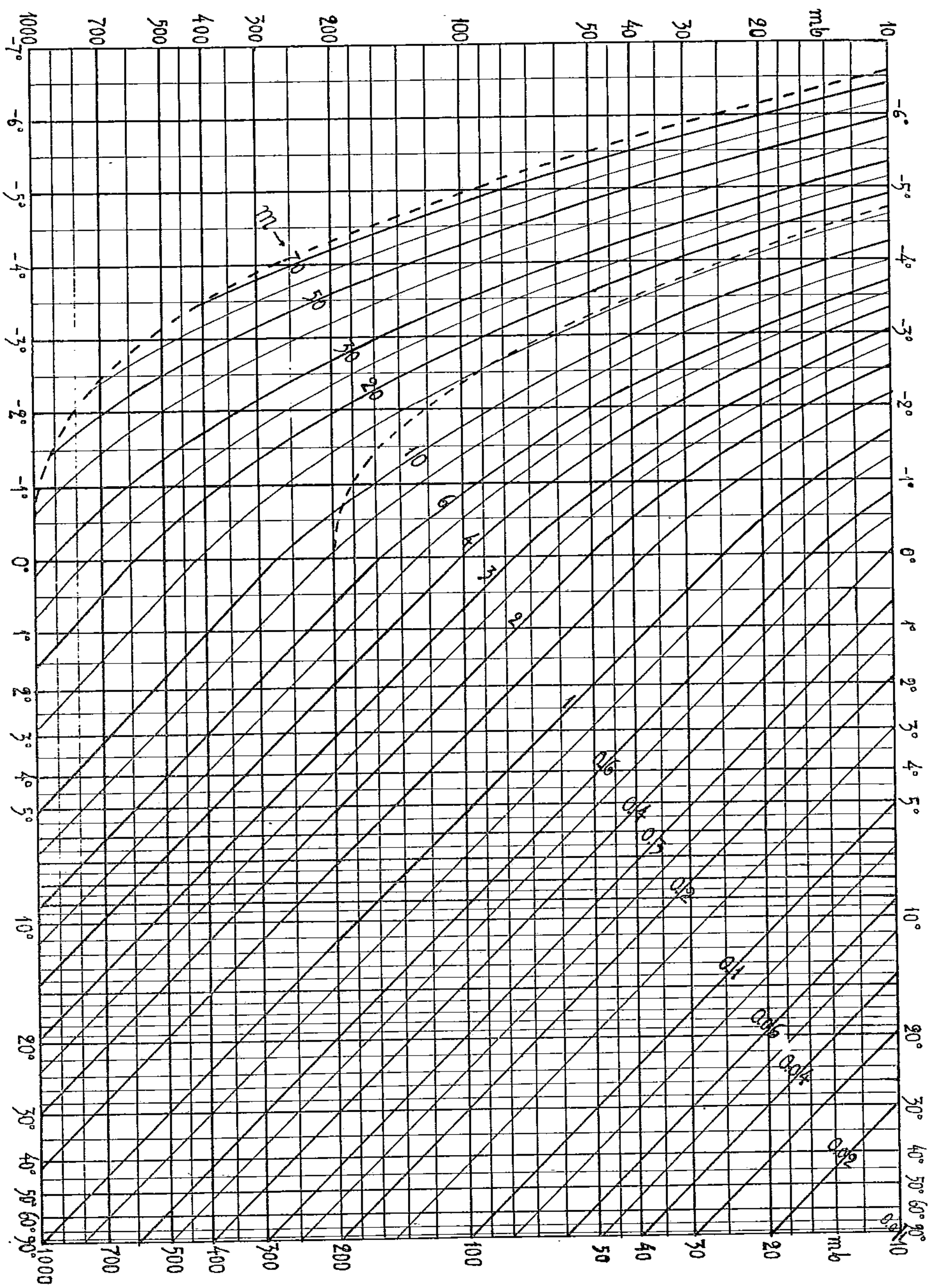

Fig. 1. Radiation Depleting Air Mass (Sloping Lines).
Abscissa: Sun's True Height Angle. Ordinate: Air pressure (mb).

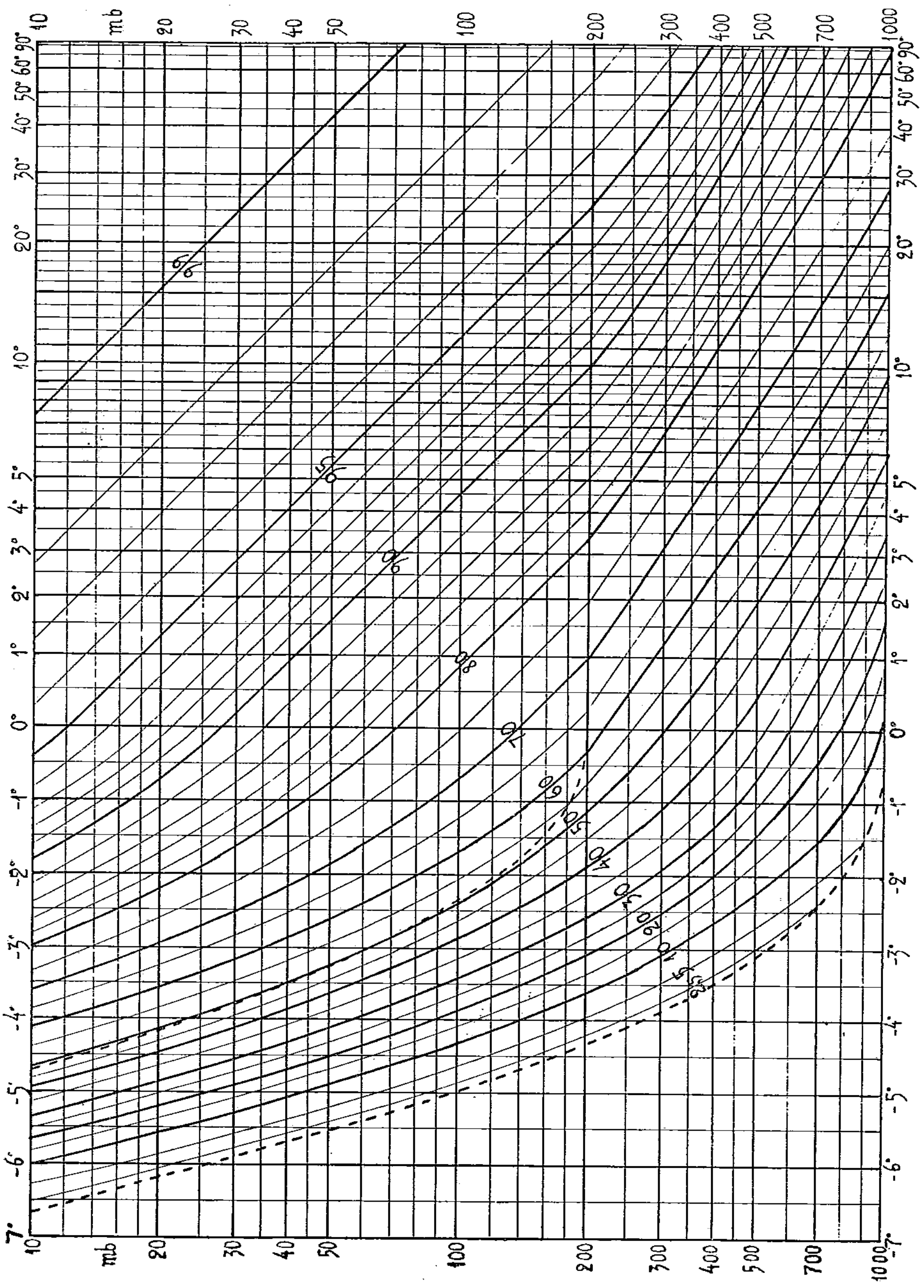

Fig. 2. Relative Radiation Intensity (in Percents) at the Ascending Radiosonde. Abscissa: Sun's True Height Angle. Ordinate: Air Pressure (mb).

Table 4 contains the relative mass $\frac{M}{\varrho_0 H_0}$. It shows that the beam curvature slightly influences the mass at the smallest height angles (0°—2°). In Table 5 we have the rel. masses, refractions and true height angles corresponding to table 4. Table 6 represents some beams in the troposphere reaching 1,000 mb level at different height angles. E.g. the beam which reaches 1,000 mb at 2° app. height passes through the tropopause (200 mb) with 3°.90 true solar height. The rel. mass of the beam at 500 mb is 7.67.

By means of these tables we plotted the rel. mass (m) as isopleths, shown in Fig. 1. Therein the pressure, in logarithmic scale, is used as ordinate. The true height angle of the Sun is taken as abscissa in a scale determined as follows: between 0° and 90° $x = m_0(\odot h)$ the air mass in isothermal stratosphere for straight beams, for negative height angles x is linear with $\odot h$.

Table 6. True Height Angles ($\odot h$), Masses (m) and Water Amounts (W) at Different Pressure Levels of Solar Beams which Reach the 1,000 mb Level at 1°, 2°, 5°, 10° Apparent Height (h)

h	1°			2°			5°			10°		
mb	$\odot h$	m	W	$\odot h$	m	W	$\odot h$	m	W	$\odot h$	m	W
			cm			cm			cm			cm
200	3°.50	2.68	0	3°.90	2.47	0	6.03	1.77	0	10.60	1.03	0
250	3.16	3.51	0.07	3.60	3.22	0.06	5.83	2.24	0.04	10.44	1.31	0.02
300	2.95	4.44	0.21	3.42	4.05	0.19	5.71	2.75	0.11	10.38	1.58	0.07
400	2.58	6.45	0.96	3.13	5.78	0.83	5.53	3.77	0.49	10.29	2.13	0.27
500	2.27	8.73	2.66	2.84	7.67	2.26	5.36	4.85	1.29	10.20	2.70	0.68
700	1.64	14.06	11.1	2.36	11.77	8.86	5.11	6.90	4.71	10.08	3.82	2.47
1,000	0.61	26.23	51.7	1.70	19.39	35.7	4.84	10.28	16.8	9.91	5.56	8.48

From fig. 1 we can read the rel. mass at each pressure from 1,000 mb (ground) to 10 mb (about 30 km) and at all true height angles of the Sun. In the figure the beams $S(1000)$ and $S(200)$ are represented by dash curves.

Mass of Water-vapor along the Solar Beam

Concerning the water vapor we assumed a relative humidity of 50 % throughout the troposphere and dry air in the stratosphere. Parallel to (15) we can write the water mass

$$M_w = \int \frac{\varrho_w \, dr}{\sin h} \tag{37}$$

where ϱ_w = density of water vapor and M_w = the water mass between the limits of the integral. As liquid water, W cm, we have

$$W = \frac{1}{10} M_w. \tag{38}$$

Table 7 shows the liquid water amounts along the horizontal and the vertical beams as well as the resp. air masses. The formulas (37) and (15) show that for greater height angles W and m are proportional because h can be considered as constant along the beam. If the mixing ratio of water vapor, $\varrho_w : \varrho$, is constant, the mentioned proportionality is valid at all values of h. In table 7 this is not the case as we can see from the values of W' horiz. $= \frac{m \text{ horiz.}}{m \text{ vert.}} w$, where $w = W$ vert. is the »precipitable» water. Hence, the absorbing water amount W along the beam can be computed by multiplying the »precipitable» water (w) with the relative air mass if the height angle is great enough. However, if the beam is nearly horizontal this is not possible unless e.g. the mixing ratio is constant. It is to be noticed that the mixing ratio can not be constant through all the troposphere because of the great decrease of the temperature. In the stratosphere, on the contrary, the mixing ratio can very well be constant but not the relative humidity because in such a case the water vapor would in great heights be the prevailing gas.

Tables 3 and 6 also contain the water masses on the different solar beams in our special atmosphere.

Table 7. Water Mass (W cm) along the Horizontal and Vertical Beams between 200 mb and the Pressure $\bar{p}$ as well as the resp. air masses (m) in our special atmosphere. $W'_{\text{horiz.}} = w \frac{m \text{ horiz.}}{m \text{ vert.}}$.

$\bar{p}$ mb	W cm		m		$W'_{\text{horiz.}}$	$\frac{W_{\text{horiz.}}}{W'_{\text{horiz.}}}$
	horiz.	vert. w	horiz.	vert.		
250	0.42	0.004	5.02	0.05	0.40	1.05
300	1.01	0.012	7.56	0.1	0.91	1.11
400	3.32	0.046	12.0	0.2	2.76	1.20
500	7.71	0.116	16.1	0.3	6.62	1.24
700	25.6	0.413	23.9	0.5	19.8	1.29
1,000	83.4	1.39	35.6	0.8	62.0	1.34

Table 8. Relative Intensity of Solar Radiation after passing through the mass m in dry, pure atmosphere according to Kastrow.

m	$I : I_0$	m	$I : I_0$
0.01	0.998	6	0.677
0.03	0.995	8	0.625
0.1	0.988	10	0.582
0.2	0.976	15	0.504
0.5	0.948	20	0.449
1	0.907	30	0.373
1.5	0.872	40	0.320
2	0.840	60	0.246
3	0.788	75	0.206
4	0.745	100	0.156

Extinction of Radiation Intensity by Pure Air

When calculating the depletion of the solar radiation intensity we take into consideration the extinction by the molecules of the permanent gases and water vapor as well as the absorption by water vapor but not the extinction by dust. The extinction by the molecules of dry air is computed from the formula of KASTROW [3]. We transformed it taking in account that our unit air mass is that above 1,000 mb. The selective absorption, called Φ by Kastrow, we have given in the form $0.011\sqrt{m}$ Thus

$$(39)\quad I : I_0 = 1.112\,(1 + 0.2665\,m)^{-0.325} - 0.011\sqrt{m} - 0.112,$$

where I_0 = solar constant, I = the intensity after passing through the mass m. The table 8 contains some values of $I : I_0$. The formula (39) is also used for extrapolating the rel. intensity for very great air masses. This extrapolating cannot be very accurate.

Extinction and Absorption of Radiation Intensity by Water-vapor

The depletion of solar radiation due to extinction and absorption by water vapor is taken into consideration according to KIMBALL [4] (curves in his Fig. 3, p. 176). The slight influence of dry air upon the extinction coefficient in question has been neglected. In this manner we got the following coefficient of extinction due to scattering in water vapor:

(40)
$$\varepsilon_v = 0.024 \text{ for } W \leqq 1 \text{ cm},$$
$$\varepsilon_v = 0.024 - 0.004 \log W, \text{ for } W \geqq 1.$$

This formula was used also for extrapolating the extinction in very great amounts of water mass, needed for horizontal solar rays penetrating the troposphere. Table 9 contains the relative intensity

(41)
$$I : I_0 = q_v^W = e^{-\varepsilon_v W}$$

when only the extinction in water vapor is considered.

Table 9. Relative Radiation Intensity after passing through the Water Mass W cm.
Q_v^W: with regard to the extinction,
$1 - A_v$: with regard to the absorption A_v,
$Q_v = q_v^W (1 - A_v)$ the total transmission.

W cm	q_v^W	$1 - A_v$	Q_v	W cm	q_v^W	$1 - A_v$	Q_v
0.01	1	0.997	0.997	8	0.851	0.878	0.747
0.02	1	0.995	0.995	10	0.818	0.873	0.714
0.05	0.999	0.987	0.986	15	0.749	0.864	0.647
0.1	0.998	0.975	0.973	20	0.687	0.857	0.588
0.2	0.995	0.960	0.955	30	0.581	0.848	0.492
0.5	0.988	0.940	0.928	40	0.494	0.842	0.416
1	0.976	0.924	0.902	60	0.363	0.832	0.302
2	0.955	0.909	0.867	80	0.269	0.826	0.222
3	0.936	0.899	0.841	100	0.202	0.821	0.166
4	0.918	0.893	0.820	150	0.102	0.812	0.083
6	0.882	0.884	0.780	200	0.052	0.806	0.042

Considering the extinction both by dry air and by water vapor we have

(42)
$$I : I_0 = q_a^m \, q_v^W$$

where q_a is the transmission coefficient of dry air.

As to absorption by water vapor, we got the following formulas for the relative amount of the energy absorbed

(43)
$$A_v = 0.25\, W, \quad W \leqq 0.1 \text{ cm},$$
$$A_v = 0.076 + 0.0514 \log W, \quad 0.1 < W.$$

Eq. (43) is also used for extrapolating the absorption in very great water masses. Table 9 contains numerical values of $1 - A_v$.

Combining the extinction and absorption we write the whole transmission by water vapor as

$$(44) \qquad Q_v = q_v^W \, (1 - A_v).$$

Note, that the

$$\left\{\begin{matrix}\text{influence of} \\ \text{extinction is}\end{matrix}\right\} \begin{matrix} \leq \\ \geq \end{matrix} \left\{\begin{matrix}\text{influence of} \\ \text{absorption}\end{matrix}\right\} \text{ for } W \begin{matrix} \leq \\ \geq \end{matrix} 5.84 \text{ cm}.$$

Table 9 also contains numerical values of Q_v.

Taking in account all the three depleting causes we obtain the final relative radiation intensity as follows:

$$(45) \qquad I : I_0 = q_a^m \, Q_v.$$

For the purpose of testing this formula we begged measured radiation intensities at very low Sun heights from Dr. MÖRIKOFER Davos. I have taken two observations he kindly sent and computed the corresponding intensities according to (45):

1. Basel 2.5.1933, $\odot h = 1^\circ.5$, $I_{\text{meas.}} = 0.10$ cal., $p = 973$ mb, $W \sim 56$ cm.
   $I : I_0 = 0.26 \times 0.38 \times 0.835 = 0.082$,
   $I = 1.94 \times 0.082 = 0.16$ cal.
2. Basel 3.9.1932, $\odot h = 3^\circ.1$, $I_{\text{meas.}} = 0.29$ cal., $p = 980$ mb, $W \sim 59$ cm.
   $I : I_0 = 0.52 \times 0.37 \times 0.83 = 0.16$,
   $I = 1.94 \times 0.16 = 0.31$ cal.

As we can see, our computed values are somewhat greater than the ones observed. That is probably due to the dust which we neglected.

For lack of exact measurings at small Sun heights, especially at negative heights, our results are, of course, only approximate ones, but we believe that they are sufficiently exact for the purpose of studying the radiation error of the radiosonde.

According to eq. (45) we calculated the rel. intensity of the solar radiation at different pressures and true Sun heights in our special atmosphere. The result is shown in the isopleth plot fig. 2. The dashed curves represent the Sun beams $S(1000)$ and $S(200)$. The iso-intensity curves are broken at 200 mb because of the discontinuity of water vapor at this pressure.

Helsinki, April 1948.

REFERENCES.

1. Väisälä, V., Der Strahlungsfehler der finnischen Radiosonde (Mitt. Met. Inst. Univ. Helsinki) 1941.
2. Bemporad, A., Besondere Behandlung des Einflusses der Atmosphäre (Refraktion und Extinktion). (Encykl. math. Wiss. VI, 2, 6). 1907.
3. Kastrow, W., Durchlässigkeit der absolut reinen und trockenen Atmosphäre für Sonnenstrahlung (Met. Z. 1928, p. 377).
4. Kimball, H. H. Measurements of Solar Radiation Intensity and Determinations of its Depletion by the Atmosphere (M. W. R. 1927, p. 155).